Three phase traffic theory

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Abstract

In the three phase theory, the traffic phenomena are explained by three phase theory: free flow, synchronized flow, and moving jam. Here, this paper introduces the concept of each phase. Moreover, the paper explains the process of the phase transition. The three phase traffic theory offers qualitative explanation of real traffic.

1.Introduction

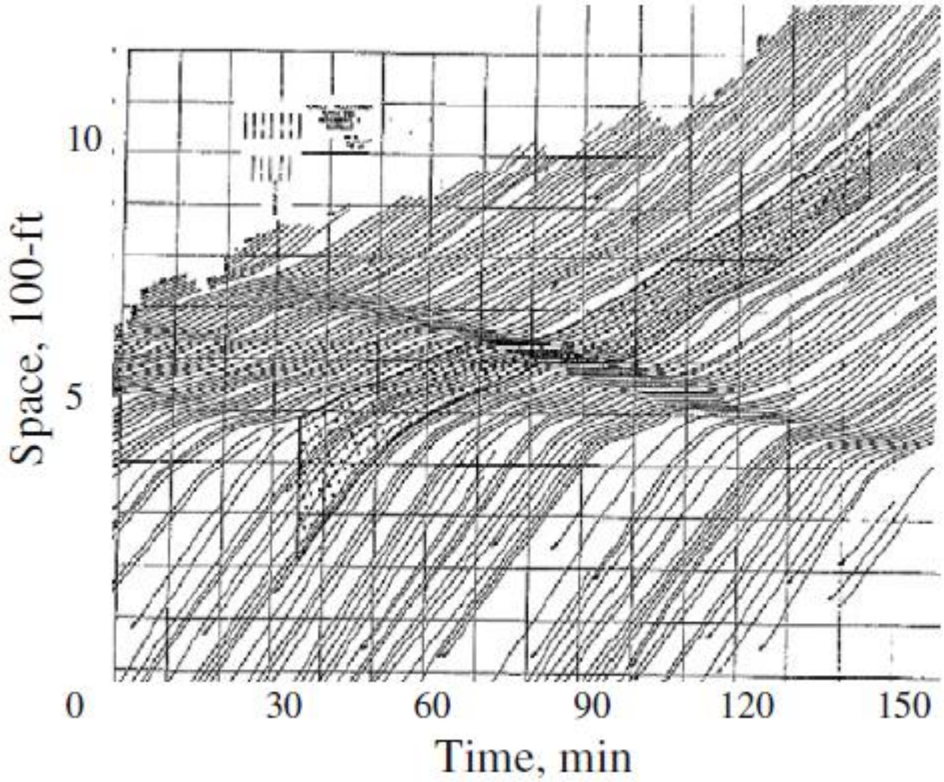

Fig.1 Space time diagram depicting traffic congestion. Each line has horizontal slope interval meaning congestion. The congestion pattern propagates backward over time.[4]

Traffic is a complex system having similarities with physical particle flow. For example, successful applications of physics to traffic involve hydrodynamic model of traffic flow. One can treat a single vehicle like a particle flowing through pipe. The compatibility of physics approach to the traffic theory attracted many physicists for decades. As many other collective physical systems exhibit emergent state, congestion emerges in traffic. Three phase traffic theory developed by Russian physicist Boris Kerner explains the congestion by the phase transition in traffic system.

In the three phases traffic theory, the three phases in traffic are consist of free flow and two congestion phases: synchronized flow and wide moving jam. The three phases offer qualitative features of traffic congestion phenomena[1]. However, because of the complexity of traffic system, such as inhomogeneity among vehicles and behavior of drivers, the theory is limited to provide quantitative correspondence with reality although recent simulations have been able to reproduce qualitative features. For this reason, this paper emphasizes the discussion on empirical data obtained from free highway in order to maximize the consistency of the theory, as the research by Kerner.

In traffic theory, a large set of parameters exist(e.g. safety gap between vehicle, average vehicle length, time delay for acceleration). Among these, it is sufficient to provide only three parameters for understanding the empirical features of three phases: Velocity v, density ρ and flow rate q of vehicles. Obviously, these are not independent each other. They are related by simple intuitive expression.

$$q = \rho v \tag{1.1}$$

That is, as long as the two values are given, the another one can be ignored. Nevertheless, the correlation of the full three variable are important to distinguish the three traffic phases.

