How our real world emergent?

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I. INTRODUCTION

Because of difficulty to work with intense gravitational field in the laboratory setting and the failure of quantum field theory in gravity, people belief that the Einstein gravity is not fundamental and try to simulate it.

From condense matter aspect, the effective metric has its concrete form and physics. Althought the dynamics of the general relativity can not be completely simulated, actually, the quantum liquid(superfluid) has many good properties that are very similar to our Universe. So, they try to use it to interpret many puzzles in our real world.

Finally, we also discuss how our four dimensional space-time emergent in Matrix model of string thery to complete our story of "how our real world emergent". We also talk a little about how gravity emergent from Matrix model. Totally, the models from condense matter and string theory are all have interesting physics picture.

II. EFFECTIVE METRIC

The first paper talking about the acoustic metric is by W. G. Unruh. [1] They deal with black hole evaporation with fluid dynamics. In the irrotational fluid with the equations

$$\nabla \times \vec{v} = 0 \tag{1}$$

$$\rho[\partial \vec{v}/\partial t + (\vec{v}.\nabla)\vec{v}] = -\nabla p - \rho \nabla \Phi \tag{2}$$

$$\partial \rho/\partial t + \nabla.(\rho \vec{v}) = 0 \tag{3}$$

where the pressure p is assumed to be a function of densityρ, and Φ is is an external force potential.

With some redifinition of velocity distribution as a gradient of scalar potential of variable

$$\vec{v} = \nabla \Psi \tag{4}$$

and mathematical manipulations, and then take linearized perturbation about some background variables (solutions) Ψ_0 with

$$\Psi = \Psi_0 + \tilde{\Psi} \tag{5}$$

then they have a new linearinzed equation of the perturbation, and the motion of the perturbation dependent on the backgroud. After some manipulation, they find the equation of the perturbation similar to the massless scalar field equation in a geometry with a metric which is indentified with corresponding factor in the perturbation fluid equation, and taking the local sound velocity as the light velocity. So, it is very obvious that the metric dependent on background where we perturb it. Then they use this metric to talk in about the blackhole. [1]

After that there are many papers begin to deal with gravity as emergent phenomenon, also mainly in the context of considering the propagation of sound waves in a moving fluid [1, 3–11].Most of them are also start with fluid model that is barotropic and inviscid. The flow is irrotational and the velocity field is described by a scalar potential. [12]

The same as the acoustic metric from the perturbation of fluid dynamics, there are some models [13–15].that using the BEC as the condensed-matter system to generate the "'effective metric "', and also try to mimick kinematic aspects of generality. This concept was first explored in paper by Goron. After that there are many similare models Among these models we take a model related to recent proposal of Garay, Anglin, Cirac, and Zoller based on Bose–Einstein condensates (BEC) [13–15].

The basic idea of a condensed matter analog model of general relativity is that the modifications to the propagation of a field/wave due to curved spacetime can be reproduced (at least partially) by an analog field/wave propagating in some material background with space and time dependent properties. [13]

As Garay *et al* have shown, perturbations in the phase of the condensate wavefunction satisfy, in the low-momentum regime, an equation equivalent to that of a massless scalar field in a curved spacetime (the d'Alembertian equation

$\Delta\phi = 0$), but with the spacetime metric replaced by an effective metric that depends on the characteristics of the background condensate.(this is similar to the acoustic metric from fluid theory) [13]

They start with the many-body Hamiltonian of the form [16]:

$$H = \int \mathrm{d}\mathbf{x}\ \widehat{\Psi}^{\dagger}(t,\mathbf{x}) \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{x}) \right] \widehat{\Psi}(t,\mathbf{x}) + \frac{1}{2}\int \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{x}'\ \widehat{\Psi}^{\dagger}(t,\mathbf{x})\ \widehat{\Psi}^{\dagger}(t,\mathbf{x}')\ V(\mathbf{x}-\mathbf{x}')\ \widehat{\Psi}(t,\mathbf{x}')\ \widehat{\Psi}(t,\mathbf{x}). \quad (6)$$

Here $V_{\rm ext}(\mathbf{x})$ is some confining external potential, $V(\mathbf{x}-\mathbf{x}^{'})$ is the interatomic two-body potential (other possible multibody interactions are neglected at this stage), and m is the mass of the bosons undergoing condensation (in current experiments these bosons are actually alkali atoms). Finally, $\widehat{\Psi}(t,\mathbf{x})$ is the boson field operator. [13]

AS Bogoliubov [17, 18] mean field approach,they peparate bosonic field operator $\widehat{\Psi}(t,\mathbf{x})$ into a classical condensate contribution $\psi(t,\mathbf{x})$ plus excitations $\widehat{\varphi}(t,\mathbf{x})$:

$$\widehat{\Psi}(t,\mathbf{x}) = \psi(t,\mathbf{x}) + \widehat{\varphi}(t,\mathbf{x}). \quad (7)$$

