464 Term Essay
Traffic Flow

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Abstract

Traffic flow impacts most people, both directly and indirectly—through economics, politics, and the environment. Optimal traffic flow is in a critical state, where small perturbations cause a phase transition from free flow to congested flow. Experimental understanding and predictive modeling are of great practical importance. The ideas of physics, particularly of critical behavior, provide tools for understanding traffic flow. Conversely, traffic flow provides a concrete realization of a self-organized critical phenomenon which may provide insights into other areas of physics.

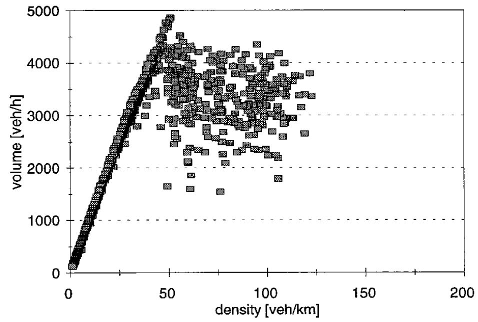


Figure 1: The *Fundamental Diagram*[4]

1 Introduction

The ideal of individual freedom has been embedded in the automobile for most of this century. American yearning for the open road has determined a development and infrastructure policy which has resulted in an expensive national nightmare. In spite of many billions of dollars worth of improvements, many drivers routinely face miserable congestion on a daily basis. Even discounting this suffering, the economic toll—estimated at $48 billion annually [1]—cannot be ignored.

But what can be done, and why are physicists interested in a problem nested somewhere between sociology, economics, urban planning, and civil engineering? Intelligent policy cannot be formulated without an understanding of observable phenomena. For practical purposes this means modeling and then predicting. Physicists were drawn to the problem because the tools of fluid flow and kinetic theory provided the best means to analyze traffic flow without a computer. They remain because computational models, particularly cellular automata, describe critical behavior observed in traffic flow—and offer the best prospects for deeper understanding.

Traffic flow is described and analyzed on two levels. Macroscopically, flow is characterized in terms of flow rate Q as a function of car density ρ and velocity V.

$$Q(\rho) = \rho V(\rho) \tag{1}$$

Plots of this map are referred to as the *fundamental diagram*, an example of which is shown in Figure 1. Qualitatively, two features are evident: a *free flow* phase at low density, in which throughput increases linearly as a function of density, and a *jammed* phase at high density. This agrees with the everyday experience of any commuter.

Traffic flow is also studied on a microscopic level. There are innumerable quantities which may be measured, all of which attempt to characterize the behavior of an individual driver. Experimental data on driver behavior is statistical in nature; people measure a velocity probability distribution. While the qualitative form of $V(\rho)$ is intuitive (see Figure 2), it is difficult to ascertain the other independent variables $V = V(\rho, V, x, t, \ldots)$. Consequently, theoretical work trails experiments, and is largely an effort to formulate a minimal model.

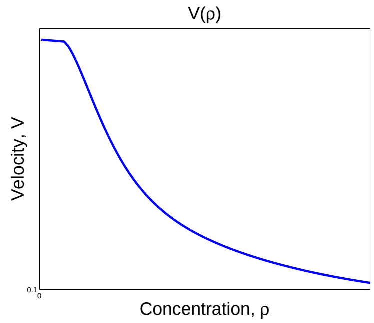


Figure 2: The qualitative form of $V(\rho)$

The transition between freely flowing and jammed phases is of particular interest. There is evidence that systems of cars self-organize to maximize throughput.[22]. Experiments and models[3, 4] suggest that this regime is inherently instable, and that slight perturbations can cause a traffic jam to occur. Thus optimal use of roadways lies in a critical region between different phases. This regime is of interests to engineers trying to best use available resources, and to physicists, as a laboratory for self-organized criticality.

2 Experimental Data

There are many different models, all of which qualitatively predict the essential behavior of traffic flow. Assessing the relative virtues of these models, particularly in the critical region, is the task of experimental investigations. Excellent summaries can be found in the works of Kerner[5] and Rehborn[8]. In addition to underpinning all theoretical efforts, experiments offer one epiphany. What we naïvely identified as the *jammed* phase is actually composed of two distinct congested phases: *jammed* and *synchronized* flow. The former is characterized by heavy, stop and go traffic. In the latter, unexpected, phase, cars in adjacent lanes more at nearly the same speed. Vehicles move more slowly than in free flow, with a high density, but with comparable flux.