2.Free flow and Congestion

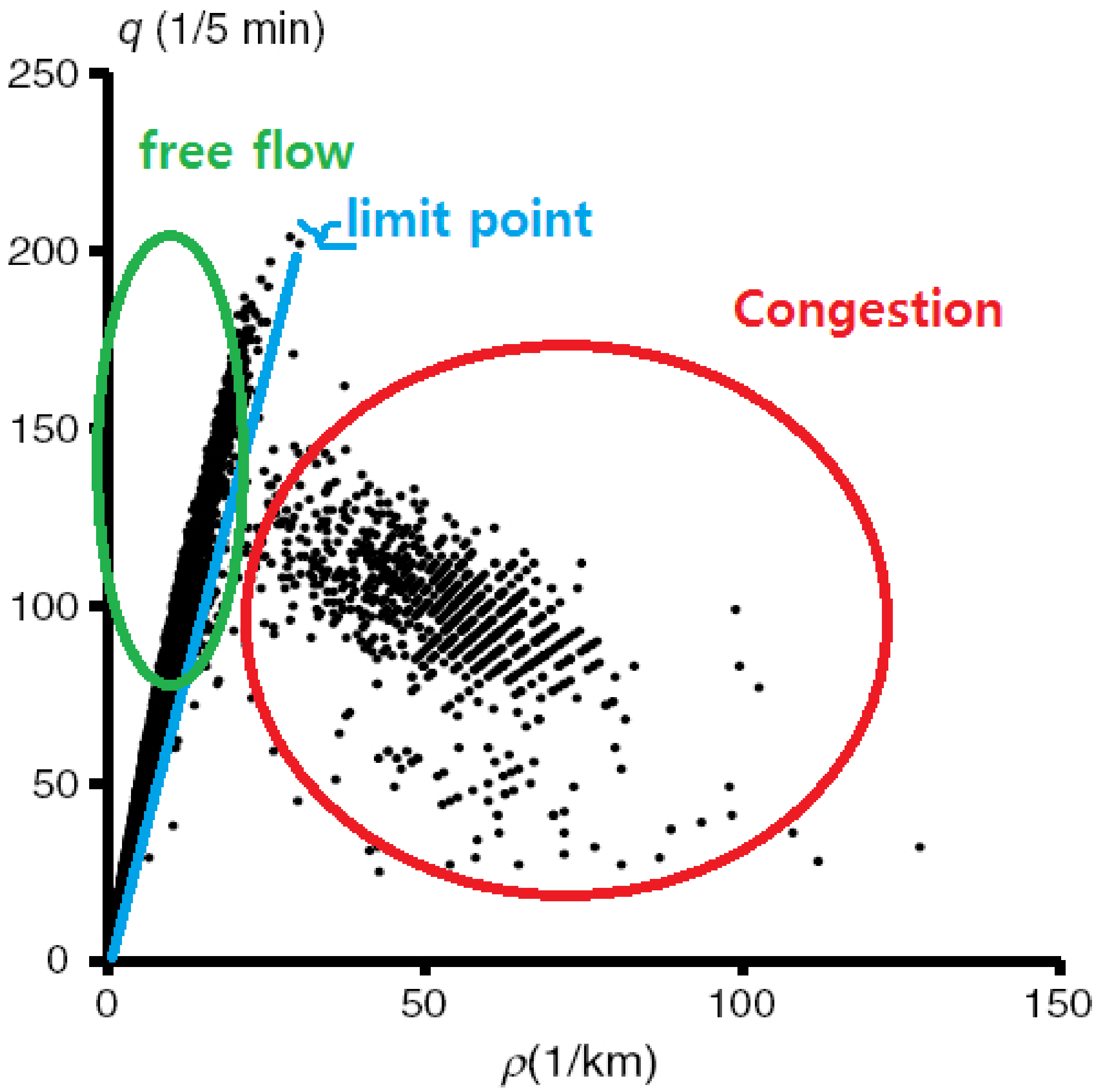

Fig2. Fundamental diagram. The data measured in Japan highway. notations are added in this paper[5]

