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The superfluid phases of ^3He

Abstract

In this essay we shall discuss some of the physical properties of the superfluid phases of ^3He obtained at ultra low temperatures of a few mK and zero magnetic field.

Introduction

Helium being a principal constituent of stellar matter is the second most abundant element in the universe after hydrogen. It has dominated the scene in cryogenics as the most important refrigerant for attaining very low temperatures ever since it was liquefied for the first time in 1908 by Kamerlingh Onnes when at 0.9 K he was also able to attain the lowest temperature ever reached before that time. It exists in two isotopic forms namely ^{3}He and ^{4}He.

The small mass of the He atom is primarily responsible for many of its very interesting and unusual physical properties. It is extremely difficult to solidify Helium due to the fact that it has significant zero point motion even at absolute zero since it is extremely light. This makes it possible to have He in the liquid phase up to very low temperatures. Hence the liquid state of He is a very good example of a quantum liquid whose phase diagrams demonstrate richness in their variety of physical properties.

Out of the two isotopes of He, only the heavier isotope ^{4}He had been in the focus of attention of physicists for a long time. The reason for this is that ^{4}He shows a transition to the superfluid phase on cooling below 2.17 K which is known as the lambda transition one of whose signatures is a characteristic peak in the specific heat. The superfluid phase, along with other strikingly unusual properties has the ability flow without friction ^{3}He on the other hand shows no such interesting property even at temperatures close to 1K. It is only at much lower temperatures (of the order of a few mK) that liquid ^{3}He undergoes transition to the superfluid state.

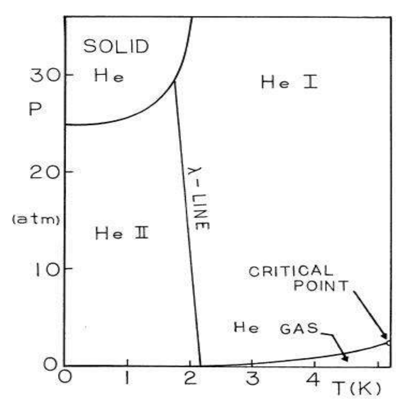

Fig 1 Phase diagram of ^{4}He

That the two isotopes of He are strikingly different in their properties has to do with the fact that ^{4}He is a boson since it has an even number of spin ½ particles (electrons and nucleons) while ^{3}He being deficient of one neutron has an odd number of nucleons in its nucleus and hence behaves like a fermion. While in ^{4}He superfluidity is associated with Bose - Einstein Condensation (BEC) of the bosonic atoms, in ^{3}He the fermionic atoms form Cooper pairs very similar to how electrons pair up in a conventional BCS superconductor [1].

Normal state behaviour of liquid ^{3}He

We shall now start by looking at the physical properties of the normal' phase of liquid ^{3}He. Experiments reveal that between 100 mK and 3mK the liquid behaves like a weakly interacting degenerate Fermi gas. Two of the notable features in the experimental data are 1) specific heat being linear in temperature and 2) spin susceptibility being independent of temperature as pointed out by Leggett [2]. Given that the hard core radius is about 70% of the interatomic spacing, it is expected that the liquid be strongly interacting.

Landau's Fermi liquid theory seems to provide a good description of the normal phase of liquid ^{3}He. Although we shall not go into the theoretical details of Landau's theory here, the essential physical idea is the following. As a result of the inter -particle interaction, each atom is 'dressed' by a screening cloud around it. Such dressed states called quasiparticles take over the role of the bare particles. The effect of interactions appears as a renormalization of the effective mass of the particles. Thus the ground state of such a gas is a filled Fermi sea of quasiparticles of effective mass m* which differs from m, the mass of the bare particle. An important point to note here is that apart from the change in effective mass, the interactions are also responsible for introducing an effective interaction between quasiparticles themselves via a parameter $f(\mathbf{p},\sigma,;\mathbf{p}`,\sigma`)$ which is a measure of the interaction energy between quasiparticles of momenta **p, p`** and spins σ and σ` respectively.

