

# Superfluid Helium 3

## Topological Defects as a Consequence of Broken Symmetry

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### Abstract

Superfluid phases in helium 3 were firstly discovered in 1972. They arise from the pairing of two fermionic helium 3 atoms to a composite boson, although there is a strong, short range repulsion between the atoms. These effects can be described using the Fermi liquid theory developed by Landau. But I will choose a different approach to the problem. Superfluidity is a macroscopic quantum phenomenon exhibiting spontaneous symmetry breaking. This characteristic can be understood by introducing an order parameter. Analyzing the structure of the order parameter you can explain the appearance of different superfluid phases. Furthermore, due to its complexity, superfluid helium 3 has a rich structure of topological defects. In the following I will give a short introduction explaining some general properties of superfluid  $^3\text{He}$ . After that its broken symmetry and the macroscopic order will be analyzed. Finally, using the concept of order parameter, I will investigate the rich structure of topological defects.

# 1 Introduction

Helium 3 and helium 4 are remarkable systems in condensed matter physics. Unlike all other fluids, they do not solidify if the pressure is below some 30 bar. The origin of this phenomenon is given by the uncertainty principle. Particles can never be at rest, but they have always a zero-point motion. Since the helium mass is very small and the attractive interaction between two atoms very weak, the helium atoms cannot form a lattice due to strong oscillations even at zero temperature. This characteristic makes helium unique among all liquids and allows us to observe quantum effects on a macroscopic scale. In  $^4\text{He}$  one observes a second-order phase transition into a superfluid state at around 2.2 K, in which the liquid can flow frictionless through narrow pores. In  $^3\text{He}$ , such a transition was not found for a long time. Finally, in 1972, Lee, Osheroff and Richardson found superfluid phases in  $^3\text{He}$  and in 1996 they received the Nobel Prize for this discovery. The transition temperature is around 2.7 mK. The striking differences in the behaviours of both helium isotopes is a consequence of quantum statistics.  $^4\text{He}$  is a Boson and therefore Bose-Einstein statistics apply. At low temperatures particles condense into the ground state and superfluidity is believed to be a manifestation for Bose-Einstein condensation. In contrast to that,  $^3\text{He}$  is a Fermi system, governed by the Pauli exclusion principle, which forbids a macroscopic occupation of the ground state. As shown by Leggett [1], the atoms in  $^3\text{He}$  form a pair, which has many similarities with the condensation of Cooper pairs in superconductivity. The main difference is the fact that in superconductors, the attractive interaction between the two electrons is due to the exchange of a phonon. Whereas in  $^3\text{He}$ , there are no phonons available. To overcome the strong repulsive interaction at short distances in this case, the Cooper pairs consisting of two  $^3\text{He}$  atoms must be in a state with non-zero angular momentum  $\mathbf{L}$ . Thus its wavefunction vanishes at the origin. Theoretically the pairing of two helium 3 atoms can be explained very good applying the Fermi liquid theory developed by Landau. It turns out that p-wave pairing is favourable and that there must be two distinct superfluid phases (Fig. 1), which were discovered 1972 as mentioned above. Those are referred to as A and B phase. Applying a magnetic field, another phase appears, the so-called  $A_1$  phase.

In the following I will choose another approach to explain the appearance of three superfluid phases in  $^3\text{He}$ . It is one of the many systems, where you can observe spontaneously broken symmetry. Obviously superfluidity, i.e. the formation of Cooper pairs, is a new state of order which appears when cooling under the transition temperature. This transition into an ordered state is continuous. You can introduce an order parameter which expresses quantitatively the state of order. The concept of symmetry breaking applies to many systems in condensed matter physics. The most common example is a ferromagnet, where a transition from paramagnetism to ferromagnetism takes place. Other examples are superconductors and superfluid helium 4. But this concept is not only restricted to condensed matter