the free flow and the congestion is more intuitive to distinguish than the three phases. The free flow and the congestion can be easily defined by the fundamental diagram in density-flow rate plane. In figure 2, the points are divided by two regions. one with positive slope line corresponds to the free flow. In free flow since there is no significant speed drop, the flow rate is nearly proportional to the density(its slope tends to decrease as the density increases). however, when the density reaches the maximum density for the free flow, the transition to the congestion must occur. this maximum point in the fundamental diagram is called limit point. Another set of points corresponds to the traffic congestion states. the congestion mostly occur at the bottleneck. In the congestion, the average speed of vehicles drops. Moreover the variance of the points in congestion states are much larger than free flow state. The synchronized flow are the source of variance(i.e. this large variance set of points are in the synchronized flow state, while the wide moving jam states has linear relationship called J line.[7]

3.Three phase theory

3.1 Synchronized flow

The three phase traffic theory divides the congested states by the two; synchronized flow and wide moving jam. after the congestion transition occur, free flow change to synchronized flow. In the synchronized flow, the speed of vehicles drops significantly, but there is no noticeable change in the flow rate is observed[7]. this is due to the increase in the density of vehicle so that the product of the speed and the density remains nearly the same. the term synchronized reflects the synchronization of speed of the vehicles in different lanes. The downstream front is mostly fixed at the bottleneck. The synchronized flow can be divided by three patterns according to the evolution of the downstream front and the upstream front.[1]

(1) localized synchronized flow : the downstream front is fixed at the bottleneck. the upstream front oscillates but the mean width of pattern do not change.

(2) widening synchronized flow : the downstream front is fixed at the bottleneck, the upstream front continuously propagate backward.

(3) moving synchronized flow : a whole pattern propagates however it cannot penetrate next adjacent bottleneck(catch effect). the catch effect distinguish the moving synchronized flow with wide moving jam
[3]