Cooper pairs and BCS theory

As a prelude to the description of the superfluid phases of ^{3}He, we briefly review the theory of conventional superconductivity in metals Bardeen, Cooper and Schrieffer [1]. Cooper showed that a pair of electrons of opposite momenta lying outside the Fermi surface in the presence of (even weak) attractive interactions is unstable towards the formation of a bound state known as Cooper pairs. In the full BCS theory, this idea is exploited to variationally find the ground state of such a system with pairs of electrons of opposite spin (spin singlet) and opposite momenta. The attractive interaction is provided by coupling between electrons and lattice vibrations (phonons).

The ground state of the BCS superconductor is characterized by the formation of an energy gap at the Fermi level in the superconducting state which means that quasiparticle excitations of the ground state have a finite lower bound in energy. The gap is thus a measure of the strength of the superconducting pairing. The quasiparticle energy spectrum is given by

$$E(\mathbf{k}) = (\epsilon(\mathbf{k})^2 + \Delta^2)^{1/2} \qquad (1)$$

where $\epsilon(\mathbf{k})$ is the kinetic energy of the electron measured with respect to the Fermi level and Δ is the superconducting energy gap.

Interatomic interaction potential in liquid ^{3}He

The interatomic interaction potential in liquid ^{3}He is characterized by a strongly repulsive hard core at small distances (less than 2 A^o) and an attractive Van der Waals interaction at larger distances as shown in the figure below.

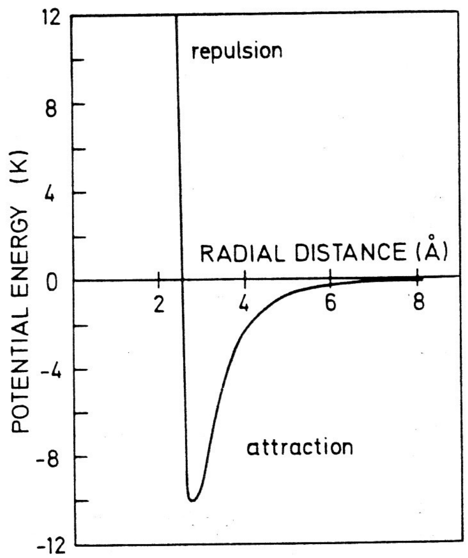

Fig 2 The interatomic potential in liquid ^{3}He (reproduced from [5]).

Shortly after BCS theory was developed [1], and applied successfully to the metallic superconductors, attempts towards applying the theory to other fermionic systems were initiated.The first thing that was noticed about ^{3}He was that the atoms could not possibly pair up in the *s*-wave state (zero angular momentum) since the hard core repulsion ensures that the wavefunction vanishes for small interatomic distances. One of the earliest works was done by Brueckner et. al. [7], who stated that the $l = 2$ state in ^{3}He can form a condensate at a temperature less than 0.07 K. Later, the work of Anderson et. al.[6], predicted an even lower upper bound of 0.02K. It was clear that ultra low temperatures were needed to be attained in order to observe condensation in ^{3}He.

At this point it would be worthwhile to mention few of the advances made in the experimental scene as well. In the late sixties and early seventies active experimental research was conducted on ^{3}He and its mixture with ^{4}He, along with important advances in refrigeration techniques, as Wheatley [3] points out. Dilution refrigeration was perhaps the most significant development. This technique enabled providing temperatures of as low as 10mK with further reduction possible by using other techniques. The other refrigeration techniques that were developed included nuclear refrigeration and adiabatic compressional cooling. It did not take too long for successful observation of superfluid condensation in ^{3}He to happen.

Experimental observation of superfluid condensation in ^{3}He

In 1972, Osheroff, Richardson and Lee [4a,b] reported some interesting observations regarding liquid ^{3}He at temperatures below 3 mK. In their first experiment [4a] they had Helium in a Pomeranchuk cell which is essentially a compressional cooling cell. Inside the cell they cooled ^{3}He under pressure so that they had the solid phase coexisting with the liquid phase. They observed two distinctive features on the pressure Vs time curve (also known as the pressurization curve) occuring below 3mK. They called these features A and B. At point A the rate of change of pressure with time (dP/dt) fell discontinuously by a factor of 1.8 at a temperature of about 2.65 mK. At point B another singularity was observed at a temperature of less than 2mK. This was initially attributed to nuclear spin effects in the solid phase until subsequent NMR experiments [4b] showed that the A and B features had to do with dynamic magnetic effects in the liquid phase.