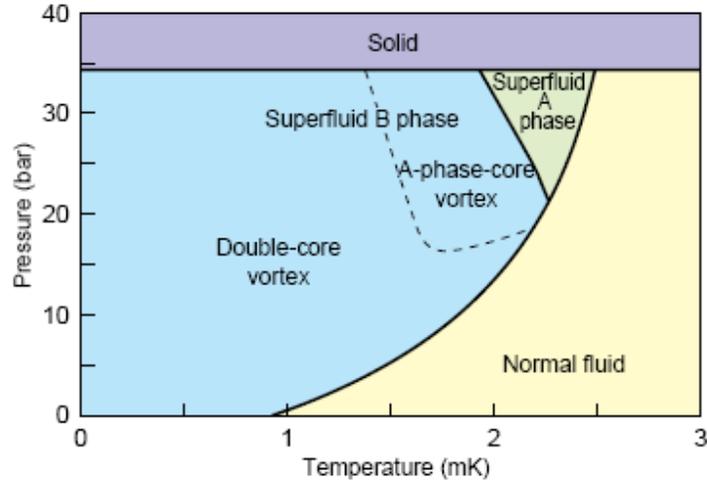


Figure 1: Phase diagram of  $^3\text{He}$  without external field. There are two superfluid phases A and B. In  $^3\text{He-B}$  a phase transition occurs represented by the dashed line.

physics. Also in particle physics, cosmology, the early universe and in neutron stars there are many examples of spontaneous broken symmetry.  $^3\text{He}$  turned out to be the ideal test system to understand this concept. As we will see later, the structure of  $^3\text{He}$  Cooper pairs has internal degrees of freedom. This is the origin for the appearance of remarkable phenomena in  $^3\text{He}$ , as for example topological defects. They are usually localised regions in which the superfluid density  $\rho_s$  vanishes. Thus there is no order in this particular region. Well known are the examples of vortex lines in  $^4\text{He}$ . But in addition to that, we see in  $^3\text{He}$  also defects without a normal core, planar defects and much more. In this paper I will first discuss the concept of spontaneous symmetry breaking in  $^3\text{He}$  and introduce the order parameter for the different superfluid phases. After that I will give an introduction into the vast field of topological defects in superfluid helium 3.

## 2 Spontaneous Broken Symmetry in $^3\text{He}$

As already mentioned, the superfluid phases of Helium 3 can be understood if you consider symmetry arguments. By cooling under the transition temperature  $T_c$ , there is a new state of order, because Cooper pairs have formed. To describe this property one introduces an order parameter, which is zero above  $T_c$  and non-zero below the transition. In order to gain a full understanding of all the aspects of symmetry breaking in  $^3\text{He}$ , you have to apply homotopy group theory. But here I will just present the main outline and results which can be understood heuristically.

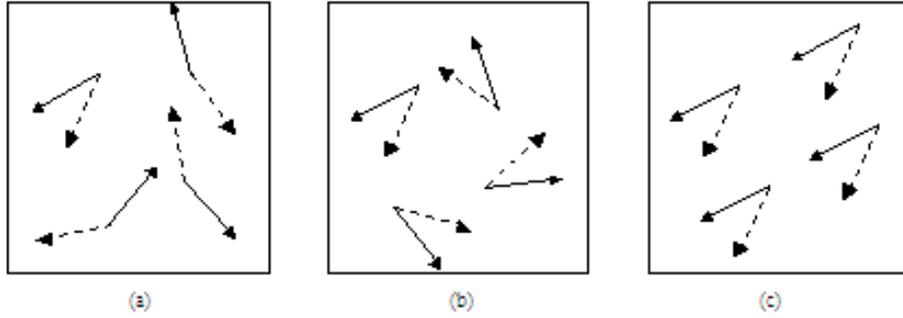


Figure 2: Illustration of a two-dimensional model of  ${}^3\text{He}$ : (a) isotropic paramagnetic liquid, (b) B-phase, (c) A-phase

At first we have to examine our system concerning its symmetry. To avoid the strong repulsive interaction at short distances, the Cooper pairs are in a p-wave state. They must form a spin triplet because their overall wavefunction must be antisymmetric. Therefore we have three degrees of freedom in spin ( $S_z = -1, 0, 1$ ) and orbital ( $L_z = -1, 0, 1$ ) space, in total  $3 \times 3 = 9$  internal degrees of freedom. The symmetry group  $G$  can be written as