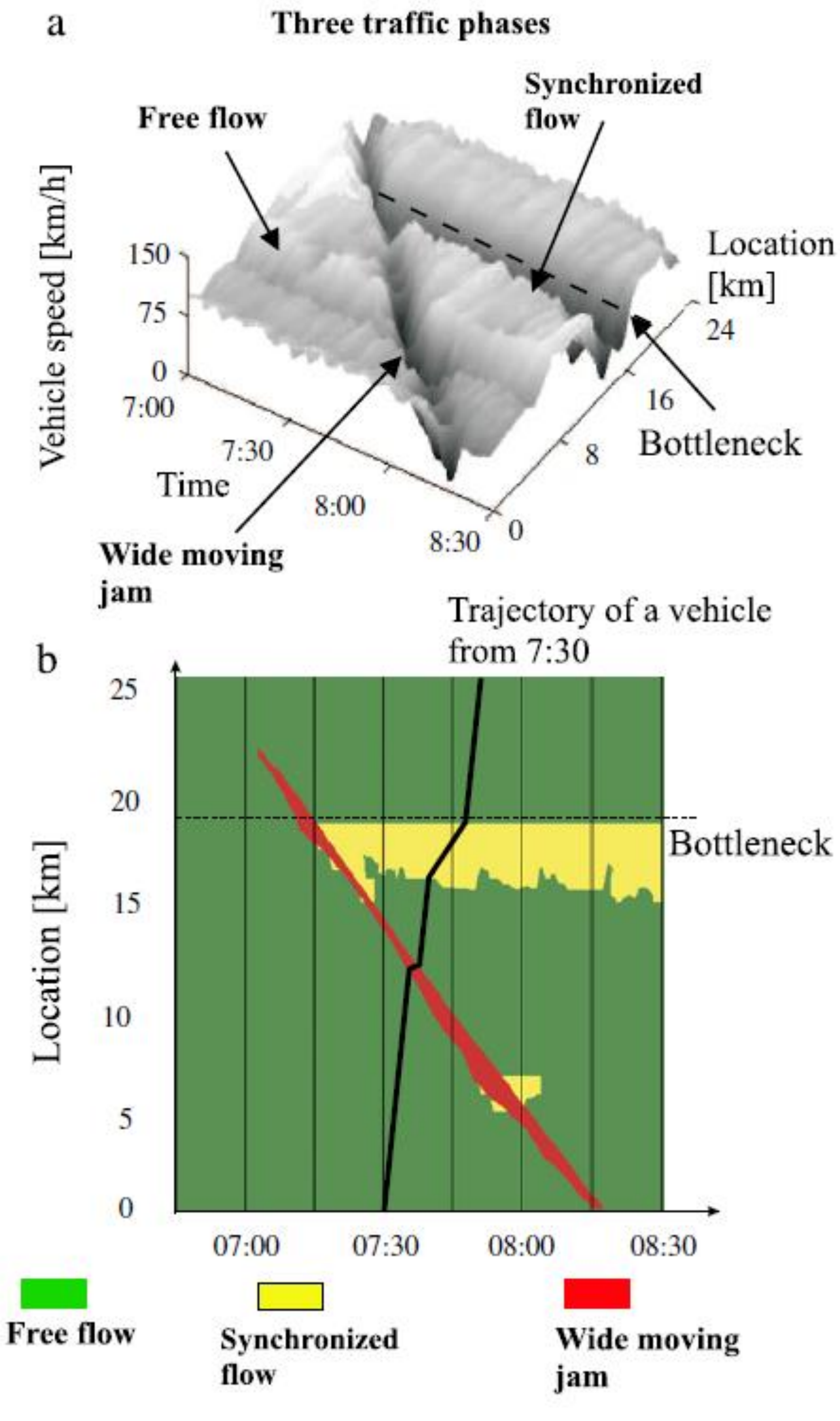

Fig3. Representation of three phases in space-time plane. synchronized flow is fixed over time in the front of bottleneck while the wide moving jam propagates to upstream[1]

3.2 Wide moving jam

The wide moving jam can only spontaneously occur through synchronized flow. At this stage, both flow rate and velocity drops significantly, and they become relatively uniform than synchronized flow. Therefore, empirically, large variance in the density-flow rate plane is consist of synchronized flow. wide moving jam propagates backward with mean velocity vg. it can pass through next bottle neck. The line formed by points in the density-flow rate plane is called J line. The slope of J line corresponds to the pattern propagation speed vg[1]

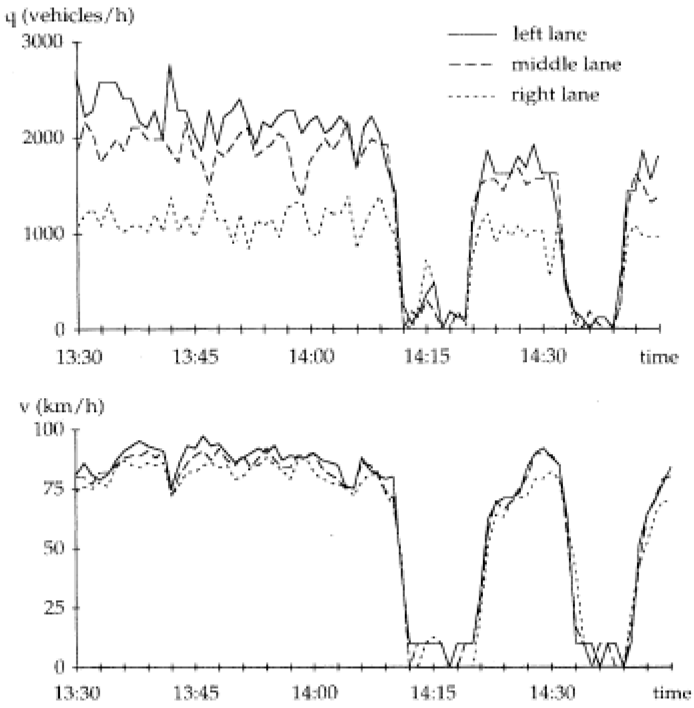

Fig 4. parameters at fixed point. Rapid drop corresponds to the wide moving jam. Both flow rate and speed drop significantly at the moving jam[2]

Table1. summary of three parameters in each phase

	Speed	Flow rate	Density
Free flow	High	High	Low
Synchronized flow	Low	Close to free flow	Middle
Wide moving jam	ignorable	ignorable	Highest

3.3 F->S transition

The speed drop occur at the transition from free flow to congested state(synchronized flow) is called breakdown phenomenon. In figure 2, after the density reaches limit point(maximum density), the transition from free flow to synchronized flow occur. This is not the whole scenario in real situation. Random fluctuation plays a role for the transition in real case. deceleration of vehicle, lane changing, any random behavior of drivers can cause this fluctuation. small enough fluctuation will decay over time. This is called metastablility of the free flow. However if the fluctuation exceeds critical value, it will grow over time and the transition from the free flow to congested state will occur. Therefore, the transition has probabilistic nature. At bottleneck, the probability of fluctuation (deceleration, lane changing) grows, so the transition mostly occur at bottleneck. [3]