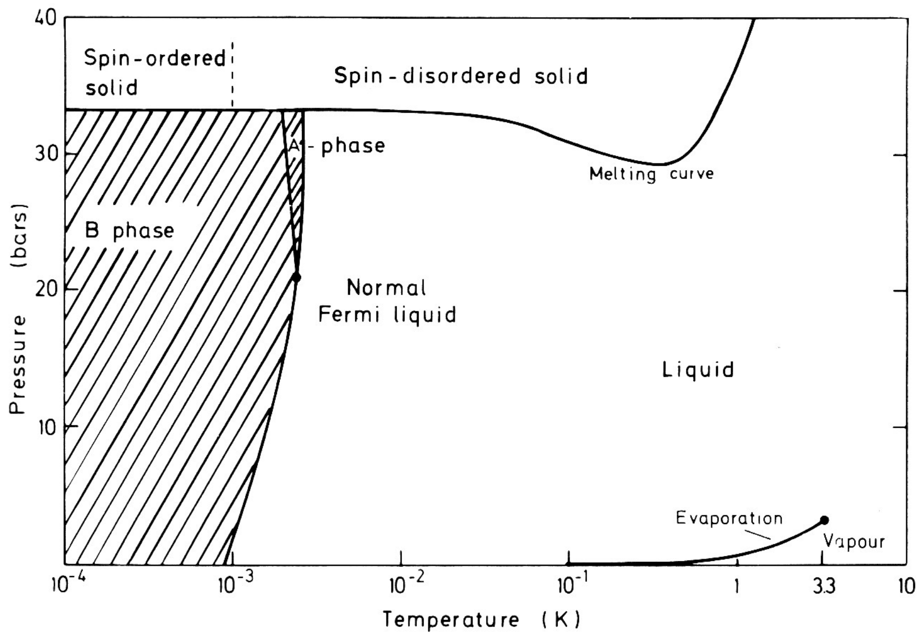

Fig 3 The phase diagram of ^{3}He drawn in logarithmic temperature scale and zero magnetic field.(reproduced from [5])

In the above figure the zero field phase diagram of ^{3}He is shown. The new phases A and B are the superfluid phases that are achieved at very low temperatures under pressure. The microscopic structure of these phases will be discussed in some detail later in this essay. In this phase diagram there are two transition lines that are of concern in the light of the present discussion. The first is the T_c that separates the normal and superfluid phases. As pointed out by Wheatley [3], the transition is second order. The second transition is the A to B transition which is first order .

We now look at few of the physical properties of liquid ^{3}He in the transition regime.

Specific heat capacity

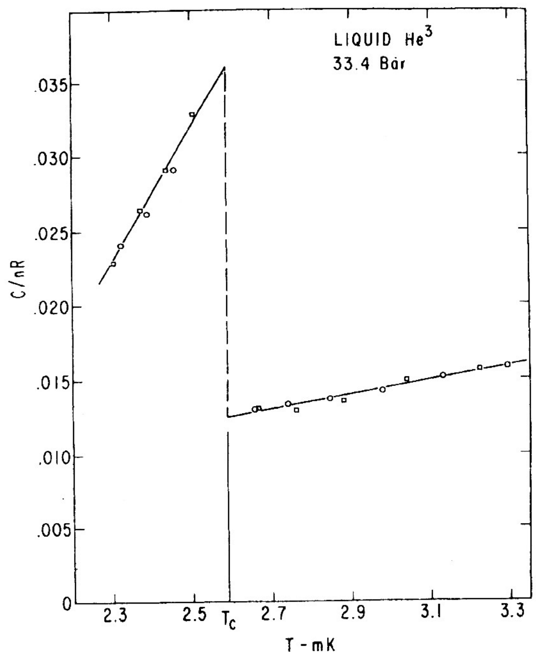

Fig 4 Molar specific heat of liquid ^{3}He relative to the gas constant R at 33.4 bar pressure near T_c (from [3])

The specific heat capacity of liquid ^{3}He is plotted around the second order normal to superfluid transition temperature. The discontinuity in the specific heat at the transition temperature is consistent with the weak coupling BCS expectation and is a signature of the energy gap formed below the transition. There is however a point of difference in the case of the anisotropic superfluid (A phase) compared to the BCS prediction. This difference lies in the behaviour of the specific heat in the zero temperature limit. In the BCS case the gap is uniform while in the anisotropic superfluid it may have nodes along certain directions in momentum space. This in turn causes the specific heat to fall off as a power law rather than the characteristic exponential behaviour observed in BCS as pointed out by Leggett [2].