$$G = (SO_3)_L \times (SO_3)_S \times U(1). \quad (1)$$

The groups  $(SO_3)_L$  and  $(SO_3)_S$  describe three dimensional rotations in orbital space and in spin space respectively. The last term, i.e. the unitary group  $U(1)$ , expresses the gauge invariance of the system. Above the transition the system is invariant under separate rotations in all three groups. There is no long range order and our liquid is isotropic. Symmetry breaking implies, that below the transition the group  $G$  is not invariant any more under its full symmetry group, but just under a subgroup. The breaking of the gauge symmetry  $U(1)$  is fundamental for superfluids and superconductors. The Hamiltonian for liquid helium 3 above  $T_c$  is invariant under global gauge transformation. But below the transition temperature this is not the case any more, i.e. the symmetry of the overall phase of the order parameter is broken. Consequently there is long range order. But what about the other symmetries, namely the angular momentum and spin? They could be also broken, either independently of each other or together in a relative sense. We will consider a two-dimensional system of  ${}^3\text{He}$ , as can be seen in Fig. 2, in order to visualize the symmetry breaking in the orbital and spin group.

The dashed vectors represent the spin degree of freedom  $\hat{\mathbf{d}}$ , whereas the straight vectors stand for the angular momentum  $\hat{\mathbf{l}}$  of the Cooper pair. Both can vary only in the plane. Picture (a) illustrates a possible configuration without any broken symmetries, thus in two dimensions

$G$  is given by

$$G = U(1)_L \times U(1)_S \quad (2)$$

The other two pictures both show examples of broken symmetries in the 2D configuration. In picture (b), the combined symmetry of angular momentum and spin is broken. If you think of both degrees of freedom as independent of each other, there is no state of order. Just in the common sense symmetry is broken, since the relative orientation of spin and angular momentum is always the same. This type of symmetry breaking is characteristic for the so-called B phase in  $^3\text{He}$ . But nevertheless a linear combination of  $\hat{\mathbf{d}}$  and  $\hat{\mathbf{I}}$  is still isotropic. For this reason this phase is sometimes referred to as pseudo-isotropic. In picture (c) the symmetry breaking is obvious, both angular momentum and spin are ordered. Consequently there is no symmetry remaining in the system. This particular symmetry breaking occurs in the A phase. Apparently the internal degrees of freedom of a spin-triplet p-wave state would allow a lot more configurations, where the symmetry is spontaneously broken. Why are only two phases present in zero magnetic field? To answer this, the Landau free energy of the different configurations has to be considered. Only the phase with the lowest free energy will be realised in practice. It was shown by Balian and Werthamer [2] using the weak coupling approach, that the B-phase is the most stable one. If one goes beyond this regime and also includes strong coupling effects, then there is also another stable phase referred to as the A phase. This was shown by Anderson and Brinkman [3]. If we bear in mind, that also the gauge symmetry in superfluid helium is broken, we see that we have broken symmetry in combined degrees of freedom. This property distinguishes  $^3\text{He}$  from superconductors or  $^4\text{He}$  and gives rise to the very rich structure of topological defects.

### 3 Order Parameter

So far, we have only considered a two-dimensional model of superfluid helium. Now this view will be extended into three dimensions and I will introduce the order parameter for this system. The expressions for the order parameters can be derived rigorously using methods of group theory [4]. I will just present the results and some useful interpretation. A superfluid is a liquid consisting of two components; a normal component and a superfluid component. The superfluid fraction behaves like a single macroscopic particle with a wavefunction  $\Psi(\mathbf{r})$ , which is called order parameter. It expresses quantitatively the state of order. In systems having one internal degree of freedom, e.g. superconductors or  $^4\text{He}$ , the order parameter is just a complex number and is given by  $\Psi(\mathbf{r}) = |\rho_s| e^{i\Phi(\mathbf{r})}$ . But as already mentioned, superfluid helium 3 in three dimensions possesses more internal degrees of freedom. Spin and orbital rotations each contribute three degrees of freedom, which makes nine in total. Additionally we have the gauge invariance. Hence the order parameter  $A_{ij}$  will be a complex  $3 \times 3$  matrix.

In general, it can be written as a linear combination of spherical harmonics  $X_{S_i}$  and  $\Upsilon_{L_j}$ . Consequently we obtain

$$A_{ij} = \sum_{ij} a_{ij} X_{S_i} \Upsilon_{L_j} \quad (3)$$

In the B phase, angular momentum and spin are isotropic, if you view them as independent of each other. They couple to a total angular momentum  $\mathbf{J} = \mathbf{S} + \mathbf{L} = 0$  [5]. All states  $L_z = \pm 1, 0$  and  $S_z = \pm 1, 0$  are equally occupied, which is reasonable, because of the isotropy constraint. Therefore there are three equal components of the order parameter:  $a_{+-} = a_{00} = a_{-+}$ . It can be shown [6], that it has the form

$$A_{ij} = |\Delta_B(T, P)| e^{i\Phi} R_{ij} \quad (4)$$

where  $\Phi$  is the overall phase and  $R_{ij}$  describes rotation of spin and orbital spaces in a common sense.  $\Delta_B(T, P)$  is a pressure and temperature depending amplitude. As you see, the order parameter is also invariant under a simultaneous three-dimensional rotation in spin and orbital space, as claimed above.

