The Emergence of Oscillons in Granular Media

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Abstract

A vibrating plate of granular material can exhibit unexpected emergent behavior, including the formation of oscillons. Oscillons are stable, localized, particle-like clumps of vibrating material that are capable of drifting across the plate as well as attracting and repelling other oscillons. This paper details the characteristics of these strange particles and the specific circumstances required for their formation.

1 Introduction: Shaking Sand

Granular materials, such as sand, may seem too commonplace to be of much interest to modern physics. They fill beaches, slip through hourglasses, erode mountains, and trigger avalanches. The flow of granular materials plays a role in industries from mining to agriculture to construction.[1] Yet granular flow remains poorly understood.[2]

Surprising, considering that interactions between grains appear far less complex than most interacting systems. The flow of granular material generally obeys the following.[1]

1. Grains are noncohesive, with only repulsive interactions
2. The flow is independent of temperature
3. Grain interaction is dissipative

The last property results from grains colliding inelastically and experiencing static friction. This means that to stay in motion, the grains must be driven externally. In the case considered here, a layer of grains lies on a horizontal plate in vacuum and is vibrated in the vertical direction. Depending on the amplitude and frequency of the vibrations, standing waves will emerge. These are known as Faraday waves and they can take the form of stripes, squares, or hexagons.[2] But in special situations, these give way to a different phenomenon: oscillons.

1.1 What are Oscillons?

Oscillons are a particle-like excitation arising from the cooperative behavior of colliding grains. They are small, only the width of about 30 grains.[2] Unlike waves, they do not propagate, but they can drift across the plate. A flat layer of grains cannot produce oscillons without local perturbation. Reducing the driving frequency from a state with standing wave patterns can also create them.

Oscillons were given their name by Paul Umbanhowar and Harry Swinney at the University of Texas-Austin and Francisco Melo at the Universidad de Santiago in Chile. The name is apt: the most interesting behavior of these excitations is directly due to the fact that they oscillate at half the frequency of the driving force. (Figure 1 shows oscillons in action.) This condition allows oscillons of opposite phase to exist simultaneously. And when two oscillons of opposite phase come close together, they attract each other. Likewise, two oscillons of the same phase will repel.

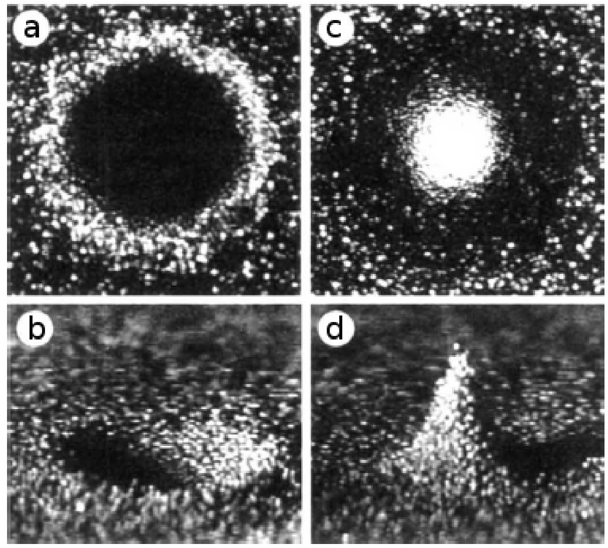

Figure 1: *Oscillons of opposite phase.* During one complete oscillation of the plate, an oscillon exists as a crater, as shown from above in (a) and from an angle in (b). During the next oscillation of the plate, the oscillon is peaked, as shown from above in (c) and from an angle in (d). Image reproduced from [2].

Oscillon attraction can lead to rich complex phenomenon: the formation of tetramers, polymeric chains, and lattice structures. These are only possible because oscillons are relatively stable, existing for more than 5×10^5 container excitations.[2]

1.2 Oscillons in Theory and Experiment

Oscillons were discovered in 1996 by Umbanhower, Melo, and Swinney[2]. Their behavior has since been studied in experiments as well as modelled theoretically[3, 4, 5, 6]. In this paper, I will highlight experimental results that both reveal the characteristic behavior of oscillons, and hint at a theoretical picture to describe them.

