Disorder Induced Critical Phenomina Far From Equilibrium

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Abstract

The observation of Barkhausen Noise in magnetic systems and related scaling suggests the possibility of the systems proximity to a critical point. This turns out to be consistant with the zero temperature Random Field Ising Model [RFIM] with a critical point designated by the width of the uncorrelated (long wavelength) random field distribution over a d-dimensional cubic lattice and a given external field.

Introduction

If one were to scour all of the scientific literature over the lifetime of science one would generally find an aversion to impurity. Noise, dirt, disorder, structural impurities anything that gets in the way of the beautiful theories constructed by the most notable scientific intellects of history usually plays the role of spoiler. Where the effects of disorder are important good practitioners are sure to qualify at least and quantify when possible these effects but thier influence on the outcome is rarely the primary focus of investigation. There are however increasingly more studies of systems in which the disorder *is the source* of the behavior of interest. In this paper we present one such study.

The behavior of interest can be couched in terms of a discovery made in 1919 by H. Barkhausen and results from simply magnetizing a ferromagnetic material. Upon increasing the external magnetic field one might look at the resulting M vs. H curve and conclude that this is a continous process, but we note upon further inspection that this curve is made up of a set of descrete jumps (fig. 1) . One can actually hear these jumps with the aid of a pickup coil and some speakers as a sort of static "like the noise of sand grains falling over each other as a can of sand is tilted"[1]. This static is known as Barkhausen Noise (BN) and is useful because when coupled with a mesoscopic theory it gives us means of studying aspects of mesoscopic structure of the ferromagnetic materials (among others) in a nondestructive way through macroscopic measures such as noise spectra [7]. These materials are of great technological interest because they are materials that "remember" what has happened to them or more technically they

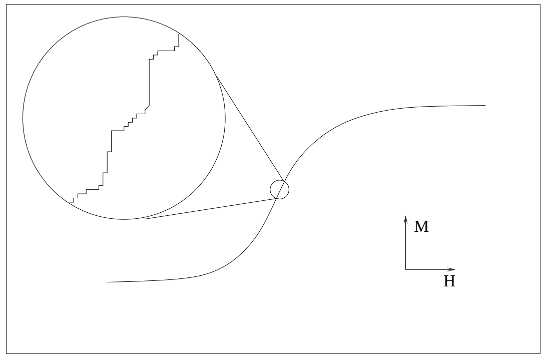

Figure 1: Detailed view of magnetization curve, magnified region shows Barkhausen Jumps

display hysteresis on long time scales. This property is manafestly tied to the pinning effect of the disorder.

That disorder is a factor in this phenomena seems clear given the non-deterministic nature of the BN as well as the polycrystaline structure of these ferromagnets but extracting its precise role is by no means a trivial task. And as of now it seems for generic materials and parameters there does not exist an tacitly accepted theory. For instance one theoretical treatment (ABBM model)[11]reduces the system to one degree of freedom modeling the system as "the" domain wall being dragged through a random potential. Another treatment (Cizeau et.al.)[12]takes into account the surface tension of the domain wall and models the phenomenon as a flexible domain wall being dragged through random pinning sites. While both appear to be consistant with some experiments the collapse to one degree of freedom of the ABBM model is not understandable in terms of the mesoscopic structure. That is it is hard to tie the correlation of the disorder in the 1d pinning potential to the large number degrees of freedom the system is supposed to have. The smooth domain wall picture given by Cizeau et.al. is limited to soft ferromagnetic materials and may correspond to a different universality class than is presented in this paper (i.e. long range forces).

Dispite the differences in many theories one curious fact is observed when analysing BN. That is that the frequency spectrum and amplitude distribution exhibits power law scaling over many decades. This experimental fact has been interpreted as Self-Organized-Criticality (SOC) due to decades of scaling apparently without tuning any parameters[1] (observed cutoffs were attributed to finite size effects). There is evidence however to suggest that the scaling may be the result of the system being near a "plain old" critical point. Here I present a

[1]The defining criteria "criticality without fine tuning" seems to have need of amendment [3] but this is material for another paper.

model that I hope will convince you of this.

The proximity to a critical point is stated not in terms of temperature and external field (as in the standard Ising model) but in terms of the "strength" of the disorder and the external field. The disorder is responsible for pinning the system in local energy minima resulting in a system far from equilibrium. In the present discussion it is assumed that the barriers posed by the disorder do not allow for the observation of relaxation to the equilibrum state on the time scales of the experiments. In this paper I introduce a cartoon picture of the mesoscopic physics and show that it goes a long way into explaining scaling found in experiments. The model that applies to these sketched out properties and will carry us for the rest of the paper is the zero temperature Random Field Ising Model (RFIM).