In analyzing models of traffic flow phase transition, it is not "clear enough in what degree the *modeled dynamical effects* are related to the *real dynamics* of traffic flow."[5]. To ground my examination of models, I summarize the basic aspects of the three possible phase transitions.

Synchronized Transitions

The transition from free flow to synchronized flow is characterized by an abrupt reduction in both the mean and variance of velocity, with minimal diminution of total flux. This change frequently is caused by "localized perturbations of finite amplitude"[6], such as a highway on-ramp, although the transition is also observed without obvious causes. Once formed, the region of synchronized flow

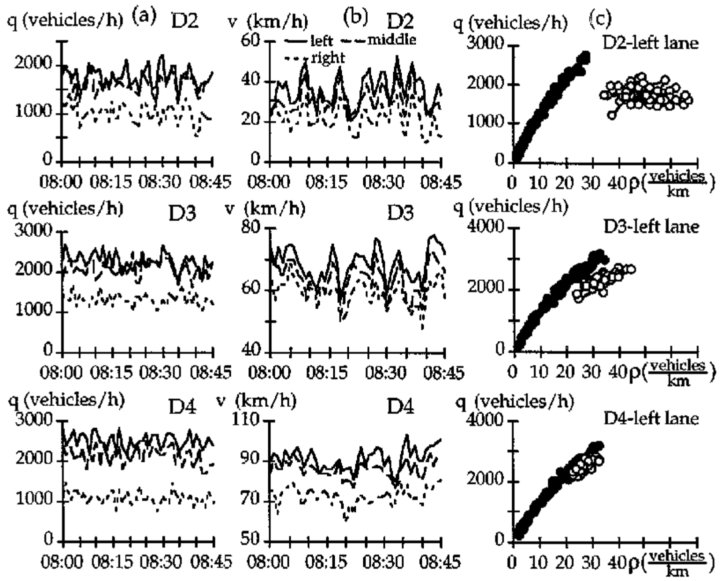


Figure 3: The transition between from synchronized to free flow. Three detectors, D2 $\to$ D4 arrayed in sequential order were used to measure the time dependence of flow flux (q) and velocity (v). The three different lines correspond to the three lanes of this autobahn.

can induce a traveling wave of synchronized flow both upstream and downstream. Downstream there is typically a gradual spatial transition back to free flow. "After the phase transition has occurred the synchronized flow can further be *self-maintained for several hours.*"[6]

An illustrative data set is shown in in Figure 3. The free flow phase is signified by the black circles in the $q-\rho$ plane, and is shown for reference only. The synchronized flow data is signified by white circles. The spatial transition from synchronized to free flow can be seen through the plots for different diagrams.

Jamming Transitions

The traditional traffic jam is shown in Figure 4. These jams arise in transitions from both free flow and synchronized flow, although the latter are "considerably more frequent"[7], and—when it occurs—free flow transitions are often separated from the jam by a short region of synchronized flow. This agrees with intuition: Traffic jams form most easily when density is higher. Once formed, jams tend to propagate upstream. This can be seen in Figure 4, where the zero velocity pocket (jam) propagates upstream, from D14 $\to$ D9.