Then I will present an overview of the theoretical work of Crawford and Rieke[5], who developed a two-dimensional Swift-Hohenberg model that produces oscillon solutions. Their model illustrates a key insight: that oscillons are *not* dependent on granular media itself. Rather, oscillon solutions are a general result for *any* media that produces square patterned states that undergo large transition hysteresis[7, 5].

2 Experimental Observations

2.1 Conditions for Oscillon Production

The three factors that mediate oscillon production are the depth of the granular material, the plate's vibrational frequency, f, and a factor called the *dimensionless acceleration amplitude*, labelled Γ [2]. For a driving force that results in displacement $z = A \sin 2\pi f t$, the dimensionless acceleration amplitude is given by $\Gamma = 4\pi^2 A f^2 g^{-1}$, where A is the displacement amplitude and g is the acceleration due to gravity. When $\Gamma < 1$, the sand does not leave the plate, but for $\Gamma > 1$, the sand undergoes free flight.[2]

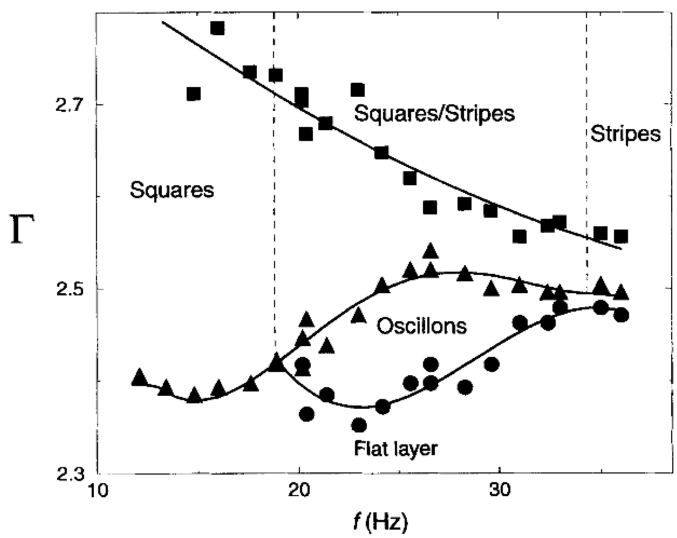

Figure 2: *The phase diagram for a vibrating plane of sand.* The squares indicate increasing Γ and the triangles and circles indicate decreasing Γ. Hysteresis is clearly visible for the transition from no pattern to the square or stripe pattern, but is much less pronounced for stripes. Image reproduced from [2].

Free flight is necessary for oscillon production. When the sand collides with the plate, the oscillon changes from a peak to a crater and *vice versa*. Umbanhower, *et al.* experimented with sand to determine the phase diagram of a vibrating plate [2]. They found that a stable oscillon region forms when the sand is more than 13 grain layers deep, and expands in f as the depth increases. The sand does not enter a pattern forming region below $\Gamma \approx 2.5$. Umbanhower, *et al.* created oscillons by lowering Γ below this value within the necessary frequency range. A phase diagram showing the stable oscillon region is shown in Figure 2.

The diameter of oscillons varies with Γ. The number that appear, however, varies randomly, with 0 to 50 oscillons produced each time Γ is lowered across the oscillon phase threshhold. They show no preference for where they appeared on the plane.[2].

2.2 Oscillon Molecules

When multiple oscillons move within close range, a distance of about 1.4 diameters[2], they interact. Two oscillons of opposite phase will attract. One might expect them to annihilate one another, but instead they retain their phase and shape, forming a stable dipole. Multiple oscillons will form tetramers, chains, and lattice structures (Figure 4). The chains are closely linked, with oscillons sitting only about a radius apart from each other. This means that they feel repulsion from their next-to-nearest neighbor. As a result, they form chains in straight lines. Umbanhowar *et al.* suggest that these interactions might be mediated through the chain.