The Model

The RFIM consists of descrete up down spins on a d-dimensional hypercubic lattice. The general hamiltonian for the RFIM which includes all of what is assumed to be an effective charactiture of the basic physics for both long and short range forces is given by-

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{i,j} S_i S_j - \sum_i (H + h_i) S_i \underbrace{- \sum_{i,j} J_{dipole} \frac{3cos(\theta_{ij}) - 1}{r_{ij^3}} S_i S_j + \sum_i \frac{J_\infty}{N} S_i}_{omitted\ in\ present\ treatment}$$

It includes the dipole interaction term and a infinite range so called demagnetization term which in this paper will be neglected. In order to investigate the short range force universality class I will instead focus on the system dominated by nearest neighbor (nn) interaction ($J_{i,j}$ summed over nn $\langle i,j \rangle$), interaction with the external field (H), and the local quenched disorder represented by the R.V. h_i with the following gaussian distribution-

$$\rho(h_i) = \frac{1}{R\sqrt{2\pi}} e^{\frac{-h_i^2}{2R^2}}$$

The disorder is spacially uncorrelated which in the long wavelength limit seems reasonable. R, the standard deviation of the distribution, is what I am referring to when I talk about the "strength" of the disorder (fig 2).

We take the long time (compared to the time it takes a spin to flip given it is energetically favorable to do so) limit and obtain spin flips in the simulation upon the reversal of the sign of the net local effective magnetic field $h^{eff} = H + h_i + \sum_{\langle j \rangle} J_{ij} S_j$. Qualitatively one can imagine the resulting hysteresis loops obtained for extreme values of R assuming we begin with all spins pointed down and ramp the field from -∞ up. When R is small the system looks like a regular Ising model at zero temperature, that is upon ramping the field the system undergoes one large jump when the external field goes from $H < 0$ to $H > 0$.

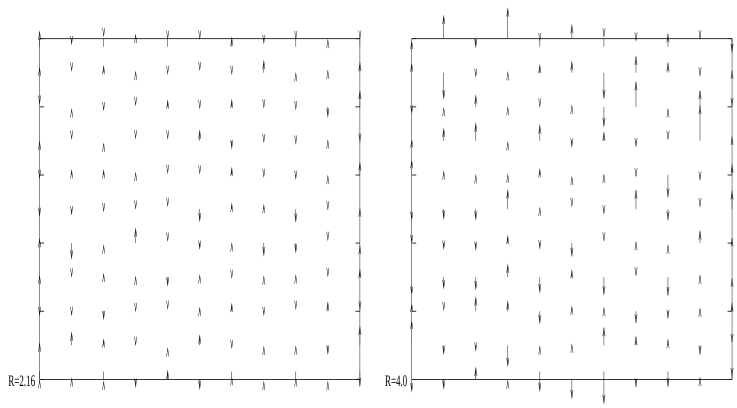

Figure 2: Picture of the quenched random fields present for different values of R. Note this is a side view of a 3d cube, the random fields point along the axis of symmetry i.e. the z-axis

When R is very large the random fields dominate the dynamics overriding the nn interaction and the hysteresis curve looks like a gaussian intetgrated from -∞ to H. Somewhere in between the loop goes from having a discontinous jump to a smooth curve having a point of infinite slope (fig. 3).

From simulations as well as mean field theory we discover the system near this point turns out to behave much like a traditional equilibrium system near a critical point. The role of the barkhausen jumps is played by "avalanches" which have a characteristic size (fig 4) that diverges as one would expect, the distribution of avalanche sizes scales, the susceptability diverges by definition and so on.