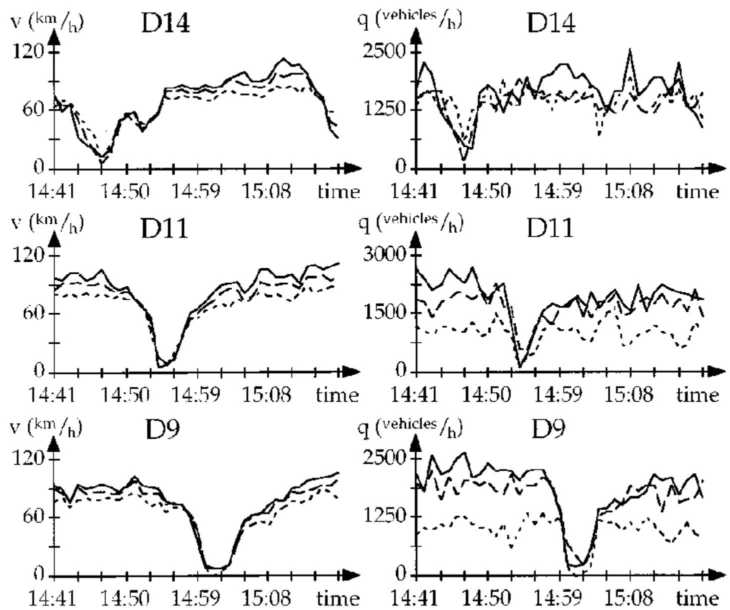


Figure 4: A traffic jam[8]. Three detectors, D9, D11, and D14, arrayed in sequential order, as in Figure 3.

Common aspects of phase transitions

There are several features observed in both kinds of transitions. Both kinds of transitions arise due to localized perturbations "whose amplitude exceeds some *critical amplitude*"[7]. These local changes lead to cluster formation, which acts as a nucleus for the formation of the congested phase (either jammed of synchronized). After formation, properties of the phase do not depend on the characteristics of the initial perturbation. It should be emphasized that for a given initial state of free flow, the transition to either kind of congested phase is equally likely. "It seems that the resulting kind of traffic ... depends on small peculiarities of the initial state"[6].

In both cases, the reverse transition generally occurs due to a decrease in traffic volume coming from upstream. At a given density, the flux necessary for the reverse transition is considerably less than what is required for the forwards process—a *hysteresis effect.*

3 Models

Even my cursory overview of experimental results is not encouraging to most model builders. The urgent demand for successful models, however, has spawned innumerable candidates. I follow the historical development of these models, beginning with hydrodynamic models. While the most developed (and useful) models are one dimensional, i.e. traffic flowing on a road, rather then two roads intersecting, I also consider one two dimensional model. There are two macroscopic approaches to modeling traffic flow, both of which are derived from simple analogies with fluid mechanics.

Macroscopic Models

In 1955, Lighthill and Whitman approximated the discrete flow of vehicles with continuous functions $\rho(r,t)$ and $V(r,t)$. This *continuum approximation* lends itself to a *hydrodynamic* description of traffic flow, wherein we have a continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial r}. \tag{2}$$

To proceed, one needs to hypothesize a relationship $V(r,t) = V_e(\rho,t)$. The simplest ansatz is that $V_e(\rho,t)$ is slowly varying, and always near an equilibrium value. This leads to solutions

$$\rho = \rho(r - vt), \tag{3}$$

where $v \equiv \frac{\partial(\rho V_e(\rho))}{\partial \rho}$. These solutions describe traveling *kinematic waves* in the flow of traffic, an effect which is actually observed. The assumptions made, however, lack verisimilitude, as they fail to include acceleration, inertia, or any dynamics which could lead to phase transitions.

These weaknesses can be remedied, leading to a modern macroscopic theory, most recently formulated by Helbing[11, 5], Kerner and Konhäuser[19, 21]. The continuity equation (2) is preserved, but $V(r,t)$ is described by a more complicated, Navier-Stokes like equation,

$$\frac{\partial V}{\partial t} + \underbrace{V\frac{\partial V}{\partial r}}_{\substack{\text{convection}\\ \text{term}}} = \underbrace{-\frac{1}{\rho}\frac{\partial P}{\partial r}}_{\substack{\text{anticipation}\\ \text{term}}} + \underbrace{\frac{1}{\tau}(V_e - V)}_{\substack{\text{relaxation}\\ \text{term}}} + \underbrace{\xi(r,t)}_{\substack{\text{fluctuation}\\ \text{term}}} . \tag{4}$$

- The *convection* term reflects velocity changes due to the average velocity $V(r,t)$.
- The *anticipation* term accounts for drivers reactions to upcoming traffic. P is called *traffic pressure*, and is best understood in the context of forthcoming microscopic explanations.
- The *relaxation* term, and *relaxation time*, τ, describe the equilibration of the average and equilibrium velocities.
- The *fluctuation* term describes random accelerations.