Density velocity diagram depicted in figure 5 is Z shaped line. The first free flow branch has negative slope as the slope in the density-flow rate(speed) decrease. The dashed line is the critical transition speed which divide the free flow and the congestion phase. Since the transition must occur at ρmax, the dashed line meet with free flow branch. With the similar reasoning, the dashed line must meet with synchronized flow branch at ρth. [3]

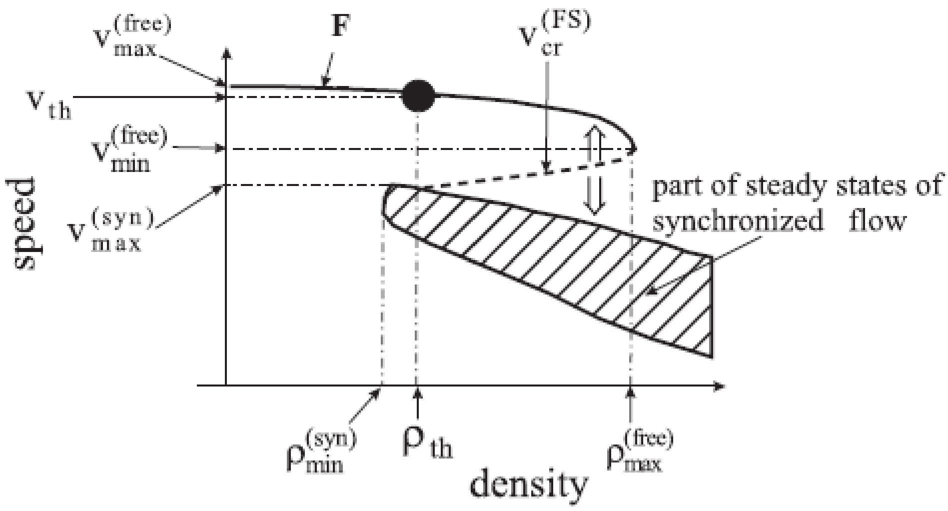

Fig 5. Z shaped speed-density line. dashed line means critical speed. i.e. if the speed gets lower than critical speed, the transition from free flow to synchronized flow occur[3]

3.4 S->J transition

While other traffic theories had contradiction by implying the existence of emergence of wide moving jam from free flow, which is not observed in real traffic, The three phase theory resolved that direct spontaneous transition from free flow to wide moving jam is not possible[1]. In the three phase theory, the critical fluctuation amplitude for transition to wide moving jam is much lower than the critical fluctuation amplitude for transition to synchronized flow, so the probabilistic nature disturbs the direct emergence of the wide moving jam from the synchronized flow.[3]

while the F->S transition has z shape the density-speed plane, the S->J transition has double z shape. In the wide moving jam branch, the average velocity is assumed to be zero, so dashed critical velocity line start from zero, and meet synchronized branch at its limit velocity since the transition from S to J must occur.[3]