According to our investigation above, the A phase is anisotropic in spin space as well as in angular momentum space. The Cooper pairs are in a state with  $L_z = 1$  and  $S_z = 0$ , so that only  $a_{0+} \neq 0$  [5]. Its order parameter is given by

$$A_{ij} = |\Delta_A(T, P)| \hat{d}_i (\hat{m}_j + i\hat{n}_j) \quad (5)$$

with  $\hat{\mathbf{l}} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$ . It depends on the unit vectors  $\hat{\mathbf{l}}$  and  $\hat{\mathbf{d}}$ , representing the direction of the angular momentum of the Cooper pairs and the orientation of the spin wave function respectively. All Cooper pairs in this phase have the same orbital axis determining the orbital anisotropy and simultaneously its spontaneous ferromagnetic moment. Another example, where an orbital anisotropy along  $\hat{\mathbf{l}}$  occurs, are liquid crystals. A magnetization along  $\hat{\mathbf{l}}$  however is characteristic for orbital ferromagnets. An interesting result of this behaviour is the fact that many properties, for example viscosity, are different in directions parallel or perpendicular to  $\hat{\mathbf{l}}$ . In addition to that we find magnetic anisotropy, described by the vector  $\hat{\mathbf{d}}$ , which is sometimes referred to as spontaneous magnetic anisotropic axis. This axis is common in spin antiferromagnets. You see, that <sup>3</sup>He-A inherently contains special features of other systems in condensed matter physics, making it unique among all of them.

The third experimentally observed phase is the A<sub>1</sub> phase, which is only present at non-zero magnetic field. The order parameter looks like the following:

$$A_{ij} = |\Delta_{A_1}(T, P)| (\hat{d}_i + i\hat{e}_i) (\hat{m}_j + i\hat{n}_j) \quad (6)$$

The Cooper pair is in the state  $L_z = 1, S_z = 1$ . Thus it shows like the A phase orbital ferromagnetism, but also ferromagnetic ordering of the spins.

## 4 Investigation of Topological Defects

Now we are prepared to investigate some of the many phenomena arising from the spontaneous broken symmetry in  $^3\text{He}$ . As already mentioned, the broken gauge symmetry gives rise to the phenomenon called superfluidity. What effects are caused when symmetries in orbital and spin space are broken? Obviously the preferred directions of spin and angular momentum can never be uniform in the whole systems, because there are walls, boundaries or other limiting effects. But a varying order parameter leads to an increase of the free energy, or in other words, spatial variation of long range order in the condensate is not favorable [4]. Therefore the order parameter contains a certain rigidity, which tries to prevent a nonuniform state of order. The competition of these two forces leads to new configurations called textures. Especially singular textures, where the superfluid density vanishes, are interesting. In these regions the fluid is normal and those textures are called topological defects. We will see, that defects in  $^3\text{He}$  can take the form of points, lines and planes.

In the following I will mainly focus on line defects, also called quantum vortices. Their appearance can be predicted theoretically, but they have also been observed experimentally using nuclear magnetic resonance (NMR) measurements on rotating  $^3\text{He}$ . Superfluidity is the result of the helium atoms forming a macroscopic quantum state. Mathematically spoken, it appears because the order parameter is a complex number. To describe the motion of the superfluid component, you introduce a superfluid velocity. In isotropic superfluids, like superconductors or  $^3\text{He-B}$ , it is given by:

$$\mathbf{v}_s = \frac{\hbar}{2m} \nabla \Phi(\mathbf{r}) \quad (7)$$

$\Phi(\mathbf{r})$  is the phase of the governing order parameter, and  $m$  is in our case the mass of a Helium atom. Since  $\mathbf{v}_s$  is a gradient field, its rotation vanishes. Therefore a superfluid cannot rotate like a normal liquid, but vortex lines will develop. This fact gives rise to a quantized circulation  $\kappa$  in a rotating superfluid. In a simply connected, i.e. vortex free, liquid the circulation is

$$\kappa = \oint \mathbf{v} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{v}) \cdot d\mathbf{S} = 0. \quad (8)$$