Here $\psi(t,x)$ is the expectation value of the field $\psi \equiv \langle\widehat{\Psi}(t,\mathbf{x})\rangle$. It is sometimes referred in the literature as the "wave function of the Bose–Einstein condensate". Its modulus fixes the particle density of the condensate, $\rho = N/V$, in such a way that $|\psi(t,\mathbf{x})|^2 = \rho(t,\mathbf{x})$. [13]

Taking "zeroth-order approximation" and short-distance delta-like interaction assumption [13], They finally have Gross–Pitaevskii (GP) equation, (sometimes called the nonlinear equation, or even the time-dependent Landau–Ginzburg equation),

$$\mathrm{i}\hbar\ \frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{x}) + \lambda\ |\psi(t,\mathbf{x})|^2 \right)\psi(t,\mathbf{x}). \quad (8)$$

and the associated effective action has the form

$$\mathcal{S}\ =\ \int \mathrm{d}t\ \mathrm{d}^3x \left\{ \psi^* \left(\mathrm{i}\hbar\ \partial_t + \frac{\hbar^2}{2m}\nabla^2 - V_{\rm ext}(\mathbf{x}) \right)\psi - \frac{1}{2}\lambda\ |\psi(t,\mathbf{x})|^4 \right\}, \quad (9)$$

the time-dependent Landau–Ginzburg action.

Actually if we start from this equation, and take the linear perturbation procedure, we can also find the effect metric as the acoustic metric as before.

A. Generalized Gross–Pitaevskii equation [13]

But in order to explore the limit of BEC systems to mimic general relativistic, we have to generalize our Gross–Pitaevskii equation by symmetry arguments. [13] for example:

—(1) to replace the quartic $\frac{1}{2}\lambda|\psi(t,\mathbf{x})|^4$ by an arbitrary nonlinearity $\pi(\psi^*\psi) = \pi(|\psi|^2)$.

—(2) permit the nonlinearity function to be explicitly space and time dependent: $\pi \to \pi(x,[\psi^*\psi])$ with $x \equiv (t,\mathbf{x})$.

—(3) to permit the mass to be a 3-tensor: $m \to m_{ij}$. This anisotropic form of effective mass many examples in condense matter physics.

—(4) also permit the 3-tensor mass to depend on position (both time and space). It is convenient to introduce an arbitrary but fixed (time and space independent) scale μ with the dimensions of mass and then write

$$m_{ij} = \mu\ {}^{(3)}h_{ij} \quad (10)$$

with ${}^{(3)}h_{ij}$ being a properly dimensionless 3-metric. Note that the introduction of μ is a *mathematical convenience*, not a *physical necessity*, and that all properly formulated physical questions will be independent of μ.

—(5) allow time-dependence for the confining potential $V_{\rm ext} = V_{\rm ext}(t,\mathbf{x})$. (This is already implicit in the analysis of Garay *et al*, but is somewhat non-standard from the usual condensed-matter viewpoint.) [13]

With above extensions, generalized action [13] is

$$\mathcal{S}\ =\ \int \mathrm{d}t\ \mathrm{d}^3x\ \sqrt{\det\left[{}^{(3)}h\right]} \left\{ \psi^* \left(\mathrm{i}\hbar\ \partial_t + \frac{\hbar^2}{2\mu}\ \Delta_h + \frac{\xi\hbar^2}{2\mu}\ {}^{(3)}R(h) - V_{\rm ext}(t,\mathbf{x}) \right)\psi - \pi(x,|\psi|^2) \right\}. \quad (11)$$

Here Δ_h is the 3-dimensional Laplacian defined by

$$\Delta_h \psi = \frac{1}{\sqrt{\det\left[{}^{(3)}h\right]}} \; \nabla_i \left(\sqrt{\det\left[{}^{(3)}h\right]} \; \left[{}^{(3)}h^{-1}\right]^{ij} \; \nabla_j \psi \right), \tag{12}$$

where $[{}^{(3)}h^{-1}]^{ij}$ is the inverse of the 3-metric ${}^{(3)}h_{ij}$. Additionally note the presence of the DeWitt term

$$\frac{\xi \hbar^2}{2\mu} \; {}^{(3)}R(h) \tag{13}$$

involving the dimensionless parameter ξ and the 3-dimensional Ricci scalar—this term arises from operator-ordering ambiguities in going from the "flat space" metric to "curved space"(going from position-independent m to a position-varying effective mass) [19, 20]. They include the DeWitt term here for completeness, and because it should be included as a matter of principle, but note that it is unlikely to lead to experimentally measurable effects. [13]

