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Models of Social Phase Transitions

Abstract

Social phenomena can exhibit drastic changes with small changes in external parameters—phase transitions. Furthermore, physical models can be applied to social systems because most of the small details can be ignored just as in physical systems. Many models have been used and developed, most notable network models. However empirical data and their interpretation are not yet good enough to make definitive judgments about the validity of these models. This is a good direction for further research.

Introduction

A social system is an interacting group of actors that have complex individual behavior. Although animals and their behavior can form social systems, this paper will only look at human social systems. Everyone is aware of several examples of social systems around him, from groups of friends, employees of the same company, members of the same organization, or citizens of the same country, as well as many others. In each of these systems, and as a general property of social systems, not only do the actors influence each other, but there is usually also a source of outside influence on them.

Social systems usually react gradually to gradual changes in external forces, but as in physical systems, there also can be phase transitions in which the behavior of the individual actors, taken collectively, exhibits a dramatic change with a small or even immeasurable change in external conditions [1]. Political revolutions are a good example of this phenomenon, where a large fraction of the population changes from apparent support to protest within a short period of time, and for reasons that may have seemed rather insignificant before the revolution.

Such problems are interesting to physicists because their methods, which are successful in studying phase transitions of physical phenomena, turn out to be very useful for modeling social systems as well. While it was once scandalous to represent complex human beings by a simple model such as Ising spins, these models are useful because with large systems the individual behaviors become unimportant compared to the collective behavior, and this is true both for physical systems and social systems. In actuality, the physical systems modeled by simple Ising models are already substantial simplifications of the true microscopic behavior, so even averaging out nearly all the complexities of human behavior in simple model is actually quite reasonable.

Models

Unlike physical systems, however, social systems can not be easily experimented on, so one must rely on observational data such as surveys. Since the usual timescale of social behaviors is on the order of months, it is difficult to collect a significant amount of statistics. Finally, because of the great complexity of the human constituents of social systems, there is potential for many competing explanations. These reasons were all given in [2].

This creates a lack of compelling numerical data, which causes the theorists to depend on qualitative descriptions to motivate their models, though observational evidence is also given. This also means that there exist many models for the same kind of phenomenon, each with its own set of assumptions and complications. This paper will summarize and review the major types of model used in the literature for social phase transitions.

Mean Field Theory

The first type of model consists of models based on mean field theory. Two nearly equivalent models are represented well by two papers: one by Levy [1] and one by Durlauf [3]. In Durlauf's model, actors interact with each other and an external field. Each actor's choice in a particular issue is determined by a utility function which is determined by a set of interaction coefficients between this actor and all others as well as between the actor and the external field. Each actor also has internal, in the sense that they are not visible to an experimenter, degrees of freedom. While being honest enough to show this, the author quickly shoves this issue under the rug through the assumption that these internal degrees of freedom are logistically distributed. Then the model is made tractable through the assumption that the actors are completely rational in their decision making. This is a classic economics assumption, though it is not clear that this assumption is justified. His model actually allows for each interaction coefficient to be different, but to do any calculations, he takes the coefficients to all be constant. This is equivalent to the Curie-Weiss model of ferromagnetism because each actor interacts equally with every other actor.

In Levy's model [1], the results are similar, but the method is different. Levy also makes a distinction between observable and unobservable factors. The difference between his and the previous model is that this one takes the variable x, the mean value of choice of a system, as a special variable. For a high enough x, most people will eventually cave in due to peer pressure. Each person has his own threshold x, which leads to a distribution of thresholds. This distribution of thresholds can be gaussian, so as to recover the Curie-Weiss model, or have some other shape. The model produces graphs like this.