Versions of modern hydrodynamic models mainly differ in their specifications of P, τ, V_e, and ξ. The final components of these models are an implicit dependence on *velocity variance*, $\Theta(\rho)$, and particle conservation,

$$\int_0^L \rho(r,t)dr = N. \tag{5}$$

These models, like all models treated herein, have had some measure of success. They are particularly useful for real-time simulations, due to their limited number of degrees of freedom, and are useful for their predictive power, i.e. forecasting traffic volume, or the effects of additional roads. The most sophisticated models begin to describe behavior near transitions[9] and the formation of clusters[20]. These models are, however, derived from microscopic models of vehicular kinetics.

Microscopic Models

Gas-Kinetic Theory

There are obvious shortcomings in any purely macroscopic description: cars occupy a finite volume, drivers adapt their behavior to varying conditions, etc. Attempts to remedy these weaknesses follow the fluids analogy, and turn to a statistical theory for deeper insights.

A gas kinetics based approach was first developed by Prigogine and Herman[2]. This approach tries to derive the parameters of equation 4 from considerations of the details of real driver behavior. The first notion is that the $V(r,t)$ follows a probability distribution, $f(r,v,t)$, the *kinetic equation.* A second distribution function, $f_0(r,v)$, which describes the *desired speed distribution* is also introduced, and represents the first (time-independent) idealization of driver behavior. After any perturbations, the system will eventually relax back into equilibrium, i.e.

$$\lim_{t\to\infty} f(r,v,t) = f_0(r,v).$$

The simplest dynamical approximation is an exponential relaxation, with a characteristic time, τ

$$\frac{\partial f}{\partial t} = -\frac{f - f_0}{\tau}. \tag{6}$$

The correspondence between this equation, true for distributions, and the relaxation term in equation (4) can be seen just by taking expectations. Typically f_0 is assumed to follow a Gaussian distribution. The final conceptual step in the approach of Prigogine is to introduce an interaction term which affects the distribution f, through the equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + v\frac{\partial f}{\partial r} = \left(\frac{\partial f}{\partial t}\right)_{rel} + \left(\frac{\partial f}{\partial t}\right)_{int}. \tag{7}$$

This foundation, combined with a constraint on the total number of cars, Eqn. (5), and with simple interaction model, serves to predict kinematic waves, the transition to synchronized flow, and the generic form of the fundamental diagram, as in Figure 5. This work served to provide a unifying framework for an entire class of microscopic models, as well as a basis for improving hydrodynamic models.

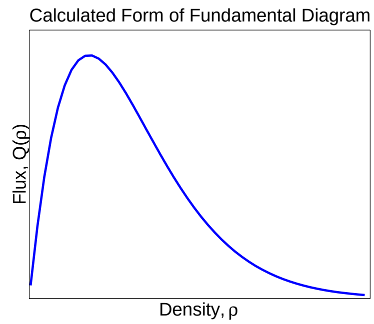


Figure 5: The typical form of solutions to Eqn. (7)

This basic approach has been improved and extended by many workers over the past thirty years, although the essential concepts remain unchanged. The *phase space density*, $\tilde{\rho}(r, v, v_0, t)$, where v_0 is the desired velocity distribution, supplants f as the fundamental distribution. Equation (7) is replaced with the *Paveri-Fontana equation*,

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial r}\left(\tilde{\rho}\frac{dr}{dt}\right) + \frac{\partial}{\partial v}\left(\tilde{\rho}\frac{dv}{dt}\right) + \frac{\partial}{\partial v_0}\left(\tilde{\rho}\frac{dv_0}{dt}\right) = \left(\frac{\partial \tilde{\rho}}{\partial t}\right)_{rel} + \left(\frac{\partial \tilde{\rho}}{\partial t}\right)_{int}. \quad (8)$$

This starting point, together with a Gaussian assumption for v_0, reproduces the mean value equations for density, Eqn. (2), and velocity, Eqn. (4)[1]. Additionally, it yields an equation of motion[2] for the variance, $\Theta(r, t)$, which permits calculation of the *traffic pressure* used in Eqn. (4):

$$P(r, t) = \rho(r, t)\Theta(r, t) \quad (9)$$

The gas kinetic approach has done well in predicting numerous qualities of traffic flow. In particular, workers have explained:

- The formation and evolution of clusters[21]
- The behavior velocity variance near phase transitions[11]
- The spontaneous appearance of jams due to slight inhomogeneities[19].
- Multi-lane velocity dynamics[12]
- The formation of synchronized traffic flow due to perturbations[9, 10].