Mean Field Theory

This model is exactly solvable in the Curie-Weiss type mean field theory where the nn coupling is replaced by the net magnetization of the system and $J_{ij} \longrightarrow \frac{J}{N}$ where N is the system size. This results in the following Hamiltonian

$$\mathcal{H} = -\sum_i (H + h_i + JM)S_i$$

Most of the qualitative properties of the system are given by MFT. For instance the system displays hysteresis for $R < R_c$, there is a discontinuity in the slope of $\frac{dM}{dH}$ and the avalanche size distribution scales as R approaches R_c from the right. This is a great help since it gives us a starting point for analytic treatments but how does this apply to our system? After all details given by

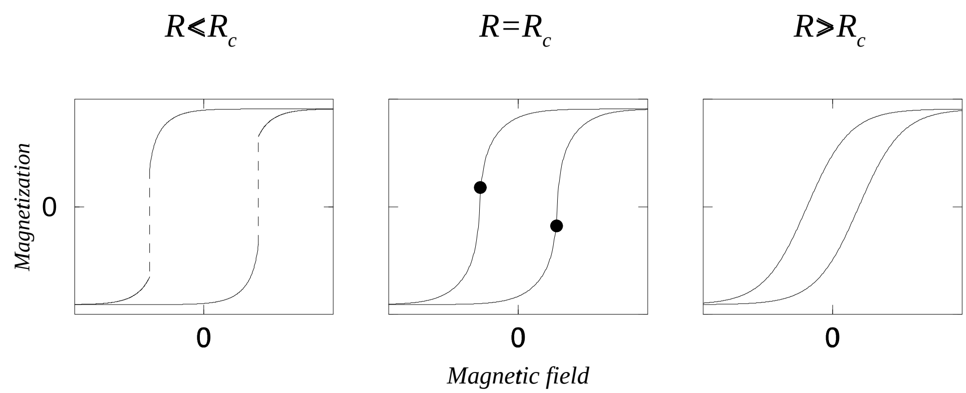

Figure 3: Schematic drawing of loops for various levels of disorder with the critical point illustrated in the center. [5]

MFT are notoriously unreliable. We do know however that if we increase the dimension of our system the number of nearest neighbors increase giving the effect of each spin "feeling" the mean field. To find out how high the dimension must be for MFT to apply to our system the following argument has been presented.

Suppose you are approaching the critical point by varying h along the $r = 0$ line (where $h = \frac{H-H_c}{H}$ and $r = \frac{R-R_c}{R}$). For the transition to be well defined the fluctuations in h, Δh must be less than the value of h or $\frac{\Delta h}{h} \ll 1$ as h approaches zero. We know[2] that the correlation length $\xi \sim h^{\frac{-\upsilon}{\beta\delta}}$ near the critical point and that $\Delta h \sim \xi^{-\frac{d}{2}}$. This yield the following inequality

$$\frac{\upsilon}{\beta\delta} \geq \frac{2}{d}$$

and with the MF values of $\upsilon = 1/2$ and $\beta\delta = 3/2$ we obtain the **upper critical dimension** to be 6. This is an unphysical dimension but as you will see d=6 provides a useful starting point around which one can use perturbation methods in the dimension of the system to probe more realizable dimesions.

Scaling forms and Critical Exponents

From MFT we can construct scaling forms for various quantities and use them as ansatz' below the upper critical dimension to test how the exponents change as we approach more physical dimensions. Here, we simply quote a couple of scaling forms and define the exponents of interest in order to demonstrate scaling collapse and the extraction of critical exponent values from simulations. There are other useful scaling forms which the interested reader can investigate[2],[9].

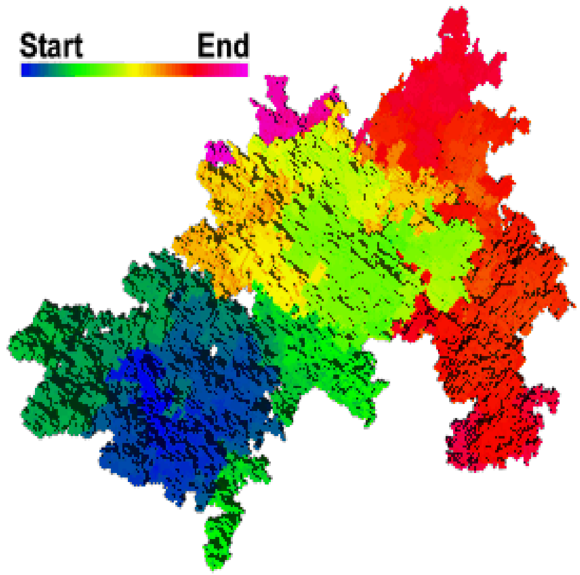

Figure 4: Illustration(from [8]) of 3 dimensional avalanche near the critical disorder. The colors indicate the order of flipping events.