Effect of applied magnetic field on the phase diagram

Since ^{3}He has a nuclear spin of ½ it is affected by a magnetic field unlike ^{4}He. The most striking effect of magnetic field is in the splitting of the phase boundary between normal and superfluid components. In the presence of a magnetic field the T_c line splits into two transitions with temperatures T_{c1} and T_{c2} , the difference between these temperatures being proportional to the applied magnetic field [2].

Structure of the A and B phases

In this section we will be discussing about the structure of the two principal superfluid phases of ^{3}He and the nature of order in these phases. We shall start by briefly discussing about a very important experimental fact regarding the two phases. [2]

The spin susceptibility of the A phase is temperature independent and is close to its value in the normal state . In the B phase susceptibility approaches a finite value as the temperature approaches zero . This finite value is nearly one third of the normal state value. This contains very crucial information regarding the possible pairing symmetries in each of these phases. In a spin singlet paired superfluid the spin susceptibility is expected to vanish as T goes to zero, because in that limit all the atoms are bound into singlets and hence are insensitive to magnetic perturbations. Hence neither of these phases is a spin singlet.

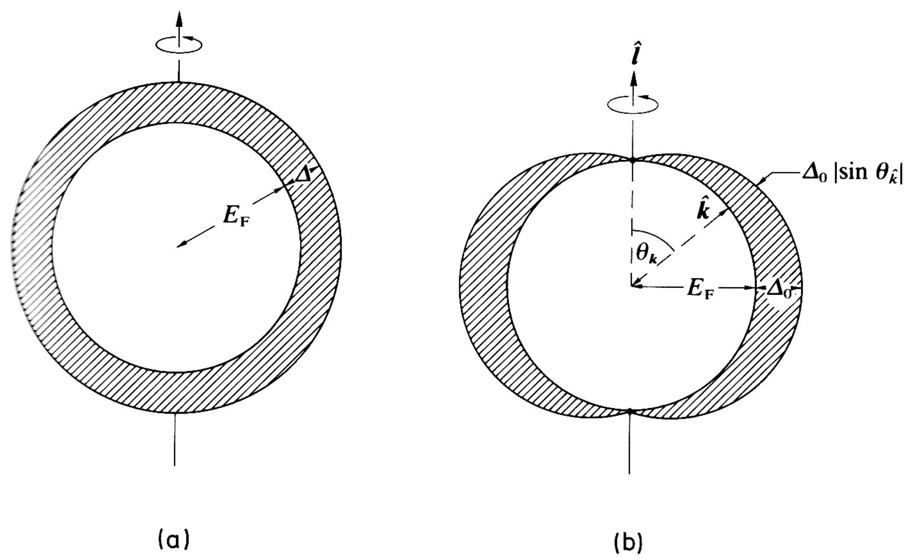

Fig 5 Schematic diagram of the energy gap of a)the BW (B phase) and b)the ABM(A phase). Notice that the gap vanishes along l direction in the ABM phase.(from [5])

The BW phase

We start out by describing the "isotropic " BW phase first. Balian and Werthamer [8] showed using a variational scheme that in case of BCS like pairing in the p-wave case (in zero field) the ground state is a superposition of all the three S_z channels (1, 0 and -1) unlike the ABM phase that we are yet to discuss.The BW phase has the following essential features.

1. The single particle spectrum (and the energy gap)is isotropic (like the BCS singlet case)
2. The total orbital and spin angular momenta (S and L) are each equal to 1

3. The spin susceptibility is finite at zero temperature. (unlike BCS)

It is because of its "apparent" isotropy that the BW state shares quite a few similarities with the BCS state. For instance it is impossible to distinguish between the two states only with the aid of themodynamic data.