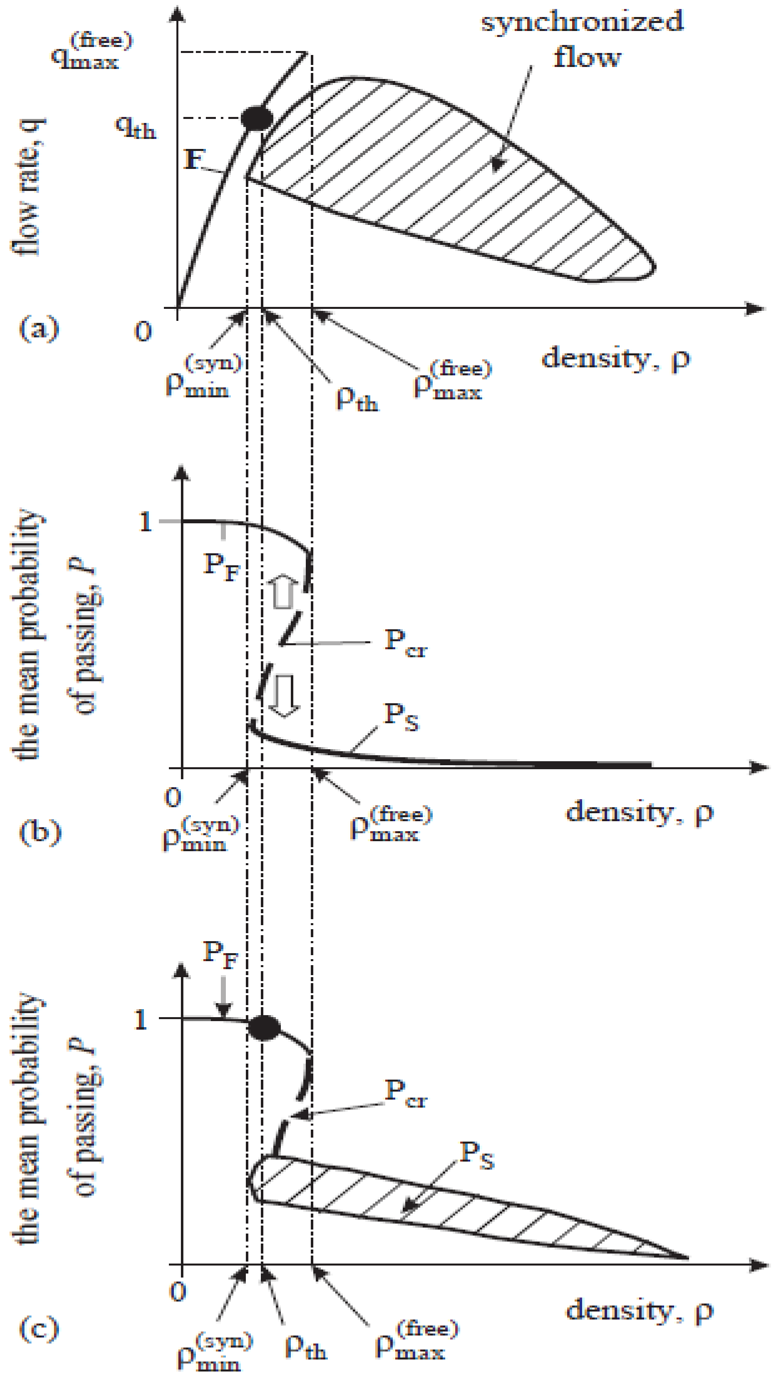

Fig 6. Probability of passing also have Z shape line by the same reason with Fig.5. Here, passing means that a driver pass using other lane on multilane highway when approaching the preceding vehicle. In congested flow, the probability decrease as density increase since lanes are more likely to be occupied[3]

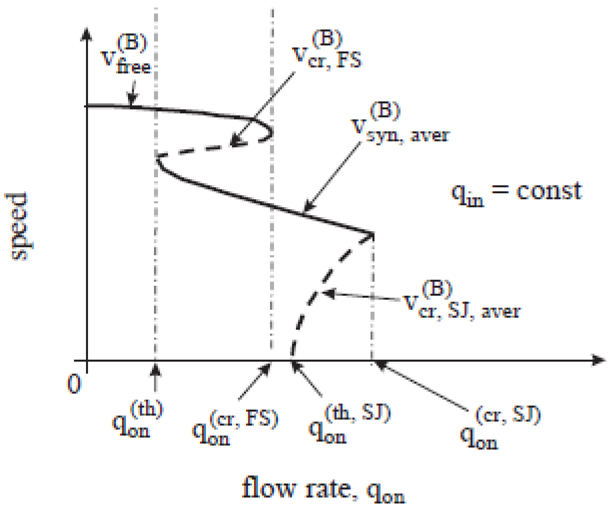

Fig7. Double Z shaped line for transition to wide moving jam. [3]

4. Analysis

One can demand conservation law of vehicle number. since the free high way is one dimensional coordinate.