Magnetization Curves

Near the critical point (r & h small) M(H,R) is found to have the much more restrictive form

$$M(H,R) - M_c \sim |r|^{\beta} \mathcal{M}_{\pm}(h/|r|^{\beta\delta})$$

where $M_c = M(H_c, R_c)$ and $\mathcal{M}_{\pm}$ is the universal scaling function ((+) denotes above R_c and (-) is a potentially different scaling func. below R_c). Both β and $\beta\delta$ are defined by this relation and have MF values of $1/2$ and $3/2$ respectivly.

To remove the non-universal M_c as well as investigate the divergence in the slope we take the derivative w.r.t. H to obtain

$$\frac{dM(H,R)}{dH} = |r|^{\beta-\beta\delta} \mathcal{M}'_{\pm}(h/|r|^{\beta\delta})$$

Avalanche Size Distribution

In general the Avalanche Size Distribution is formally D(S,R,H), near the critical point MFT gives

$$D(S,R,H) \sim S^{-\tau}\mathcal{D}_{\pm}(S^{\sigma}|r|, \frac{h}{|r|^{\beta\delta}})$$

where as usual r & h are small and S is large. The mean field values of τ and σ are $3/2$ and $1/2$ respectively. One can integrate this over all fields and by noting that the integral will only have notable contributions near h=0 we obtain the following so called "integrated" avalanche size distribution.

$$D^{int}(S,R) \sim S^{-(\tau+\sigma\beta\delta)}\mathcal{D}_{\pm}^{int}(S^{\sigma}|r|)$$

with $\tau + \sigma\beta\delta = 9/4$ in MFT.

Scaling Collapse

The constraint of the scaling form imposed by nature allows us to extract exponent values from the experimentally obtained or simulated functions by collapsing the data. For example knowing that $\frac{dM(H,R)}{dH}$ has the form given above leads us to take $\frac{dM(H,R)}{dH}$ as data and plot $\frac{dM(H,R)}{dH}|r|^{\beta\delta-\beta}$ vs. $h/|r|^{\beta\delta}$ for different values of r. The universality of $\mathcal{M}_{\pm}^{'}(h/|r|^{\beta\delta})$ implies these data will collapse onto one curve if we get the exponents right. Unfortunately this requires taking a range of r values and we know we will only get a collapse for r close to zero so we are torn. For scaling to be convincing i.e. have well defined exponents for a given fit, the range over which r is taken must be significant. That is if r = 0.1,0.1001,0.1002 we surely expect to see collapse but the parameter space over which we get collapse is huge. On the other hand if we choose r =0.1,0.5,1.0 we are likely to not get collapse at all since some of the values in the range fall outside the region described by the critical exponents. This is precisely the problem experienced with $\frac{dM(H,R)}{dH}$. I mention this because in the case of $D^{int}(S,R)$ we get bailed out by a very curious situation.

It seems that the scaling form holds for conventionally large r values but the exponent values are not the same as you increase r, and they seem to vary smoothly. This allows a wonderfully simple trick. By taking a set of small sets with non-trivial but small ranges centered around an increasing r so you have $\{r_{i-}, r_i, r_{i+}\}$, collapsing the small sets to end up with a new set composed of ordered pairs of r and corresponding exponents $\{r_i, exponents_i\}$ and by extrapolating the processed data to $r = 0$ we extract the true critical exponent (fig 5) . Now for MFT we already know the answers so this is just a demonstration of method but when simulating finite systems this technique will prove quite useful.

Simulation

Before I sketch any of the analytical aspects of this problem it will be good to introduce you to some computational aspects as well as the results of the simulation. In principle the simulation is very straight forward so you may wonder why I discuss the computational aspects at all. Indeed these are well covered in [8] but you will hopefully note in the implimentation of the model that there are some details, notably true infinitely slow sweeping rate, that may

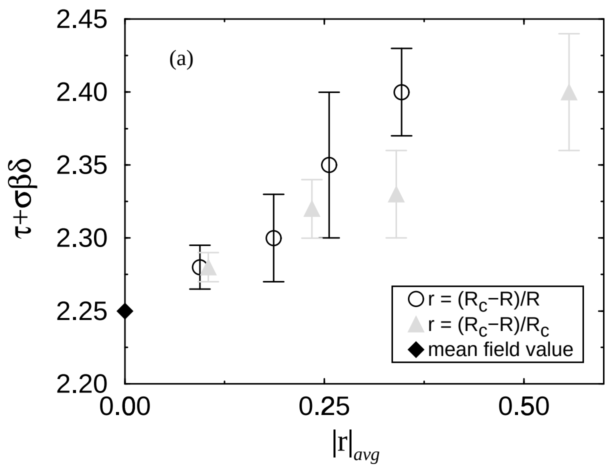

Figure 5: MF example of critical exponent determination from integrated avalanche distribution.

aid in the understanding of the physics as well as give limits (mostly hardware) imposed by the model.