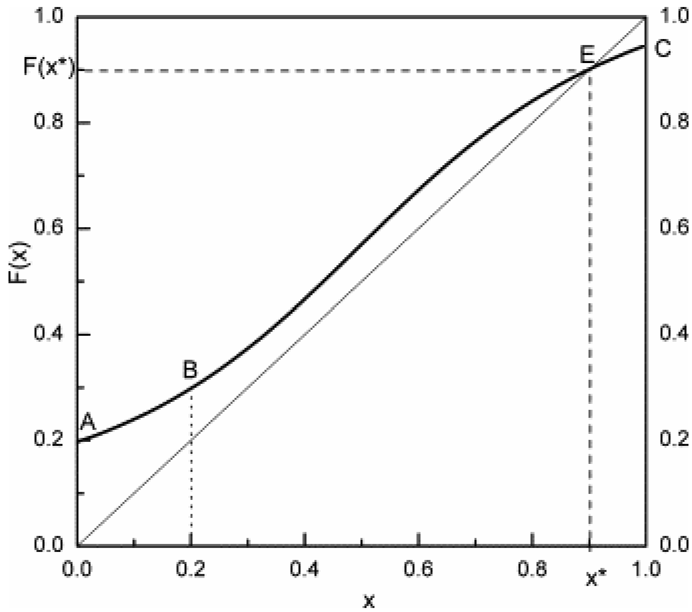

Fig 1. Fraction willing to choose in light of seeing *x* fraction actually choosing [1]

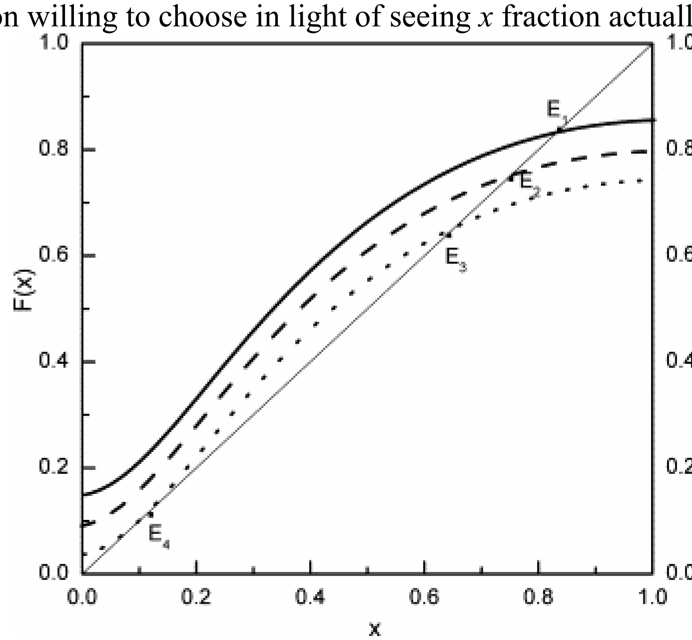

Fig 2. Shift in *F(x)* due to external forces and the appearance of a new fixed point. [1]

F(x) is the fraction of people that would make a certain choice given the fraction *x* that have already made the choice. The places where this curve crosses the 45 degree diagonal line are fixed points because *F(x) = x*. It is clear that with a different curve *F(x)*, a different distribution of thresholds *x*, that there could exist more than one fixed point. As in the Curie-Weiss model, for three fixed points, the outer two would be stable, but the central one would be unstable, directing anyone upwards or downwards. With more than one fixed point, we have the possibility of phase transitions between the multiple equilibria. Which fixed point is the actual equilibrium will depend on the exact utility function at the time of measurement.

The main conclusion of Levy's was that the relative wideness or narrowness of the distribution of threshold is what determined how radical a phase transition was going to be. The narrower the distribution, the more radical the change will be. This makes sense because if all the actors had the same threshold x, then the whole population would change at once if the threshold could be reached. And on the other hand, a wide distribution of the threshold x, meaning that there are people who will not say yes no matter how many others do, and some people that will say yes no matter who does not, leads to a gradual change.

The merits of this model are that it quickly produces the qualitative behavior of phase transitions in a model which is quite intuitive. Not only can this model qualitatively produce the phenomenon that a minority can have a quick revolution, but also that sometimes promising revolutions are defeated [1]. One weakness to this theory is that it is a mean field theory, so that each actor feels the opinions of all others, and feels them all with equal strength. This is clearly not correct, though it could be sufficient. Another more important weakness is that Levy provides no good suggestion on how to determine the effects of external factors, like a government policy, affect the distribution of thresholds. Levy shows the graph $F(x)$ simply being slid up and down as one of the other parameters changing. This is equivalent to every actor having his preferences shifted by the same amount. Since an individual's threshold is a complex function of parameters, the response to an external change could distort the shape of $F(x)$ as well as shifting it.