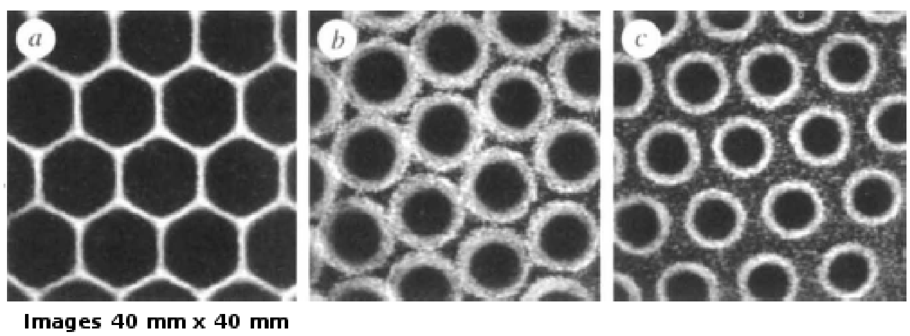

Figure 3: *Oscillon formation from a standing wave state.* The sand is driven with two-frequency plate vibration, a quarter wavelength out of phase. One frequency is at $f_1 = 26$ Hz and the other at $f_2 = 13$ Hz. The ratio between the driving amplitudes is $A_2/A_1 = 0.04$. A hexagonal standing wave emerges at $\Gamma = 2.70$, shown in (a). The pattern begins to separate into connected cells in (b), when $\Gamma = 2.50$, the oscillon threshhold. Below the threshhold, at $\Gamma = 2.45$, the cells separate into individual oscillons that are all in phase, as shown in (c). Image reproduced from [2].

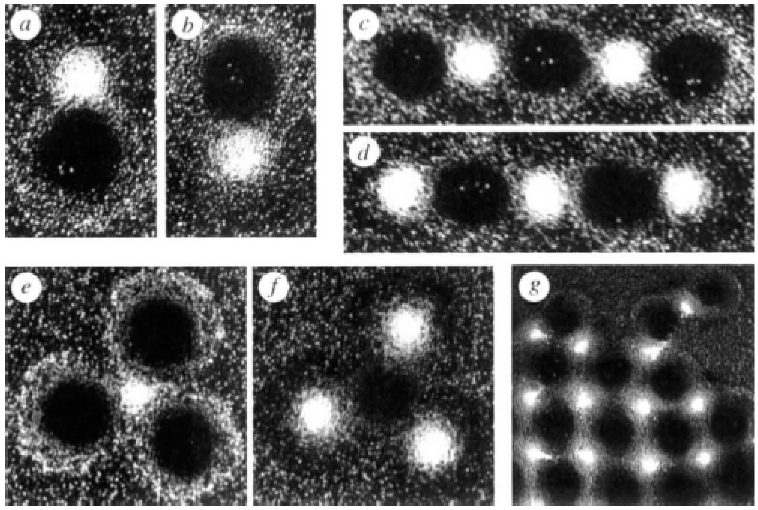

Figure 4: *Chains of oscillons.* Pictures (a) and (b) show an oscillon dipole in its two phases. The light portion indicates a peak, the dark portion a crater. Pictures (c) and (d) show a chain of oscillons. Notice how the chains form straight lines. Pictures (e) and (f) show a tetramer. Picture (g) shows a crystal lattice, with some oscillons breaking free. Image reproduced from [2].

3 Theoretical Work

Many different models have been shown to produce oscillon solutions. Tsimring and Aranson[3] developed a Ginsberg-Landau equation for wave amplitude that is coupled to the local height of the granular layer. Their model assumes small wave amplitudes and weak damping. Barashenkov, Alexeeva, and Zemlyanaya[6] used a model comprised of coupled, nonlinear oscillators to produce oscillon solutions in two dimensions, as well as show that there is no stable oscillon solution in three dimensions.

Crawford and Rieke[5] took a Swift-Hohenberg (S-H) approach. Unlike the amplitude in a Ginzburg-Landau model, the S-H order parameter ψ is allowed to vary quickly in space. The model relies on non-adiabatic effects, which couple slow and fast space dependence, to produce the oscillon state.

3.1 A Swift-Hohenberg Model

Let's consider the form a S-H equation must take in order to exhibit the behavior found experimentally. The fact that one oscillon can have a "positive phase" (as a peak) and another can simultaneously have a "negative phase" (as a crater) means that the S-H equation used to model them must remain unchanged under amplitude reflection ($\psi \to -\psi$). Thus only odd powers of ψ can be included.