Computational Aspects-

There have been two primary algorithms used to study this system[8]. The first uses a sorted list structure with runtime scaling of O(NlogN) requiring asymptotically $N \cdot (double_{size} + integer_{size})$ bytes of memory. This allows for system sizes of approx 80,000 sites per megabyte and takes about 10min for one run of a 200^3 system (RS6000 43P 260). The memory required by this algorithm limits the system sizes to approx 500^3 for a machine with 2gigs of memory. A benefit of this algorithm is that you can sweep the field back a forth to study proporties of the hysteresis loop. The other algorithm used takes advantage of the fact that if you only go one way in the hysteresis loop you do not have to store information about the individual random fields on each site. Thus you only have to store the spin state (up or down) or one bit per spin. This allows for much larger systems ($\sim 1000^3$ and larger) and also runs in O(NlogN) time. The drawback is that you can't go backwards.

All of the results quoted in this paper are quoted for infinitely slow sweeping rate and indeed that is what is implemened in the simulation. In schematic form the algorithm goes as follows (both follow this in spirit).

- The external field is raised until one spin flips.
- The spin may or may not cause others to flip.
- If it does then we propogate the avalanche until no more spins want to flip, all the time the external field has not increased.
- Start over.

measured exponents	3d	4d	5d	mean field
$1/\nu$	0.71 ± 0.09	1.12 ± 0.11	1.47 ± 0.15	2
θ	0.015 ± 0.015	0.32 ± 0.06	1.03 ± 0.10	1
$(\tau + \sigma\beta\delta - 3)/\sigma\nu$	-2.90 $\pm$ 0.16	-3.20 $\pm$ 0.24	-2.95 $\pm$ 0.13	-3
$1/\sigma$	4.2 ± 0.3	3.20 ± 0.25	2.35 ± 0.25	2
$\tau + \sigma\beta\delta$	2.03 ± 0.03	2.07 ± 0.03	2.15 ± 0.04	9/4
τ	1.60 ± 0.06	1.53 ± 0.08	1.48 ± 0.10	3/2
$d + \beta/\nu$	3.07 ± 0.30	4.15 ± 0.20	5.1 ± 0.4	7 (at $d_c = 6$)
β/ν	0.025 ± 0.020	0.19 ± 0.05	0.37 ± 0.08	1
$\sigma\nu z$	0.57 ± 0.03	0.56 ± 0.03	0.545 ± 0.025	1/2

Table 1: Values for the exponents extracted from scaling collapses in 3, 4, and 5 dimensions. The mean field values are calculated analytically[2, 10] ν is the correlation length exponent and is found from collapses of avalanche correlations, number of spanning avalanches, and moments of the avalanche size distribution data. The exponent θ is a measure of the number of spanning avalanches and is obtained from collapses of that data. $(\tau + \sigma\beta\delta - 3)/\sigma\nu$ is obtained from the second moments of the avalanche size distribution collapses. $1/\sigma$ is associated with the cutoff in the power law distribution of avalanche sizes integrated over the field H, while $\tau + \sigma\beta\delta$ gives the slope of that distribution. τ is obtained from the binned avalanche size distribution collapses. $d + \beta/\nu$ is obtained from avalanche correlation collapses and β/ν from magnetization discontinuity collapses. $\sigma\nu z$ is the exponent combination for the time distribution of avalanche sizes and is extracted from that data.

Simulation Results-

Numerical results measured by Perkovic et al are given in the following table and caption found in [9].

Renormalization Group Results

This system is at zero temperature far from equilibrium, there is no partition function to be had, so the usual step of writing down the partition function cannot be done. Instead an analog to the partition function is created. It is noted that given a configuration of random fields the system follows a deterministic path upon sweeping the external field. By assigning a δ function weight to that path for that configuration and then averaging over all possible configurations we obtain a distribution for the possible paths of the system. So the "possible states" of good old thermodynamics goes to "possible paths" in this system and the "sum over states" of the traditional partition function goes to a "weighted sum over paths" in this system.