But in a rotating system containing singularities, we obtain:

$$\kappa = N \cdot \frac{h}{2m} \quad (9)$$

$N$  is called the topological charge or winding number, classifying the given defect. Thus a vortex represents a field configuration which is characterized by a non-zero winding number. The situation is different in  $^3\text{He-A}$ . Since the A phase is not isotropic the expression for  $\mathbf{v}_s$

differs from the one above. You can work out using the order parameter given above (5) that the superfluid velocity yields [7]:

$$\mathbf{v}_s = \frac{\hbar}{2m} \sum_i \hat{m}_i \cdot \nabla \hat{n}_i \quad (10)$$

Thus the curl of the velocity field does not vanish in general. The superflow is only irrotational, when  $\hat{\mathbf{l}}$  is a planar field. In this case the rotation  $\nabla \times \mathbf{v}_s$  is equal to zero. But usually a superfluid current is globally unstable in the A phase, if you do not take interactions or boundaries into consideration.

In order to see, which vortices are present at certain conditions, the following strategy is inevitable. At first, the textures have to be grouped into classes defined by their topological charge. Secondly, within these classes you subdivide into symmetric classes. At last, to obtain the stable textures, the Landau free energy has to be minimized. The results provided by theory can then be compared to experiments using NMR. A comprehensive discussion about the topology of vortices in  $^3\text{He}$  can be found in [8]. Here I will present some results showing which vortices are stable or at least metastable and can be observed in rotating superfluid  $^3\text{He}$ .

Let's start with the A phase. Although the curl of  $\mathbf{v}_s$  is non-zero, there is homogenous rotation possible. This is because there are inhomogeneous textures in the  $\hat{\mathbf{l}}$  field. All in all four topologically different vortex structures occur [9], whose existence depends crucially on the external field  $\mathbf{H}$  and the angular velocity  $\Omega$  of the rotating vessel. Moreover the dipole interaction between the Cooper pairs tends to align  $\hat{\mathbf{l}}$  and  $\hat{\mathbf{d}}$ . If this happens you talk about a locked configuration. Basically there are two different types of vortices possible, namely singular ones and continuous vortices. Singular linear defects have a normal core and cannot be dissolved. In continuous vortices, the order parameter vanishes nowhere and thus  $\mathbf{v}_s$  is defined everywhere, also in the core. It can be shown, that line defects with even number of circulation are continuous. But these vortices are not stable because they can be transformed into a configuration with continuous vorticity [10]. In contrast, odd charged topological defects are always singular and therefore stable.

Which vortices occur in  $^3\text{He-A}$ ? Firstly there is a continuous unlocked vortex (CUV) with topological charge  $N = 2$ . Moreover a singular vortex (SV) with  $N = 1$  can be observed, especially at low angular velocity. This vortex consists of two cores, one hard core with radius of the order of the coherence length,  $\xi \approx 10 \text{ nm}$ , and a soft core with a radius of the order of  $\xi_D \approx 6 \mu\text{m}$ . In the hard core  $\mathbf{v}_s$  vanishes, while in the softcore always  $\hat{\mathbf{l}} \neq \pm \hat{\mathbf{d}}$  yields. This softcore contains all of the vorticity.

A very interesting kind of linear defects are vortices with topological charge  $\pm \frac{1}{2}$  (Fig. 3(a)). Those are locked vortices and they are favoured in a low magnetic field. Their appearance can be understood quite easily. The wavefunction must be single valued upon circling a