From this action, we have "generalized" nonlinear equation [13]:

$$\mathrm{i}\hbar \, \frac{\partial}{\partial t} \psi(t, \mathbf{x}) \; = \; - \frac{\hbar^2}{2\mu} \; \Delta_h \psi(t, \mathbf{x}) - \frac{\xi \hbar^2}{2\mu} \; {}^{(3)}R(h) \; \psi(t, \mathbf{x}) + V_{\mathrm{ext}}(t, \mathbf{x}) \; \psi(t, \mathbf{x}) + \pi'(\psi^* \psi) \; \psi(t, \mathbf{x}). \tag{14}$$

B. Madelung representation and the hydrodynamic limit

Actually, as using the Madelung representation [21–25] of a wavefunction

$$\psi(t, \mathbf{x}) = \sqrt{\rho(t, \mathbf{x})} \; \exp[-\mathrm{i}\theta(t, \mathbf{x})/\hbar]. \tag{15}$$

and replace it into the Gross–Pitaevskii equation, they can have a hydrodynamic like equation. And here, we have phase function $\theta(t, \mathbf{x})$that is similar to the scalar potential of velocity distribution in the acoustic metric in fluid dynamics. [13]

C. Fluctuations

And then we can follow the same steps as before, linearing the the perturbation of around the background (ρ_0, θ_0). So we have

$$\rho = \rho_0 + \epsilon \rho_1 + O(\epsilon^2)$$

and $\theta = \theta_0 + \epsilon \theta_1 + O(\epsilon^2)$(16)

and we can get equation of the perturbation θ_1 which is

$$\begin{aligned} &- \; \partial_t \left\{ \left[\pi''(\rho_0) - \frac{\hbar^2}{2\mu} \; D_2 \right]^{-1} \left(\partial_t \theta_1 + \frac{1}{\mu} \boldsymbol{\nabla} \theta_0 \cdot \boldsymbol{\nabla} \theta_1 \right) \right\} \\ &+ \; \frac{1}{\mu} \boldsymbol{\nabla} \cdot \left(\rho_0 \; \boldsymbol{\nabla} \theta_1 - \boldsymbol{\nabla} \theta_0 \; \left\{ \left[\pi''(\rho_0) - \frac{\hbar^2}{2\mu} \; D_2 \right]^{-1} \left(\partial_t \theta_1 + \frac{1}{\mu} \boldsymbol{\nabla} \theta_0 \cdot \boldsymbol{\nabla} \theta_1 \right) \right\} \right) = 0. \end{aligned} \tag{17}$$

and if we identify the four dimential metric with the differantial operator in front of θ_1 and then we will find this equation is also similar to the curved space scalar d'Alembertian.

the above wave equation (17) is easily rewritten as

$$\partial_\mu (f^{\mu\nu} \; \partial_\nu \theta_1) = 0. \tag{18}$$

Here

$$f^{00} \; = \; - \left[\pi''(\rho_0) - \frac{\hbar^2}{2\mu} \; D_2 \right]^{-1} \tag{19}$$

$$f^{0j} = -\left[\pi''(\rho_0) - \frac{\hbar^2}{2\mu} D_2\right]^{-1} \frac{h^{jk} \nabla_k \theta_0}{\mu} \tag{20}$$

$$f^{i0} = -\frac{h^{ik} \nabla_k \theta_0}{\mu} \left[\pi''(\rho_0) - \frac{\hbar^2}{2\mu} D_2\right]^{-1} \tag{21}$$

$$f^{ij} = \frac{\rho_0 \, {}^{(3)}h^{ij}}{\mu} - \frac{h^{ik} \nabla_k \theta_0}{\mu} \left[\pi''(\rho_0) - \frac{\hbar^2}{2\mu} D_2\right]^{-1} \frac{h^{jl} \nabla_l \theta_0}{\mu}. \tag{22}$$

Thus $f^{\mu\nu}$ is a 4×4 matrix of *differential operators* [13]

Notes that they already has three dimension non-flat metric(mass metric) before getting effective four dimentional one, but basically the four dimentional effect metric is the same as acoustic metric. [13]

The Gross–Pitaevskii equation describes, in a simple and compact form, the relevant phenomena associated with BEC. In particular, it reproduces typical properties exhibited by superfluid systems, like the propagation of collective excitations and the interference effects originating from the phase of the condensate wave function. It is precisely from this (non-relativistic) theory that we shall now see how an analogue of GR can be constructed, and possibly used for experimental purposes. [13]

Present-day experimental achievements, and the rapid development in magnetic trapping techniques, seem to illuminate a viable path to experimentally reproducing important general relativistic features such as ergoregions and event horizons [13–15]

III. EFFECTIVE DYNAMICS

But it is not enough to simulate the gravity by effective metric, we also need to find the dynamical equation of gravity–Einstein equation. They start from the idea of "induced gravity" of Andrei Sakharov, there is a model that they deal with the lagrangain that dependant only on a single scalar field and its first derivatives. The scalar field here is like the velocity po tential in the W.G.Unruh's paper [2].

