

## Homework #4-Physics 498Bio

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Review of papers:

+“Emergence of Cooperation and Organization in an Evolutionary Game” by D. Challet and Y.C. Zhang, adap-org/9708006 v2, 03/Sep/1997.

+“Modelling Market Mechanism with Evolutionary Games” by Y.C. Zhang, cond-mat/9803308 v1, 25/Mar/1998.

+“Algorithmic Complexity in Real Financial Markets” by R. Mansilla, lanl archive.

### **INTRODUCTION & BACKGROUND**

There's been a growing trend from the physics community to have more and more involvement in the problems of the economy: more specifically, the behavior of the financial market. However, one can never hope to capture the economic phenomena in physics type laws, as the agents in the market are more or less ignorant of the existence of such laws or theories. Closer observation leads us to a basic set of assumptions as in the following:

- 1). There is a large number of agents in the market.
- 2). Each agent can make some alternative choices.
- 3). The aggregate activity results in a market price.
- 4). Agents can keep history or record, which is publically shared.

The market is supposed to be a self-contained system without any external input. All the behavioral patterns are results of the activities of the agents.

It is the purpose of this review, to explore some existing models that introduce the concept of ‘cooperative behavior’ and ‘measures’ that extracts the information of the market.

### **DIFFERENT METHODS**

One of the simplest models is the so-called “Minority Model”. This is a ‘toy model’ where the basic assumptions of this model coincide with those mentioned in the previous section and those are:

- 1). There are  $N$  agents in the market.
- 2). Each agent can make 2 choices ( A or B => 1 or 0).
- 3). Those in the minority group win. In terms of the market, more buyers than sellers drive the market price higher thus benefiting the sellers that are minority and vice versa.
- 4).  $M$  previous steps of the market can be kept as record.
- 5). Each agent has a number  $S$  of strategies.

So the parameters of the system(market) are  $N$ ,  $M$  and  $S$ . As the strategies are drawn from the record, there are  $2^{(2^M)}$  strategies( $S$ ).

First of all, the simplest of the pay-off function is considered: agents in the winning side (minority) get the reward (1 point each regardless of how many are in the winning side). So the system is efficient (benefits more agents) when the minority group reaches the limit  $(N-1)/2$ . If the number of agents in one group fluctuates largely around the central value, the system becomes inefficient and points are being 'wasted'.

An interesting effect (it can be seen in computer simulations) is that as the record size ( $M$ ) grows, the fluctuations get smaller. This phenomenon can be interpreted as a sort of 'cooperative behavior' of selfish agents that manage to better distribute the available resources (points). In the same context, in a population of mixed memory size, those with larger record perform better than those with shorter record.

Now, instead of this simple pay-off function, we can devise a little bit more complex one where the members of smaller minority group get higher rewards than those in a larger minority. In this case the 'jackpot' would be the maximal gain for an agent although highly unlikely.

Quite remarkable is the fact that in the second case, the larger size memory ( $M$ ) worsens the success rate. This could be interpreted as if the agents can get more confused than before: said in other words those that switch less in the strategy are more likely to have better success rate than those who do switch often.

If we consider that all the strategies are more or less identically good, what makes an agent good or bad has to do with the timing of the strategy. As we can identify those good/bad agents based on the points (rewards), we can apply Darwinian selection and mutation: the bad players (agents) are weeded off and replaced by clones of the best player. Certain mutations are allowed in order not to end up with a pure state where all the agents are identical. After this Darwinian selection is introduced, the fluctuations of the minority population become smaller around  $(N-1)/2$ . So somehow, a larger number of agents are equally benefited instead of favoring a lucky one with a jackpot.

In conclusion, even with different reward criteria there seems to be some basic assumptions that lead to the appearance of 'cooperative behavior' of the agent population. This is quite a remarkable result taking into account the simplicity of the model and recognizing that no external rules are imposed on the agents other than their own record and strategy.

More elaborate study method of the evolution of market is proposed by R. Masilla. In his paper, he tries to apply the concept known as 'physical complexity'; a quantity given by Kolmogorov-Chaitin theory. In this method, a time series corresponding to the price evolution ( $\{a_i\}$ ) is considered, such that  $a_i = 1$  if the price gets higher and  $a_i = 0$  otherwise.

The definitions of the complexity sound a little 'cryptic' such as:

+Physics complexity: number of binary digits that are meaningful with respect to the environment in a string  $\eta$ . The 'environment' is a binary string given by ordering the digits of the serie  $\{a_i\}$ . It is assumed that the series  $\{a_i\}$  carries some kind of information about the behavior of the agents.

+Kolmogorov-Chaitin complexity: it is the length of the shortest program  $\pi$  that can produce the sequence  $\eta$  when run on the universal Turing machine.

These two types of complexity are related. In fact, the physical complexity can be given as an ensemble average of Kolmogorov-Chaitin complexity (given analytically).

Two different types of intervals of series  $\{a_i\}$  are considered: the ones without any ‘turbulence’ and the ones with ‘turbulence’ (that is with market crashes). The calculation of the physical complexity indicates that this quantity is lower for the intervals where no disturbance is observed than those right before the crashes. This result corresponds to the interpretation that the turbulence has more meaningful binary digits and the information content is larger. Calmer markets would correspond to having more features given by random (thus low information content).

## **DISCUSSION**

In the ‘minority model’ computer experiments were conducted using different pay-off functions and criteria for the selection of the agents. It was possible to identify the assumptions that increase the collective cooperation.

In the ‘physical complexity’-based study of the market records, it was possible to test the measures of complexity given by the statistical mechanics in the context of the market analysis and identify the situations of low/high information content.

As a final comment, it would be interesting to be able to ‘classify’ the content of that information and somehow to be able to have a predictive power.