The last prediction is of great interest, as it provides a tool for determining the effect of entering on-ramp traffic on highway flow. On-ramp traffic can be regulated so as to minimize disruptive effects. An elegant demonstration of gas kinetic predictions is shown in Figure (3). Therein a small perturbation in the flux entering from an on-ramp causes the formation of a large and persistent region of synchronized flow.

[1] This is true only if $\left\langle \xi(r, t) \right\rangle = 0$

[2] See Helbing, [5] for the details of this calculation.

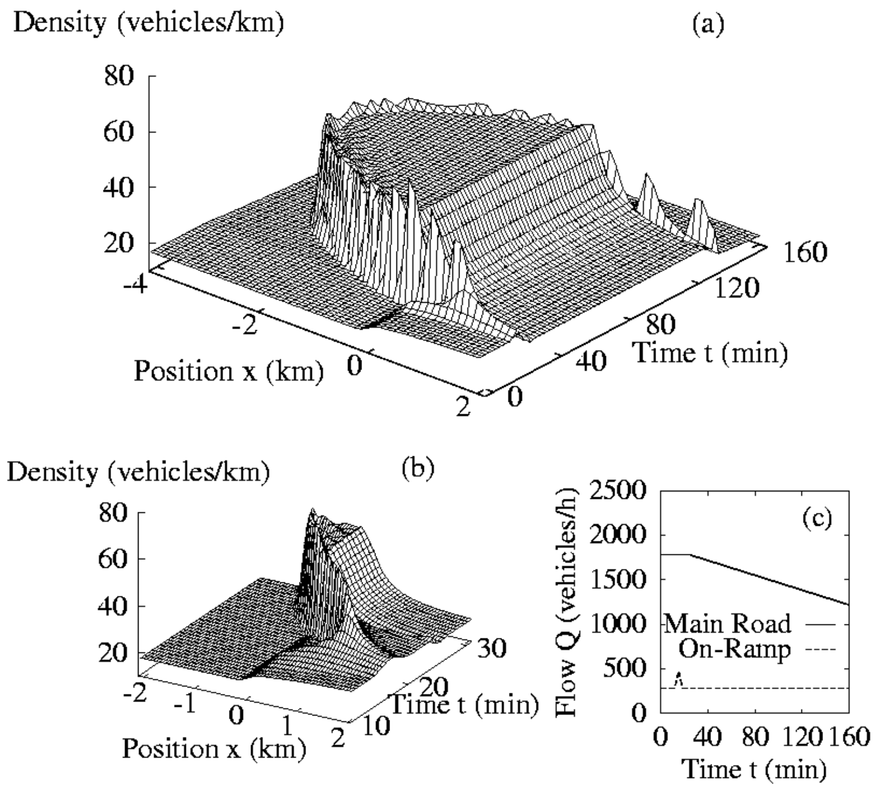


Figure 6: The evolution of lane averaged density resulting from a small peak in on-ramp inflow. (c) Shows the peak at $t \approx 10$, and the subsequent reduction in highway flux. (a) Shows the growth and death of a parabolic shaped region of *synchronized flow*. (b) Shows the initial perturbation and subsequent evolution of the synchronized flow phase evolution in greater detail. The on-ramp is located at $x = 0$, with traffic flowing from left to right.[9]

1. Acceleration: If car velocity $v_i < v_{max}$, and $d_i > v_i + 1$, the car accelerates; $v_i = v_i + 1$.
2. Deceleration: If the next car is too close, $d_i < v_i$, the car slows; $v_i = d_i - 1$
3. Randomization: If $v_i > 0$, the car decelerates with some probability p; $v_i = v_i - 1$.
4. Car Motion: Each car moves forward v_i steps.

Figure 7: Rules for a simple CA model. The i^{th} car has velocity v_i, and a distance d_i to the next car in front. Note that the *order* in which rules are implemented is important.

Cellular Automata

It is intellectually appealing to understand problems from first principles. Unfortunately, human behavior has resisted reductionism, prohibiting a 'deep' understanding of problems like traffic flow. It is nonetheless appealing to search heuristically for minimal models which reproduce observed macroscopic behavior. A convenient way to build toy models is by using *cellular automata* (CA) and powerful computers.