And also consider the linearized fluctuations around some background solution $\phi_0(t, \vec{x})$ of the equation of motion that will relate to the equation of motion of the effective action. So, the the scalar field expand in the form

$$\phi(t, \vec{x}) = \phi_0(t, \vec{x}) + \epsilon \phi_1(t, \vec{x}) + O(\epsilon^2). \tag{23}$$

The same as before, the linearized Euler-Lagrange equation has the form of second-order differential equation with coefficients that are implicit functions of the background field $\phi_0(t, \vec{x})$, with definition of the acoustic metric by

$$\sqrt{-g}\; g^{\mu\nu} \equiv f^{\mu\nu} \equiv \left. \left\{ \frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi)\, \partial(\partial_\nu \phi)} \right\} \right|_{\phi_0}, \tag{24}$$

that is,

$$g_{\mu\nu}(\phi_0) = \left. \left(-\det \left\{ \frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi)\, \partial(\partial_\nu \phi)} \right\} \right)^{1/(d-1)} \right|_{\phi_0} \left. \left\{ \frac{\partial^2 \mathcal{L}}{\partial(\partial_\mu \phi)\, \partial(\partial_\nu \phi)} \right\}^{-1} \right|_{\phi_0}. \tag{25}$$

and the equation of motion has the form like d'Alembertian wave equation(the massless scalar field equation)

$$\left[\Delta(g(\phi_0)) - V(\phi_0)\right] \phi_1 = 0, \tag{26}$$

where Δ is the d'Alembertian operator associated with the effective metric $g(\phi_0)$, and $V(\phi_0)$ is a background-field-dependent potential [26].

Also, if the metric $g_{\mu\nu}$ has Lorentzian signature, then the linearized PDE should be hyperbolic $\pm[-, (+)^d]$ [27].

From the above definition of metric, in order to take the effective metric as really "emergent" phenomenon, the way of choocing the background field become very important. We follow similar idea of "induced gravity", proposed several years ago by Andrei Sakharov [28], provides a natural framework for such an emergence of an effective geometrodynamics. So, we try to find the dynamical equation of metric from the one-loop effective action of ϕ field. [26]

First, separate the ϕ field into two parts, one is just classical background solution, another is the quantum fluctuactionϕ_b.
$\phi = \phi_b + \phi_q$
after integrate out the quantum fluctuactions to one loop level, they has

$$\Gamma[g(\phi_b), \phi_b] = S[\phi_b] + \frac{1}{2}\,\hbar\,\mathrm{tr}\ln\left[\Delta(g(\phi_b)) - V(\phi_b)\right] + O(\hbar^2). \tag{27}$$

By some manipulation of the determinant of the differential operator, defined in terms of zeta functions or heat kernel expansions, with both regularization and renormalization, they finally has (more or less equivalent to Sakharov's proposal)

$$\begin{aligned}\Gamma[g(\phi_b), \phi_b] &= S[\phi_b] + \hbar \int \sqrt{-g}\,\kappa\left[-2\Lambda + R(g) - 6V(\phi_b)\right] \mathrm{d}^{d+1}x \\ &\quad + \hbar X[g(\phi_b), \phi_b] + O(\hbar^2).\end{aligned} \tag{28}$$

from this equation we can easily find that the effective action depends on the background field in two ways: *explicitly* through ϕ_b, and *implicitly* through $g(\phi_b)$ [26].

This action includes the term of original action $S[\phi_b]$and and potential(ϕ_b),which explicitly dependent on the ϕ_b, and the terms that implicitly dependant on (ϕ_b) through $g(\phi_b)$, which are cosmological terms and Einstein-Hilbert term. The last term denotes the other finite contributions of one loop effective action. It will correspond to the curvature squared correction to the Einstein equation after being differentiated with effectivee metric.(from many aspects, people believe there should be a such correction) [26]

It is the automatic emergence of the Einstein–Hilbert action as part of the one-loop effective action. (Note that their approach is not much identical to Sakharov's idea — In Sakharov's approach the metric was free to be varied at will, leading precisely to the Einstein equations (plus quantum corrections); in their approach the metric is not a free variable and the equations of motion will be a little trickier. So, in order to get a dynamics as close as possible to that of Eistein, when we get the the EL equation from effective action, the terms that have the form of explicitly difficiated by the backgroud field ϕ should satisfy the equation

$$\begin{aligned}\left\{ \frac{\delta S[\phi_b]}{\delta \phi_b} - 6\hbar\kappa \frac{\delta \int \sqrt{-g}\, V(\phi_b)}{\delta \phi_b} + \hbar\, \left.\frac{\delta X[g(\phi_b), \phi_b]}{\delta \phi_b}\right|_{g_b} \right\} &= \hbar \frac{\delta Y[g]}{\delta g_{\mu\nu}} \frac{\delta g_{\mu\nu}(\phi_b)}{\delta \phi_b(x)} \\ &\quad + O(\hbar^2).\end{aligned} \tag{29}$$

and then we has the equation that is similar to Eistein equation.