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{4.1}$$

if there exist propagating front(boundary of different phases), in the coordinate system which moves along with the front, the front must be at rest. Therefore, the flow rates of front must be continuous[7].

$$\rho_+ u_+ = q_+ = q_- = \rho_- u_- \tag{4.2}$$

where

$$u_+ = v_+ - v_g, u_- = v_- - v_g \tag{4.3}$$

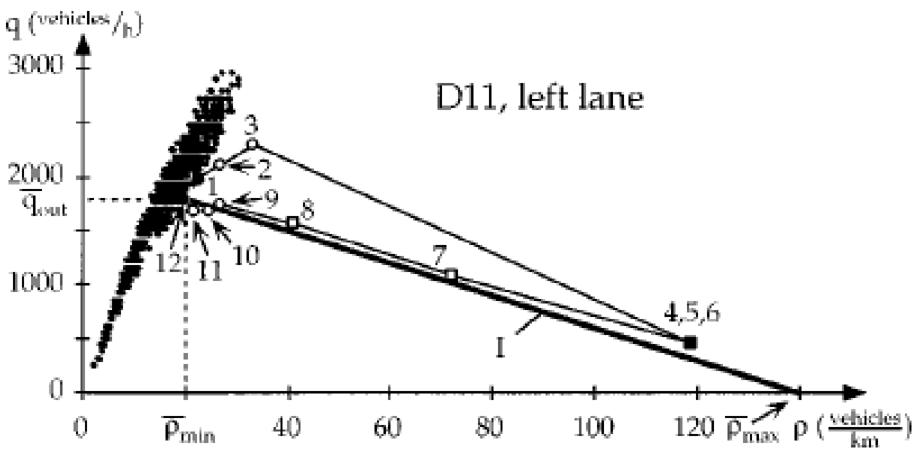

Fig 8. figure of J line.[6]

vg is propagation speed. by solving for vg, shock wave speed can be derived

$$v_g = -\frac{q_{out}}{\rho_{max}-\rho_{min}} \quad (4.4)$$

In the case of wide moving jam, the in-flow rate can be ignored. the maximum and minimum density is the extreme value given in J line.

The emergence of moving jam can be maintained only when vg(downstream)<vg(upstream) otherwise the width of jam will decay over time and vanish. from shock wave formula upstream front speed can be obtained

$$v_{g(upstream)} = -\frac{q_h}{\rho_{max}-\rho_h} \quad (4.5)$$

From 4.4 and 4.5, since vg(upstream)>vg we can conclude that the critical flow rate and the density is $\rho h = \rho min, qh = qout$.[7]

One can also derive the equation of J line. The propagation of wide moving jam is caused by time delay for a driver to accelerate after his preceding driver's acceleration. Therefore, propagation speed can be written by

$$v_g = -\frac{1}{\rho_{max}\tau_{delay}} \quad (4.6)$$

by combining this with vg

$$q_{out} = \frac{1}{\tau_{del}}\left(1-\frac{\rho_{min}}{\rho_{max}}\right) \quad (4.7)$$

since qout occur at ρmin, we can simply write the equation of J line

$$q = \frac{1}{\tau_{del}}\left(1-\frac{\rho}{\rho_{max}}\right) \quad (4.8)$$

by saying that ρmin is low enough to ignore,

$$q_{out} = \frac{1}{\tau_{del}} \quad (4.9)$$

[7]

5. Conclusion

In this paper, we showed the behavior of the three parameters(flow rate,density,speed) in traffic. The paper also explained how the random fluctuation lead to the phase transition. Three phase traffic theory offers rich qualitative argument. the three phases are well defines by its special features. each phase can be well distinguished from each other. empirical data supports the characteristics of each phase, even though quantitative information is limited.

Reference

[1]Boris S. Kerner, et al. *Physica A* **392** (2013) 221-251
[2] B.S. Kerner, H. Rehborn: *Phys. Rev. E* **53** R 1297-R1300 (1996)
[3] B.S. Kerner: *Physica A* **333**, 379-440 (2004)
[4] Treiterer, J., Myers, J.A., 1974. The hysteresis phenomenon in traffic flow. In: Buckley, D.J. (Ed.), 6th Int. Symp. on Transportation and Traffic Theory. A.H. and A.W. Reed, London, pp. 13–38.
[5]Yuki Sugiyama et al. *New J. Phys* **10** (2008) 033001
[6]B. S. Kerner and H. Rehborn: *Phys. Rev. E* **53** R4275-4278 (1996)
[7]B. S. Kerner, S.L. The Physics of Traffic Springer (2004)