What are cellular automata? They are mathematical idealizations of real systems which live on a lattice in space-time. There is a discrete variable at each site or *cell*. The entire system, is a *cellular automaton*, which is completely specified by the values of the variables at each site. The automaton evolves in discrete time steps, during which the values of all the local variables are updated according to a definite set of *local rules* **and** the values of nearby local variables[3].

In addition to providing a convenient framework for model building, CA models computationally advantageous, and have met with considerable success in modeling observed traffic flow. Qualitatively, CA models are implemented by considering vehicles as cells, and assigning local rules based on driver behavior. Traffic behavior near the critical region can then be understood as an interaction between the individual cells—as a kind of self-organized criticality.

The progenitor of many current CA traffic models is the 1992 Nagle-Schrekenberg (NA) model[15]. In this model, a lane is divided into L cells of certain length, which can be occupied by either one or zero cars. N cars are taken to be on a lane, and periodic boundary conditions are usually used (the lane wraps

[3]For more information on cellular automata, see Wolfram, S., *Reviews of Modern Physics*, Vol. 55, No. 3, July 1983.

around in a circle). At the most elementary level, cars have one internal parameter, velocity, which assumes an integral value, $v \in [1 \ldots v_{max}]$. The motion of cars is then determined by these rules given in Figure 7. There are two approaches to implementing such a model. The first is to model each site as a cell, and the second to model each car as a cell, which knows both d_i and v_i.

This simple model yields surprisingly good results. It predicts a fundamental diagram which agrees with experimental data[15, 4], as well as with other kinds of models, including multi-lane models[18]. They also predict the free flow → jam transition. An example of such a this is seen in Figure 8. Small perturbations give rise to jammed phases which propagate out in both space and time. These are seen in the figure as highly dense areas. Thus far, however, CA models have conspicuously failed to predict any transition to synchronized flow[10].

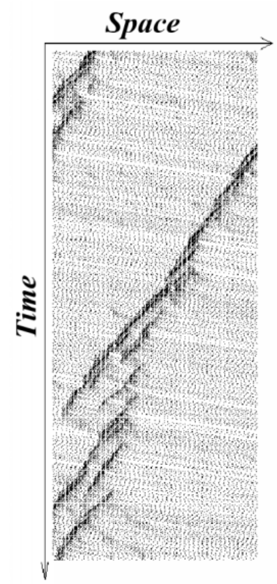


Figure 8: CA predictions showing traffic jam formation, from [18]. The vehicles, represented by dots, move left to right.

The most interesting result from 1-D CA models are their predictions near the critical region, particularly their demonstration that this region is also the regime of maximum throughput[15]. The statistical properties of a system near criticality can be studied through multiple iterations in a CA model. Nagel in particular has studied the critical behavior by analyzing the outflow from an “infinite” jam, i.e. an upstream reservoir. Small (induced) perturbations to this outflow cause new *emergent* jams, whose properties and evolution can be studied. His primary result is shown in Figure 9, which shows power law dependence for the lifetime distribution of emergent jams:

$$P(t) \sim t^{-(\delta+1)}. \tag{10}$$

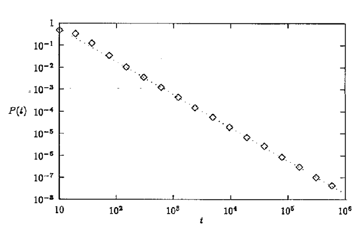


Figure 9: Lifetime distribution for emergent jams, from [22]. The dotted line has slope 1.5 ± 0.01.

This observation may be understood through an analogy to a random walk. One can quantify the number of vehicles involved in a jam through a probability distribution $P(n,t)$. The evolution of this distribution will be determined by Equation 11, where

Physical Quantity	Variable	Relation
Number of Vehicles in Surviving Jams	$n(t)$	$n \sim t^{\frac{1}{2}}$
Size of Jam	$s(t)$	$s \sim nt \sim t^{\frac{3}{2}}$
Lifetime Distributions of Jams	$P(t)$	$P(t) \sim t^{-\frac{3}{2}}$
Total Number of Jammed Vehicles	$N(t)$	$N \sim t^0$
Width of Surviving Jams	$w(t)$	$w \sim t^{\frac{1}{2}} \ln(t)$
Cutoff time	$t_{\text{CO}}(\Delta\rho)$	$t_{\text{CO}} \sim (\Delta\rho)^{\frac{1}{2}}$
Distribution of Hole Sizes	$P_h(x)$	$P_h(x) \sim x^{-2}$
Power Spectrum	$S(f)$	$S(f) \sim \frac{1}{f}$