$$\left[\kappa\left(G^{\mu\nu}(g) + \Lambda g^{\mu\nu}\right) + \frac{1}{\sqrt{g}} \frac{\delta\{X[g(\phi_b)] + Y[g(\phi_b)]\}}{\delta g_{\mu\nu}}\right] \frac{\delta g_{\mu\nu}(\phi_b)}{\delta \phi_b(x)} = O(\hbar). \tag{30}$$

But because of the the equation has the form of contraction with the $\delta g_{\mu\nu}(\phi_b)/\delta\phi_b(x)$, it is not exactly the same with Eistein equation. [26]

And they also suggest to extend the idea from single scalar field to many fields. but it is too messive [26].

Althought at this stage, most model that deal with the acoustic metric is not sufficient to simulate all properties of Einstein gravity, it still a good motivation to consider Einstein gravity as the effective low-energy and long-distance consequence of wide class of theories.

IV. PHYSICAL INTERPRETATION OF QUANTUM LIQUID AS THEORY OF EVERYTHING[29, 33]

It is well-known in both condense matter and high energy physics that the symmetry will break at low-tempuruature or low-energy limit. So, as the temperature increase, the system will has more symmetry. But there is a opposite point of view: it is argued that when we start from some proper energy scale such that we will find the symmetry(such as Lorentz invariance, gauge invariance, general covariance.) voilated when we graduately increase system energy. This is the anti-grand-unification scenario [30, 31] Actually there is an very good example: superfluid. As T near Tc, it has grand-unification scenario; as T near zero, it has anti-grand-unification scenario.

Because of the quantum liquid model(superfluid) has such good properties(It has both grand-unification and anti-grand-unification scenario), they consider the theory of every in quantum liquids.[29]

First they start with many-body Hamiltonian written in the second quantized form:

$$\mathcal{H} - \mu\mathcal{N} = \int d\mathbf{x}\psi^\dagger(\mathbf{x})\left[-\frac{\nabla^2}{2m} - \mu\right]\psi(\mathbf{x}) + \int d\mathbf{x}d\mathbf{y}V(\mathbf{x}-\mathbf{y})\psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{y})\psi(\mathbf{y})\psi(\mathbf{x}) \tag{31}$$

where m is the bare mass of the atom; $V(\mathbf{x}-\mathbf{y})$ is the bare interaction between the atoms;μ is the chemical potential – the Lagrange multiplier which is introduced to take into account the conservation of the number of atoms: $\mathcal{N} = \int d\mathbf{x}\ \psi^\dagger(\mathbf{x})\psi(\mathbf{x})$. In ^{4}He, the bosonic quantum field $\psi(\mathbf{x})$ is the annihilation operator of the ^{4}He atoms. In ^{3}He, $\psi(\mathbf{x})$ is the fermionic field and the spin indices must be added.[29]

Taking this Hamiltonian as the theory of every thing, we can understand the plank or transplank physics. At low energy the quantum liquid is well described in terms of a dilute system of quasiparticles. These quasiparticles serves as the elementary particles of the low energy effective quantum field theory, then the quantum liquid looks like the our Universe with effective background gauge or gravity field as the the collection modes of the fluid.[29] Similar to previous derivativation we can find the phono propagating in the inhomogeneous liquid are described by the effective Lagrangian

$$L_{\text{effective}} = \sqrt{-g}g^{\mu\nu}\partial_\mu\alpha\partial_\nu\alpha\ , \tag{32}$$

where $g^{\mu\nu}$ is the effective acoustic metric provided by inhomogeneity and flow of the liquid [1, 32].

This theory has a good property that the emergent phenomena do not depend much on the details of the Theory of Everything [33], the details of the pair potential $V(\mathbf{x}-\mathbf{y})$ only effect the "fundamental" parameters of the effective theory ("speed of light", "Planck" energy cut-off, etc.) and where should our effective theory exist, but notthe general structure of the theory. [29]

V. VACUUM ENERGY AND COSMOLOGICAL CONSTANT[29]

Start with this theory of every thing, they can argue that why the vacuum energy is so small.

Because of the disadvantages of calculations of vacuum energy within the effective field theory (i) The result depends on the cut-off procedure; (ii) The result depends on the choice of the zero from which; the energy is counted: a shift of the zero level leads to a shift in the vacuum energy.) they use the Theory of Everythingcalculate to calculate the energy density of the ground state exactly,

$$\rho_\Lambda = \frac{1}{V} < \text{vac}|\mathcal{H} - \mu\mathcal{N}|\text{vac} > \ . \tag{33}$$

Note that this energy does not depend on the choice of zero level: the overall shift of the energy in $\mathcal{H}$ is exactly compensated by the shift of the chemical potential μ.