Powers up to fifth order are required to see subcritical bifurcation, but more non-linearities must be included to form a stable square pattern state. Crawford and Rieke[5] used the following form for their Swift-Hohenberg model,

$$\partial_t \psi = R\psi - (\partial_x^2 + 1)^2\psi + b\psi^3 - c\psi^5 + e\nabla.[(\nabla\psi)^3] - \beta_1\psi(\nabla\psi)^2 - \beta_2\psi^2\nabla^2\psi. \quad (1)$$

The term with coefficient e introduces square patterns and the β-terms were added to further "tune" the system[5].

They applied an order parameter of the form $\psi = A\mathrm{e}^{ix} + B\mathrm{e}^{iy}$ + c.c. + higher order terms to find periodic solutions. Plugging this ψ into (1) gives two amplitude equations,

$$\partial_t A = RA - \beta|A|^2 A - \gamma|B|^2 A - \delta|A|^4 A - \rho(2|A|^2|B|^2 A - |B|^4 A) \quad (2)$$

and another in which the A's and B's are exchanged. Above, $\beta = -3b + 3e + \beta_1 - 3\beta_2$, $\gamma = -6b + 2e + 2\beta_1 - 6\beta_2$, $\delta = ((-3b - 9e - 11\beta_2)\beta_1 + 5\beta_1^2 + 480cd)/48d$, and $\rho = ((-3b - e - 7\beta_2)\beta_1 + 3\beta_1^2 + 120cd)/4d$.

3.2 Comparing Theory with Experiment

Crawford and Rieke[5] used numerical methods to find stable oscillon solutions to (1). The condition that the pattern state be unchanged under a sign change in the order parameter makes the square pattern state ideal for oscillon transitions. Hexagons, for example, break this symmetry and cannot transition into oscillons of both phases. They therefore focused on the condition $\beta < 0$, which is necessary for subcritical bifurcation to the square patterned state.

They argue that oscillons act as a small, localized patterned state, separated by interfacial fronts from the unpatterned state. These fronts are "pinned" by their interactions with the periodic pattern[8]. To test this idea, they varied the c parameter in (1), which controls how subcritical the square and stripe states are.

They found that as the square state becomes less subcritical, the R parameter must be increased to produce oscillons. R controls the radius of the oscillons. As R increases, the radius of the oscillons increase and the front locking weakens. Eventually, the oscillons give way to a periodic pattern. The locking should scale with R as $\mathrm{e}^{-\alpha/\sqrt{R}}$ according to [9]. Crawford and Rieke found that it did, with $\alpha \approx 1.8$.

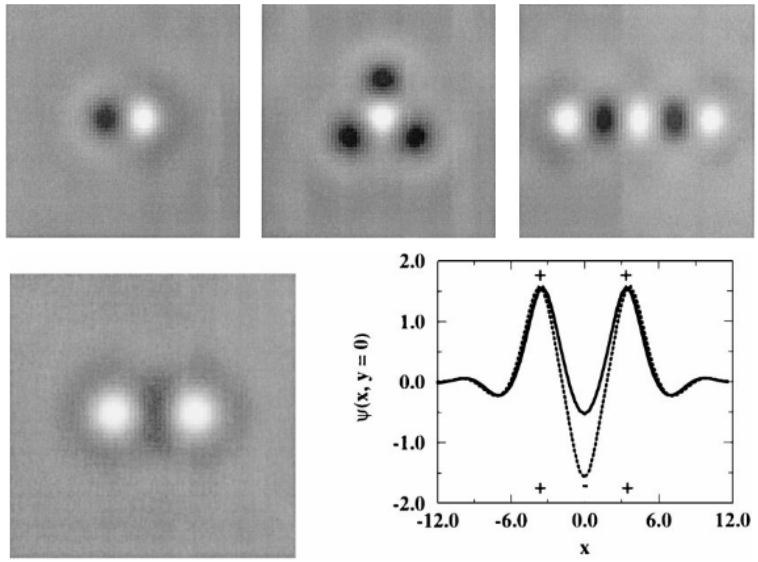

Figure 5: *Swift-Hohenberg model produces oscillon solutions.* Above, pictured from left to right: a dipole, a tetramer, and a chain of five oscillons. Compare with Figure 4. Below, a dimer of two oscillons in the same phase. This combination has not been found in experiments. The amplitude profile, $\psi(x)$, of the dimer is indicated with a solid line. Compare with that of the expected trimer, indicated with a dashed line. Image reproduced from [5].