Figure 10: Table of Critical Exponents at Maximum Throughput, from[4]. t is time, and $\Delta\rho$ is the distance from the critical density

r_{in} and r_{out} are parameters which depend on the rate at which cars enter and exit the jam.

$$P(n, t+1) = (1 - r_{in} - r_{out})P(n, t) + r_{in}P(n-1, t) + r_{out}P(n+1, t) \quad (11)$$

Note that this equation is only exact for $v_{max} = 1$. In that case it describes the growing and shrinking of the jam, in analogy to a drunk staggering from a lamp post, and—eventually— returning. The system exhibits a formal correspondence to a random walk. This immediately leads to the observed power law behavior (Eqn. 10). This random walk analogy can be extended for $v_{max} \neq 1$, and used to calculate a number of critical exponents, shown in Figure 10. Numerical simulations agree well with this crude model, with some corrections due to internal CA dynamics[4], particularly logarithmic spatial corrections ($w(t)$) and the exhibition of $1/f$ noise.

Additionally, certain quantities exhibited scaling forms and data collapse. In particular, the probability of a jam surviving after a time t, given a distance from critical density Δ, scales as:

$$P_{surv} \equiv \int_0^\infty dt' P(t') \quad \sim \quad t^{-\delta} f(t\Delta^{\nu_t}). \quad (12)$$

Simulation results exhibit data collapse as suggested by this scaling law[22], and the calculated exponents—$\nu_t = 2 \pm 0.2$ (δ is given previously)— both agree with random walk predictions.

This work, although self-admittedly "abstract" with respect to the real world, is nonetheless significant. It represents a systematic attempt to explore the details of critical behavior in traffic flow; an attempt to do more than just model traffic flow. Moreover, the definitive correspondence between maximal throughput and criticality leads to an important conclusion: The optimally efficient state will necessarily include traffic jams of all sizes. Attempts to reduce fluctuations (thereby minimizing jams), do increase flux, but they also push the system closer to criticality, rendering it less stable, and making "prediction, planning, and control more difficult."[22]

Many More Models

While I have endeavored to create an exhaustive taxonomy of models, it is inevitable that some be omitted. In particular, I have avoided analytic microscopic descriptions other than the *gas-kinetic* approach. The most general class of these is referred to as *follow the leader*, *car following*, or *dynamical* models. The idea is to construct a microscopic equation of motion for each car, i.e.

$$\frac{d^2 x_n(t+\tau)}{dt^2} = aV(x_{n+1} - x_n). \tag{13}$$

Wherein τ is a characteristic delay time, and V is an *optimal velocity* function. Models of this genre[4], as with all of the other models discussed, correctly predict many aspects of traffic flow. However, insofar as they start with behavioral model for *individual vehicles*, these models are simply a subset of the CA models, generally with fewer variables. While the analytic approach can yield new insights, these models contain no fundamental differences with the models treated earlier, and will not be discussed further.

2D Cellular Automata

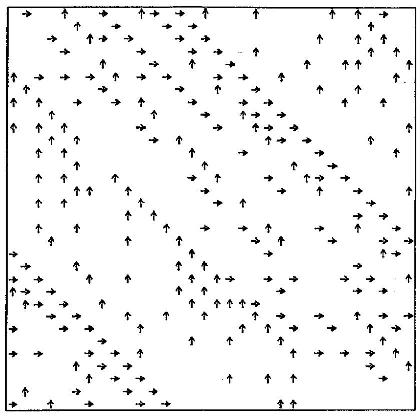
All discussion hitherto has focused on traffic moving on a single road, in one direction. Many of the real-world problems of greatest interest lie in two dimensions. It is difficult to generalize most of the 1D modeling approaches to treat 2D problems, such as a single intersection, networks of intersections such as city streets. The most fruitful approach to this problem has been the use of 2D CA models, although there has been some success with the extension of kinetic models (i.e. lattice-gas)[14], and with dynamical models[5].