The ABM phase

As we learn in the previous section, in the weak coupling BCS limit in the absence of external magnetic field, the ground state of the l =1 super fluid is the BW state. Experiments however reveal that the BW state is not the unique superfluid state in such a case. [2]. The fact that the BW state is not the universal ground state in zero field has to do with the breakdown of the BCS weak coupling assumption for high pressures. The other type of ground state that appears in superfluid liquid ^{3}He is the Anderson, Brinkman, Morel (ABM) phase.

Anderson and Morel [6] hypothesized that l = 1 Cooper pairing may be possible by pairing only same spin atoms together. Thus the S_z = 0 channel was completely left out in such a state. It is precisely because of this reason that the spin susceptibility of the ABM phase is very close to the normal state susceptibility value and is almost independent of temperature. The ABM is highly anisotropic with the gap having two nodes (along directions parallel and antiparallel to the angular momentum l). As mentioned earlier, the low temperature specific heat in the ABM phase goes down as a power law.

Symmetry breaking in the ABM phase

The NMR data of Osheroff et. al. [4b] had some very intriguing features the highlights of which are as follows. In a sample consisting of a mixture of solid and liquid ^{3}He, the part of the NMR signal attributed to the liquid phase starts shifting to higher values at the 'A ' transition (2.65 mK), while the 'solid' signal remained fixed. The spin susceptibility remains close to the normal state value. Then on hitting the point 'B ', the signal abruptly jumps back to the original value while the spin susceptibility drops by nearly a factor of 2.In addition, the difference in the squares of the solid and liquid signal frequencies was independent of external field and only dependent on pressure (or temperature since the experiment was performed along the freezing curve.

Leggett [9], showed that the intermediate phase lying in between points A and B is an ordered state in which the spin and relative orbital angular momentum directions are strongly correlated which may be alternatively termed as spontanous breakdown of spin orbit symmetry. It now remained to identify this state with the l = 1 ABM model state. From the NMR data , namely from the fact that the difference of squares of the signal frequency of the solid and liquid signal is finite and field independent, he showed that the relative angular momentum of each pair must be odd (spin triplet). However from the susceptibility data it is clear that Sz = 0 states are hardly present. It turned out that the l = 1 angular momentum ABM state was the best candidate for the anomalous phase. Thus the theory for the anisotropic phase of liquid ^{3}He was established.

Symmetry of the order parameter for ^{3}He

The symmetry group of the ^{3}He order parameter is a composed of three different symmetries namely rotation about spin axes, rotation about spatial axes and particle number conservation. The overall symmetry group is thus $SO(3)_S$ X $SO(3)_L$ X $U(1)_N$

Conclusions and overview

In this essay we reviewed some general aspects of the super fluid phases of ^{3}He .We started out by briefly comparing the two Helium isotopes and emphasized on the fact that observing superfluid Cooper pairing in ^{3}He had remained an elusive dream for a long time.

We then started to focus on the normal state properties of ^{3}He , that its description within Fermi liquid theory is satisfactory. We also looked at its interatomic potential and discussed why only s -wave pairing could possibly take place and the fact that the weakly attractive Van der Waals force would have to be responsible for the pairing. We also reviewed very briefly the rudiments of BCS theory.

Next, we picked up pioneering experimental results wherein claims of observation of the elusive superfluid condensation were made. We discussed briefly some of the experimental results including a very brief discussion of the effect of magnetic field on the phase diagram.(this essay really doesn't do any justice to magnetic field induced behaviour of ^{3}He .)

The remainder of the essay focused on the description of the two superfluid phases namely the ABM and the BW phase. The theoretical description has been very non technical. Lastly we looked at Leggett's theory of the anisotropic superfluidity in ^{3}He and the beautiful phenomenon of the breakdown of spin orbit symmetry in the ABM phase.

Some of the important things that have been left out are discussion of superflow/josephson effect, the phenomenon of spin fluctuation which apparently justifies the use of ABM like states and, as mentioned before, the detailed physics of magnetic field induced phenomena in ^{3}He .

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