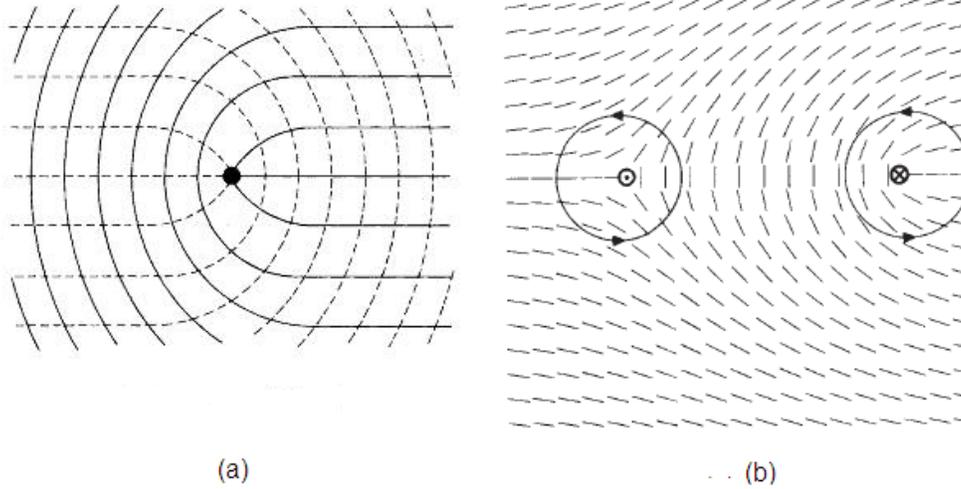


Figure 3: (a)  $\mathbf{l}$ - and  $\mathbf{d}$ -vector field around a half quantum vortex (b)  $\mathbf{d}$ -vector field around two half-quantum vortices with charge  $\pm 1/2$ . The vortices are glued together [5].

closed contour in the superfluid. So the phase of the order parameter can only change by multiple integer values of  $2\pi$ ,  $\Psi(\mathbf{r}) \rightarrow \Psi(\mathbf{r})e^{i2\pi m}$  with  $m$  being an integer. In section 3 we saw that the order parameter is a product of two spherical harmonics representing spin and orbital space. If we consider a change of  $\pm\pi$  in the phase of both parts,  $\Upsilon(\mathbf{r}) \rightarrow \Upsilon(\mathbf{r})e^{\pm i\pi}$ ,  $X(\mathbf{r}) \rightarrow X(\mathbf{r})e^{\pm i\pi}$ , and take the resulting sign change into account [ $\hat{\mathbf{d}} \rightarrow -\hat{\mathbf{d}}$ ], we get an overall phase change of  $2\pi$ . Thus half quantum vortices are stable, but they always coincide with a singularity in the spin space. Finally there exist vortex sheets (VS) having the same quantum number as the continuous unlocked vortex. The difference between them is their spatial arrangement. The CUV forms a periodic lattice while VS form a series of soliton planes. In VS the relative weak dipole interaction leads to an alignment of  $\hat{\mathbf{l}}$  and  $\hat{\mathbf{d}}$ , either parallel or antiparallel. Between two configurations of parallel and antiparallel aligned momenta a domain wall develops, called soliton wall, to which the vortices are bound. This is an example for a stable planar defect which can be seen in Fig. 4.

Many sorts of interaction between vortices and boundaries arise, which can theoretically lead to more complicated structures. I want to mention one particular interesting case, where two half quantum vortices with charge  $+1/2$  and  $-1/2$  interact and form a spin soliton as can be seen in Fig. 3(b). This configuration is analogous to the quark confinement in high energy physics. The vortices are glued together by the uniform  $\hat{\mathbf{d}}$  texture between them [5].

As seen above, the B phase is pseudo-isotropic, meaning that the superfluid is degenerate

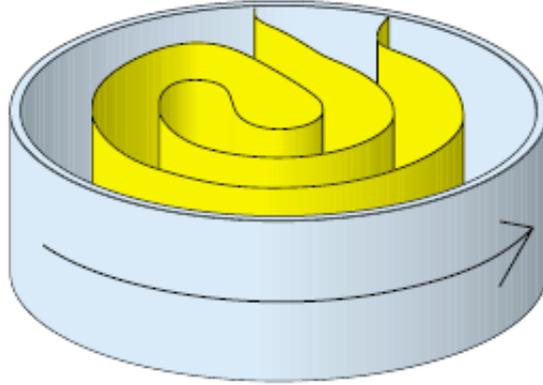


Figure 4: Vortex sheet in the A phase in a rotating cylinder [11]