Majority Rule

Another simple model is a majority-rule system. Papers by Galam [4] are good examples of this method. The basic idea is to use a real space renormalization group procedure to evaluate larger and larger groups of actors, thereby allowing them to influence one another through the majority rule process. In this paper, the actors are divided into groups of four. After one round of renormalization, each group becomes one with the value dependent on the majority present. If there is a tie, then the result can go to the incumbent. This leads to the requirement that an opposition have approximately %77 of the total population in support or the incumbents will stay in power.

The advantages with this model are that it is a clean application of renormalization group theory to a simple model. The model itself, however, is not very realistic. It requires a hierarchical structure and supposes assimilation of ideas through a process of majority-rule voting. This may be a reasonable model for a small number of political systems, but probably not for most social systems. Galam believes, however, that the applicability of this model is much wider than the assumptions would suggest, because the model can be tuned to duplicate the results of other models [5]. In that case, perhaps the attention to get the details more correct in the other models was only a waste.

Network Models

Mean field theory models are not quite correct because it is clear that not everyone should have equal influence on a particular person. The majority-rules systems likewise differ from common sense how actors influence one another. Intuition says that actors only influence one another if they have contact with one another. In an effort to more realistically model social systems, the science of networks became hotly studied. There are a plethora of different networks, each having its own strong and weak points. [6] is an excellent introduction to the use of networks in social systems, and much of this section comes from that.

Random Networks

Random networks were the first to be studied by mathematicians, and there is extensive study done on them, but they are poor models for a social system. Social systems exhibit clustering because of *homophily*, which means that like tends to attract like, so that many similar individuals would be linked together.

Lattice Networks

Some papers model social systems on a plain lattice, often 2D. [7] is an example. This paper discusses the formation of a group of followers around a strong leader, and the conditions on the group such that the strong leader and his followers could become the majority opinion.

Small-World Networks

A step up in complexity is called the small-world network. It consists of a lattice with nearest neighbors and maybe next nearest neighbors connected. A few of the edges, however, are then rewired to a different random node. This keeps the ordered nature of the network, while the long-range connections reduce the effective size of the network, because a person could find another by only traveling through a few steps.

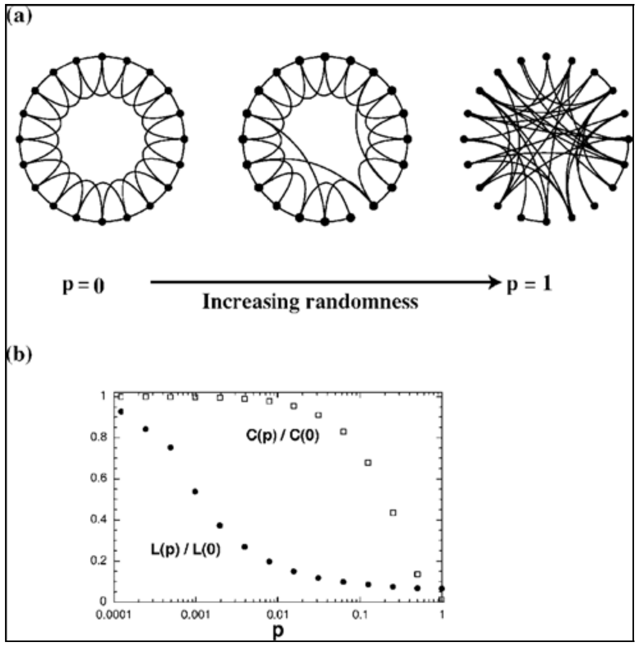

Fig 3. (a) Small-world networks, (b) Clustering and path length vs small-world randomness parameter *p* [6]

The figure shows small-world networks with a different number of random connections. The graph shows that small-world networks are both highly clustered and have low average shortest path length. Both those features are important features of real social networks.

Small-world networks are built up from a lattice which is then modified. However, it is possible to create networks with those two properties without starting with a lattice, which would seem to be a better starting point. Such networks are called *affiliation networks* because they are created by linking individuals to a number of groups. Then two individuals are considered to be in contact if they belong to the same group.