And then they use the thermodynamics arguments with the assumption that If there are no external forces acting on the quantum liquid, then at $T = 0$ in the limit of infinite liquid volume V one obtains exact nullification of the energy density:

$$\rho_\Lambda = \frac{1}{V} < \text{vac}|\mathcal{H} - \mu\mathcal{N}|\text{vac} >= 0\ . \tag{34}$$

The proof is simple.[29]

So, the only condition which we used is that the liquid exists in equilibrium without external pressure. This condition is fulfilled only for the liquid-like or solid-like states, for which the chemical potential μ is negative, if it is counted from the energy of the isolated atom. For liquid ^{4}He and ^{3}He the chemical potentials are really negative, $\mu_4 \sim -7$K and $\mu_3 \sim -2.5$K (see review paper Ref. [34]). This condition cannot be fulfilled for gas-like states for which μ is positive and thus they cannot exist without an external pressure. Thus the mere assumption that the vacuum of the quantum field theory belongs to the class of states, which can exist in equilibrium without external forces, leads to the nullification of the vacuum energy in equilibrium at $T = 0$.[29]

Thus the first lesson from condensed matter is: the standard contribution to the vacuum energy density from the vacuum fluctuations in sub-Planckian effective theory is, without any fine tuning, exactly canceled by the transk-Planckian degrees of freedom, which are not accessible within the effective theory.[29]

And they also argue how the perturbation of the vacuum come from:

1. *Vacuum energy from finite temperature*

the thermal pressure caused by quasiparticles – phonons – which play the role of the hot relativistic matter with equation of state $P_M = (1/3)\rho_M$, must be compensated by the negative vacuum pressure $P_\Lambda = -P_M$ to support the zero value of the external pressure, $P = P_\Lambda + P_M = 0$.(This is our system assumption.) the vacuum pressure and vacuum energy density has the relation

$$\rho_\Lambda = -P_\Lambda = P_M = \frac{1}{3}\rho_M = \sqrt{-g}\frac{\pi^2}{30\hbar^3}T^4 \ , \tag{35}$$

where $g = -c^{-6}$ is again the determinant of acoustic metric, with c being the speed of sound. In this example the vacuum energy density ρ_Λ is positive and always on the order of the energy density of matter. This indicates that the cosmological constant is not actually a constant but is ajusted to the energy density of matter and/or to the other perturbations of the vacuum discussed below. [29]

2. *Vacuum energy from Casimir effect*

Another example of the induced is provided by the boundaries of the system can also induce nonzero vacuum energy density. If tthere is a droplet freely suspended then at $T = 0$ the vacuum pressure P_Λ must compensate the pressure caused by the surface tension due to the curvature of the surface. This is an analogue of the Casimir effect, in which the boundaries of the system produce a nonzero vacuum pressure.

Its equation of state is $P_\sigma = -(2/3)\rho_\sigma$:

$$\rho_\sigma = \frac{4\pi R^2\sigma}{\frac{4}{3}\pi R^3} = \frac{3\sigma}{R} \quad , \quad P_\sigma = -\frac{2\sigma}{R} = -\frac{2}{3}\rho_\sigma \ . \tag{36}$$

such casimir effect is like an analogue of quintessence in cosmology[29]

3. *Vacuum energy induced by texture*

The quantum liquid may has its inhomogeneity texture.

Within the soliton the vacuum is perturbed, and the vacuum energy is induced being on the order of the energy of the perturbation. In this case $\rho_\Lambda(z)$ is equal to the gradient energy density of the texture.[29, 35]

4. *Vacuum energy due to Riemann curvature*

The analogy with general relativity is supported by the observation that the gradient energy of a twisted $\hat{\mathbf{l}}$-texture is equivalent to the Einstein curvature term in the action for the effective gravitational field in ^{3}He-A [36]:

$$\frac{1}{16\pi G}\int d^3r\sqrt{-g}\mathcal{R} \equiv K\int d^3r((\hat{\mathbf{l}}\cdot(\nabla\times\hat{\mathbf{l}}))^2 \ . \tag{37}$$

Here $\mathcal{R}$ is the Riemann curvature calculated using the effective metric experienced by fermionic quasiparticles in ^{3}He-A, $ds^2 = dt^2 - c_\perp^{-2}(\hat{\mathbf{l}}\times d\mathbf{r})^2 - c_\parallel^{-2}(\hat{\mathbf{l}}\cdot d\mathbf{r})^2$, with $\hat{\mathbf{l}}$ playing the role of the Kasner axis. That is why the nonzero vacuum energy density within the soliton induced by the inhomogeneity of the order parameter is very similar to that caused by the curvature of space-time.[29]