with respect to rotation about the orbital and the spin axis. Hence it was believed for a long time, that the vortex structure in  ${}^3\text{He-B}$  would be the same as in  ${}^4\text{He}$ . This was proved to be wrong when observing the first NMR experiments, which show that there must be a spontaneous phase transition in the vortex core. In order to understand the vortex structure of the B-phase you have to classify the vortices according to their symmetries. It turns out, that they can be divided into axisymmetric and non-axisymmetric vortices. Moreover, the nature of ordering within the cores can vary a lot. As in the A-phase, the core does not even need to be normal, but can be continuous. In this case the order parameter is "flared out" at the centre of the vortex, and the vorticity is absorbed into a nonsingular configuration. The axisymmetric vortices can be subdivided into five states depending on their internal symmetry. These are labeled with  $o$ ,  $u$ ,  $v$ ,  $w$ ,  $uvw$ . The energetically most favourable one is the  $v$ -vortex, which has a superfluid core. The existence of the many vortex core structure leads to transitions between different configurations, as can be seen in Fig. 1.

Given a superfluid in rotation, the  $v$ -vortex is the most stable one at high pressures. But as one goes to lower pressures, there is a vortex transition into a state which is called double-core vortex. This is a nonaxisymmetric vortex, consisting of two centres both carrying half a quantum of circulation, similar to Fig. 3(b). So there is a change in topology of the vortex-core matter, which is another interesting consequence of the symmetry breaking in  ${}^3\text{He}$ .

The last kind of vortex appearing in  ${}^3\text{He-B}$  is called spin-mass-vortex (SMV). But this configuration is only metastable among the two previous vortices and does not form a lattice. In addition to the circulation of mass around the vortex, there is also a spin flow. Thus particles with opposite spin move in opposite directions. SMV's usually form a composite two-quantum vortex or they are attached to the walls (5).

Summarizing, there are in total seven different line defects which can be observed experi-

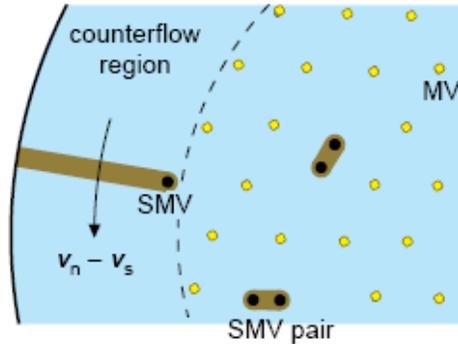


Figure 5: The yellow dots stand for the usual vortices with mass circulation. The black dots are SMVs, either attached to the wall or tied together by a soliton tail [11]

mentally. In theory, the field of quantum vortices in superfluid  $^3\text{He}$  is very vast and to cope with all appearing phenomena the application of homotopy theory is indispensable.

In addition to line defects there are many more exotic textures which can occur in such a system. Further examples are point defects. They can also be studied using homotopy theory. Here, the singularity is contracted into a point. In the B phase, point defects with a nonsingular core exist and they are being referred to as monopoles or hedgehogs [6]. These defects can be present at the edge of planar defects. The A phase supports point defects as well, but only in the spin part of the order parameter. This time, they are established as termination point of a nonsingular line defect in the spin part [4].

A very special point defect can occur in  $^3\text{He-A}$  in a spherical vessel. It is obvious, that the order parameter cannot be uniform everywhere due to boundary conditions of the orbital vector  $\hat{\mathbf{l}}$ . The shape of the vessel and the boundary conditions for the field lead to the existence of a singular defect inside. It can be either located at the wall, which is called a "Boojum", or inside the container accompanied by linear defects. This point-like defects have many similarities with a Dirac magnetic monopole comparing the velocity field around the defect with the electromagnetic field around the monopole [6].

## 5 Conclusion

Superfluid  $^3\text{He}$  is an ideal system to study spontaneous symmetry breaking in condensed matter physics. As we have seen, we can predict many effects just by considering the symmetry of the underlying system. Due to the rich structure of the Cooper pairs, i.e. the great number of internal degrees of freedom, the order parameter takes a very complicated form. Having derived the order parameter, the explanation of the many phenomena is possible.

One manifestation of symmetry breaking is the occurrence of many different kinds of topological defects. The examination of those defects is not only interesting for its own sake, but also because of its analogies to many other parts in physics. I have mentioned the similarities with quark confinement. The appearance of vortices at interfaces when cooling down the liquid might also be analogous to cosmic strings, which are expected to have nucleated in early universe [12]. All in all,  $^3\text{He}$  is a remarkable system, which allows us to study many effects occurring in condensed matter and also in other fields.

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