Scale-Free Networks

Another important feature of real social networks is that there exist a few individuals with many contacts while most others have only a few. The probability of finding a node with degree *k* is well described by a power law

$$P(k) \sim k^{-\alpha}$$

On a log-log plot, a power law is a straight line, but a Gaussian distribution would have a curve to a sharp cutoff. See the figure below.

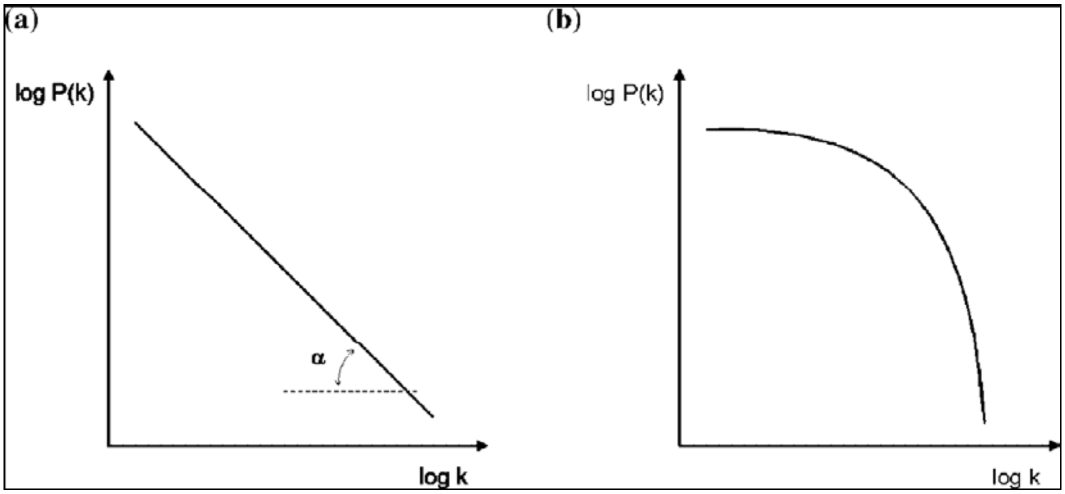

Fig 4. Log-log plots for *P* vs *k* (a) power law, (b) gaussian distribution

The lack of any special length scale for a cutoff for the power-law graph, while the gaussian graph certainly has one, is what gives networks that obey this law the name *scale-free networks*. This scale-free behavior has been found experimentally in dozens of networks as listed in [6]. Some examples are genetic regulatory networks, neural networks, collaboration networks of scientists, and the link structure of the world wide web.

Future Directions

What is currently lacking right now in this area is some hard numerical data that can be used to critically test models. While it is true that much progress has been made in the way of developing and studying networks, even to the point that there are algorithms to create networks with custom properties [8], I think this progress with networks has served to distract many from the problems yet unsolved. Just as mean field models, that completely ignore network topology, are mere qualitative descriptors, models that focus solely on the network topology can miss the fact that a network, even an exactly correct one, needs a good interpretation for it to be of use. As Barthélemy says, “purely topological models are inadequate and there is a need for a model which goes beyond pure topology” [9].

I also believe that Roehner [10] has laid out an interesting path by proposing a close analogy between the liquid state and properties of socio-economic systems. In particular, that what the field really needs is a good way to study and measure the interaction strengths in social systems. Of course it is nearly impossible to simply measure interaction strengths directly like physicists and chemists can using IR spectroscopy and the like, but other aggregate quantities could be taken as indicators of interaction strength. In his paper, he uses suicide rate as an indicator of the degree of social isolation present in certain groups. While certainly only a small minority of people commit suicide, the ones that do reveal the existence of extreme social isolation in that society, though probably mostly on the tail of a distribution of social isolation.

Therefore, the direction I feel is most needed for this field is for there to be a two-pronged attack aimed at making models empirically testable. One prong is to develop better experimental techniques to measure more accurately social variables, and the other is to research the connection between fundamental coupling constants and aggregate variables.

Conclusion

The field of social phase transitions has had quite a bit of study. Many models have been developed, and network models have been especially fruitful for research. However, the gap is still not closed between experiment and theory because the data are both hard to take and hard to interpret. I propose that closing this gap should be a priority for the field.

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