And use the equilibrium conditions of the static state of the Universe[37]

$$\rho = \rho_M + \rho_\Lambda + \rho_\mathcal{R} = 0 \ , \ P = P_M + P_\Lambda + P_\mathcal{R} = 0 \ . \tag{38}$$

The first equation in (38) reflects the gravitational equilibrium, which requires that the total mass density must be zero: $\rho = \rho_M + \rho_\Lambda + \rho_\mathcal{R} = 0$. This gravineutrality is analogous to the electroneutrality in condensed matter. The second equation in (38) is equivalent to the requirement that for the "isolated" Universe the external pressure must be zero: $P = P_M + P_\Lambda + P_\mathcal{R} = 0$. [29]

Use these conditions, they also conclude that the vacuum energy is on the order of the energy of matter obtained in the effective theory of generality(this is a very good conclusion):
For the cold Universe with $P_M = 0$, the Eqs. (38) give

$$\rho_\Lambda = \frac{1}{2}\rho_M = -\frac{1}{3}\rho_{\mathcal{R}} = \frac{k}{8\pi G R^2} \ , \tag{39}$$

and for the hot Universe with the equation of state $P_M = (1/3)\rho_M$,

$$\rho_\Lambda = \rho_M = -\frac{1}{2}\rho_{\mathcal{R}} = \frac{3k}{16\pi G R^2} \ . \tag{40}$$

Where R is the cosmic scal factor in the Friedmann-Robertson-Walker metric,$ds^2 = dt^2 - R^2(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2)$; the parameter $k = (-1, 0, +1)$ for an open, flat, or closed Universe respectively; Since the energy of matter is positive, the static Universe is possible only for positive curvature, $k = +1$, i.e. for the closed Universe. [29]

A. Discussion: Why is vacuum not gravitating?

Because of the gravity is the effective theory, it is not the foundamental theory as microscopic physics. So, for these microscopic atoms , which live in the "trans-Planckian" world and form the vacuum state there, do not experience the "gravitational" attraction experienced by the low-energy quasiparticles, since the effective gravity simply does not exist at the micriscopic scale (we neglect here the real gravitational attraction of the atoms,which is extremely small in quantum liquids). [29]

VI. HOW FOUR DIMENSIONAL SPACE-TIME EMERGENT?

The (IIB)matrix model, which was conjectured to describe (IIB)superstrings in 10 dimensions, it is the concrete propasals for nonperturbative string and has a particularly simple form. It is a supersymmetric matrix model(supersymmetry is a symmetry between fermion and boson, but we will not use this symmetry in this paper), which can be obtained from the zero-volume limit of 10d SU(N) super Yang-Mills theory(Yang-Mills theory with supersymmetry).
A large N reduced model has been proposed as a nonperturbative formulation of type IIB superstring theory[38][39]

$$S = -\frac{1}{g^2} Tr(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]), \tag{41}$$

here ψ is a ten dimensional Majorana-Weyl spinor field, and A_μ and ψ are $N \times N$ Hermitian matrices.
We must to note that the space-time is represented by 10 bosonic matrices A_ν, and hence treated dynamically.(skip the detail argument) So, the Matrix model dominates the dynamics of the spacetime. [40]
Start from this model, it allows us in particular to investigate the possibility [41] that our 4d space-time is generated *dynamically* in superstring theory in 10d. Since the model has manifest SO(10) symmetry, the emergence of 4d space-time requires the SO(10) symmetry to be spontaneously broken.And the phase of the fermion integral must play a crucial role in the SSB of SO(10) to SO(4) [40, 42–44] Some papers examine the SSB of SO(D)symmetry by means of the Gaussian expansion (Mean field approximation)
[40, 45]

VII. BOSONIC YANG-MILLS INTEGRAL[40]

In order to illustrate their method, let us consider the bosonic Yang-Mills integral defined by

$$Z = \int A^{-S} \ , \tag{42}$$

$$S = -\frac{1}{4} N\beta \, \mathrm{tr}[A_\mu, A_\nu]^2 \ , \qquad \beta = \frac{1}{g^2 N} \ . \tag{43}$$

The bosonic matrices A_μ ($\mu = 1, \cdots, D$) are $N \times N$ hermitian matrices, which we expand as $A_\mu = A_\mu^a T^a$ with respect to the SU(N) generators T^a ($a = 1, \cdots, (N^2 - 1)$) normalized as $\mathrm{tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$. The integration measure for A_μ

is defined as $A = \prod_{a=1}^{N^2-1} \prod_{\mu=1}^{D} \frac{dA_\mu^a}{\sqrt{2\pi}}$. As an important consequence of the zero-volume limit, one can actually absorb the parameter g by rescaling the dynamical variables $A_\mu \mapsto \sqrt{g} A_\mu$. Therefore, the parameter g is merely a scale parameter rather than a coupling constant. The partition function is conjectured to be finite [46] for $N > D/(D-2)$, and this conjecture was proved in [47].