This picture makes sense, given what we see experimentally. Each oscillon acts like an individual cell of the patterned state. In the oscillon regime, the cells are separated by the unpatterned state. But as we increase Γ, the cells grow until they converge into the patterned state.

The key thing to notice about this picture is that the pinning of fronts to form localized patterns is not exclusive to granular material. It happens in a class of fluid problems whenever there is structured pattern formation[8]. This suggests that oscillons could be created in fluids that exhibit pattern states. Indeed, they have since been observed in colloidal suspensions[7] and even Newtonian fluids[10].

3.3 Molecules in the S-H Model

The S-H model makes an interesting prediction about oscillon molecules not detected in experiments. For certain regions of c and R, a dimer pattern can form between oscillons of the same phase. In experiments, such pairs experienced a repulsive force[2].

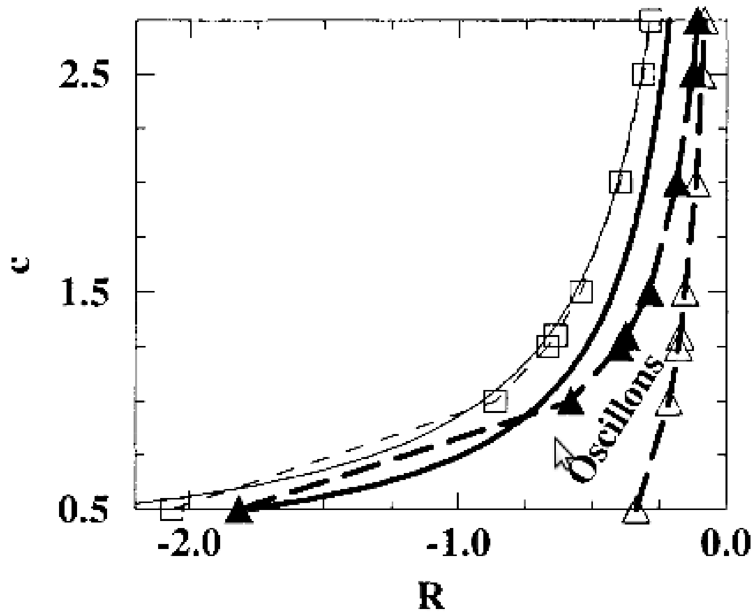

Figure 6: *Testing front pinning as a mechanism for oscillon formation.* The parameter c controls how subcritical the pattern state is, and R controls the radius of the oscillons. Image reproduced from [5].

Crawford and Rieke suggest that the reason these dimers have not been observed is that they can only exist for a small range of c values. Also, because the peak-to-peak distance between the two oscillons is larger than it would be for a traditional dipole, it is likely the binding energy is much weaker. When they added noise to the model, they found the dimers quickly decayed into normal chains[5].

4 Conclusions

Although first observed in granular material, oscillons have proven to be a general solution in media exhibiting Faraday waves, so long as two conditions are met. One, that the patterned state have a subcritical transition from the unpatterned state. Hysteresis in the transition from the patterned state appears critical to the formation of stable oscillons. Two, that the patterned state exhibit the amplitude equivariance ($\psi \to -\psi$) required for oscillons to coexist in opposing phases. This second condition rules out hexagon patterns, for example, from transitioning into oscillons of both phases.

Crawford and Rieke[5] show that a Swift-Hohenberg model can reproduce the transition from a square patterned state to an oscillon state and back again. They suggest that oscillons be considered individual cells of the patterned state that are separated from the unpatterned state by interfacial fronts.

One testable consequence of the S-H model is the existence of a state in which two oscillons of the same phase bind into a dimer molecule. The range of existence for such a state is small, and in a low frequency regime. Crawford and Rieke suggest accessing this state by starting with oscillons in a mid-frequency range and lowering the frequency.

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