When the dust settles [2, 13] we end up with an perturbation expansion in dimension $d = 6 - \epsilon$ around the mean field values of the critical exponents as well

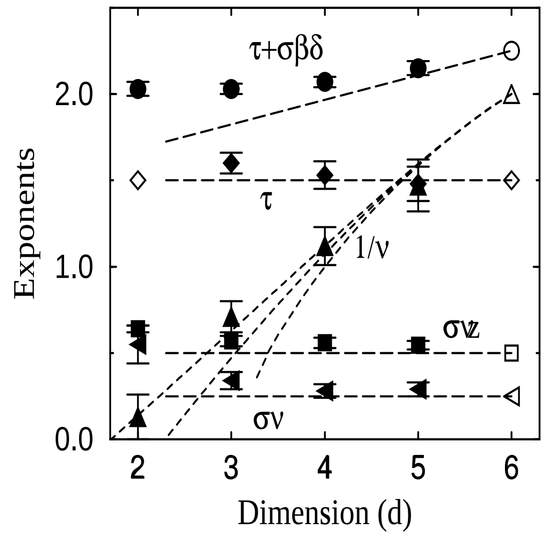

Figure 6: Results of the 6-ϵ expansion vs. numerical simulations.[14]

as a justification to the scaling forms mentioned above for our system below the upper critical dimension. The results of this calculation and corresponding phase and flow diagrams are shown in figs. 6&7. The data appear to fit surprisingly well given that the expansion was in terms of a non-physical parameter and the perturbation was in terms of a rather large number.

Conclusion

We see in this work a compelling agrument for the presence of scaling in Barkhausen noise as well as an interesting application of RG to a system far from equilibrium. The renormalization gives us a systematic way to coarse grain degrees of freedom and map the complex system onto experimentally observed phenomena like the noise amplitude distribution. What is needed is some experimental conformation in the light of the role of disorder. Unpublished experiments done by A. Berger whereby hysteresis loops are measured for various annealing temperatures of Gd films may be the beginning of this and efforts are underway to pursue this. There are also interesting questions currently being addressed regarding the connection between the equilibrium RFIM, relaxation times a finite sweeping rates. The fruits of this model are just beginning to ripen and hopefully this paper will interest you in the harvest.

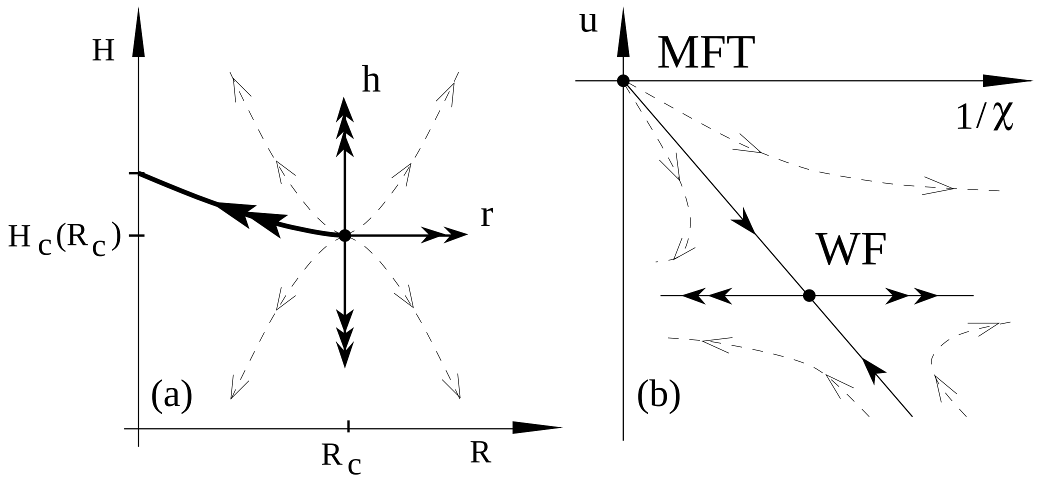

Figure 7: Left:Schematic phase diagram for our model, with arrows showing flows under coarse-graining. The dark line is $H_c(R)$, the external field at which the infinite avalanche occurs when the system is swept upwards from $H = -\infty$. Under coarse-graining, the effective external field $h = (H - H_c)/H$ grows fastest, and the effective disorder $r = (R - R_c)/R$ grows more slowly: all other directions are stable under coarse-graining. [Right is explained in reference but not discussed here. It relates to the simalarity between this system and the pure Ising model's 4-ϵ expansion... I experienced technical difficulties isolating the left figure][14] (figure and caption text in [] is mine)

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