It turns out to be convenient to introduce the rescaled dynamical variables X_μ given by $X_\mu = \beta^{1/4} A_\mu$, so that the action takes the canonical form

$$S = -\frac{1}{4} N \, \mathrm{tr}[X_\mu, X_\nu]^2 \ . \tag{44}$$

The most general SU(N) invariant Gaussian action(Mean field action) without assuming SO(D) symmetry can be brought into the form

$$S_0 = \sum_{\mu=1}^{D} \frac{N}{v_\mu} \mathrm{tr}\,(X_\mu X_\mu) \ , \qquad\qquad v_\mu > 0 \ , \tag{45}$$

by making an appropriate SO(D) transformation. Then we rewrite the partition function (42) as

$$Z \ = \ Z_0 \, \langle^{-(S-S_0)} \rangle_0 \ , \tag{46}$$

$$Z_0 \ = \ \int A^{\,-S_0} = \beta^{-D(N^2-1)/4} \int X^{\,-S_0} \ , \tag{47}$$

where $\langle \ \cdot \ \rangle_0$ is a VEV with respect to the partition function Z_0. (Because of the difficulity of getting the partition function from the S, we usually take expansion around the mean field action[45] .)

From (46) it follows that the free energy $F = -\ln Z$ can be expanded as

$$F \ = \ \sum_{k=0}^{\infty} F_k \quad ; \quad F_0 = -\ln Z_0 \ ,$$

$$F_k \ = \ -\frac{(-1)^k}{k!} \langle (S - S_0)^k \rangle_{\mathrm{C},0} \qquad (\text{for } k \geq 1) \ , \tag{48}$$

where the suffix 'C' in $\langle \ \cdot \ \rangle_{\mathrm{C},0}$ means that the connected part is taken. The first two terms of the expansion are given as

$$F_0 \ = \ \frac{1}{2}(N^2-1)\Big\{ D \ln(N\beta^{1/2}) - \sum_{\mu=1}^{D} \ln v_\mu \Big\} \ , \tag{49}$$

$$F_1 \ = \ \langle S \rangle_0 - \langle S_0 \rangle_0 \ , \tag{50}$$

$$\langle S \rangle_0 \ = \ \frac{1}{8}(N^2-1) \sum_{\mu \neq \nu} v_\mu v_\nu \ , \tag{51}$$

$$\langle S_0 \rangle_0 \ = \ \frac{1}{2}(N^2-1) D \ . \tag{52}$$

Now we have the free energy that dependent on the the parameters v_ν the next step is to find the minimun free energy for given values of v_ν of definite SO(d) symmetry. And finally they finf that the ansatz with SO(4) symmetry has the minimun free energy, the symmetry spontanuous breaking! Actually the free energy is stable at higher orders, the gaussian expansion in free energy is convergent[40, 45] .

They also calculate the extent in the μ-th direction $R_\mu \equiv \sqrt{\langle \frac{1}{N} \mathrm{tr}(A_\mu)^2 \rangle}$. Note that $R_1 = \cdots = R_d \equiv R$ and $R_{d+1} = \cdots = R_{10} \equiv r$ due to the imposed symmetry. At the first order, the ratio $\rho \equiv R/r$ is given by $\sqrt{V/v}$ and we find that $\rho > 1$. At the third order, we observe that the ratio ρ increases in all the cases except for $d = 2$[40] .

Actually it is difficult to realize the effective metric from Matrix model, but they instead deal with the general coordinate invariance of the matrix model. [48–50]

In the paper of S. Iso and H. Kawai[50], they justify the interpretation of space-time as distribution of eigenvalues of the matrices by showing that some low energy excitations indeed propagate in it. Finally they argue a possible identification of the diffeomorphism symmetry with permutation group acting on the set of eigenvalues, and show that the general covariance is realized in the low energy effective theory even though we do not have a manifest general covariance in the IIB matrix model action.

In the paper of H. Aoki, S. Iso, H. Kawai, Y. Kitazawa,and T. Tada[51], they conjecture that the general coordinate invariance is present in this model due to the following reasoning. There is the permutation symmetry S_N which permutes the color indices. It is a subgroup of the full gauge group $SU(N)$ and an exact symmetry of our effective action.(the so(9,1) Lorentz symmetry and the u(N) gauge symmetry are mixed together.) Since it does not change the density of the eigenvalues, it should be part of the volume preserving diffeomorphism group in the continuum limit. Similar argument has been made in the dynamical triangulation approach.

Because the Matrix model is the nonperturbative formulation of string theory, we have the unique vacuum, and the cosmological constant is also zero by the mechanism of Matrix model.[51, 52]

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