The simplest 2D CA model was developed by Biham, Middleton, and Levine (BML). This model consists of a square, NxN lattice with periodic boundary conditions. Each cell represents as car, which moves either to the right or to the upwards, but do not change directions. At odd times cars move upwards, and at even times to the right. The boundary conditions imply 2N conservation rules. Cars move one unit, unless *they are blocked*, in which case they do not move. The CA model is explored by considering the asymptotic states reached from an ensemble of random initial conditions. For this simple model, two different asymptotic states are reached, depending on the initial density (number of cells / number of vehicles. These states are shown in Figure 11.

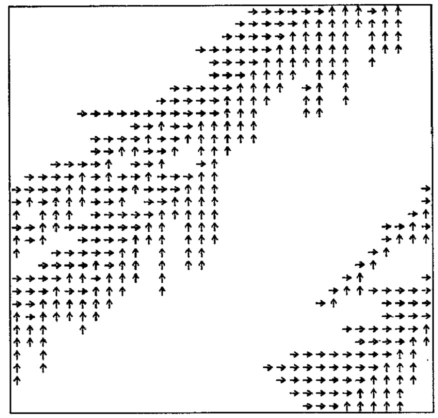
Figure 11a shows essentially free flow at low densities, which is self-organized into diagonal rows of vertically and horizontally moving cars (represented by arrows). This configuration enables the vehicles to obtain maximum flux[17].

[4]These models are discussed in many sources[2, 23, 13, 4].

[5]i.e. MIT's Intelligent Transportation Systems Program[3]



(a) Low Density: $\rho = .25$



(b) High Density: $\rho = .4$

Figure 11: Two distinct asymptotic limits for the BML 2D CA model, corresponding to two different initial densities (ρ). Taken from [17].

Figure 11b shows the jammed phase which occurs for a density higher than some critical density. The diagonal clusters in this figure prevent cars from moving at all.

There are two serious shortcomings in the original BML model. Firstly, the cars never turn. Secondly, it is difficult to determine the critical density, ρ_{crit}, as it is either slowly convergent or non-convergent as a function of system size. The model has been extended to include turning vehicles by Cuesta et. al. [14]. These extended models also predict jamming transitions, although the resultant jam phase is not unique. In addition to reproducing the structure of Figure 11b, they predict transitions to more complicated structures; the nature of the resultant phase is a function of initial density and turning probability.

Some progress has also been made in understanding the nature of the critical region by analyzing the behavior of the diagonal spatial correlations seen in Figure 11. Numerical work by Tadaki [16] has shown that the diagonal correlation length, scales as a power law in low density jams and exponentially decays in very high density jams.

The qualitatively intriguing results of these 2D models need to be connected to real dynamics and experimental results. Although the models discussed here have "fewer direct connections to real traffic flow"[16] than their 1D counterparts, the computational advantages they afford may permit realistic simulations of problems with many degrees of freedom. In fact, supercomputer tools for wide-scale transportation planning are under research and development; The Los Alamos TRANSIMS model is a CA model which aims at real-time predictions for entire metropolitan areas[3].

4 Conclusions

Despite the urgent need for understanding of traffic flow phenomena, systematic progress has been slow. Many models have been reviewed, but none stand out as definitively correct; they all qualitatively predict observed behavior, but none well enough to serve as a detailed planning tool. Continued experimentation, particularly studies designed to refine or exclude models are vital to progress.

Physicists have contributed three main concepts to the study of traffic flow:

- The analogy with hydrodynamics and gas kinetics provided a framework for conceptualizing and constructing models.
- The suggestion that people driving on roads self-organize to a state of maximum efficiency (throughput).
- The idea that this optimal state is a critical region between different *phases* of traffic flow.

The critical region in the transition between freely flowing and congested traffic is inherently unstable; traffic jams can occur as a result of small perturbations. This idea has strong analogies with other "self organized" systems, particularly in granular materials, and suggests the reason traffic flow is difficult to model—strong dependence on initial conditions.

Further study of the details of critical behavior may be the best route to a deeper understanding of traffic flow. However, as Nagel suggests, further optimization of roadways may only make them more vulnerable to catastrophic collapse from any small fluctuation. Perhaps the real problem stems from our planned, political, and ingrained dependence on the automobile.

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