Cosmic Strings

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May 11, 2021

Abstract

This essay discusses the generating, gravitational description and observable effects of cosmic strings. In this essay, cosmic strings, as linear topological defects, are shown to be generated by the spontaneously symmetry breaking of gauge fields during the inflationary era. By studying the metric around a single infinite-length straight cosmic string, we explain carefully how the cosmic string changes a flat space into a conical space and results in the deflection of light passing by. We explore the double images, accretion effects and CMB anisotropy caused by the cosmic strings and introduce the efforts to detect cosmic strings based on the observable effects.

1 Introduction

Phase transitions can happen in the early universe. During the phase transition, topological defects such as domain walls, cosmic strings and monopoles emerge as a result of the spontaneously breaking of gauge symmetries. Since the topological defects were hypothesized in 1970's (see a review like [1]), theoretical studies and observational researches have been carried out in an effort to verify their existence.

Cosmic strings are one type of the topological defects generated in the early universe phase transition. Very similar to vortex lines in superfluid Helium and superconductors, cosmic strings are linear topological defects in the 3-dimensional universe space. The linear structure of cosmic strings modifies the spacetime in an interesting way, changing a flat space into a conical space, which leads to detectable signals in the CMB, 21cm and optical astronomical observations. Since cosmic strings can be traced back as the structure in the inflationary era, detecting cosmic strings may be essential for the study of the early universe.

In this essay, I will first introduce the mechanism of phase transition in the early universe and explain the generating of cosmic strings. Then the gravitational description of cosmic strings will be discussed, serving as a bridge to connect the nature of cosmic strings and the detection. Then, I will talk about the detectable effects of cosmic strings including double imaging, accretion and CMB anisotropy.

2 Generating of cosmic strings by spontaneously symmetry breaking in the early universe

The early universe has higher temperature and denser matter distribution over the space than the universe at present. In such a situation, it is effective to use quantum fields to describe the matter and its evolution in the early universe.

Let us now study a simple case where topological defects can be generated.[1][2] Consider a complex scalar field $\phi(x)$ coupling with a $U(1)$ gauge field $F^{\mu\nu}$

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_\mu\phi^\dagger D^\mu\phi - V(\phi), \tag{1}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu - ieA_\mu$. The potential of the scalar field $V(\phi)$ has a Mexican-hat shape

$$V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - \eta^2)^2, \tag{2}$$

demonstrated in Fig.1. From the shape of the potential, we know that the minimal potential energy is achieved when the scalar field is at the bottom, $\phi(x) = \eta e^{i\alpha(x)}$. And when the complex scalar field is at the ground state (the bottom of the potential), it has to choose a phase. If we expand the scalar field $\phi(x)$ at the ground state, writing it in terms of a new field starting from the bottom $\phi(x) = \eta e^{i\alpha(x)} + \tilde{\phi}(x)$, it is impossible to multiply a random phase factor to the new field $\tilde{\phi}(x)$ such that the Lagrangian is invariant. In this sense, the $U(1)$ gauge symmetry is broken at the ground state.

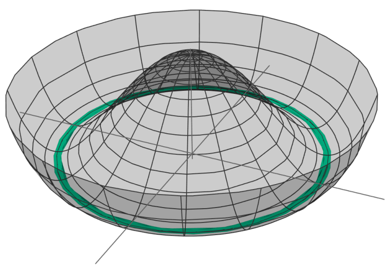


Figure 1: The Mexican-hat potential with the ground state in green colour.

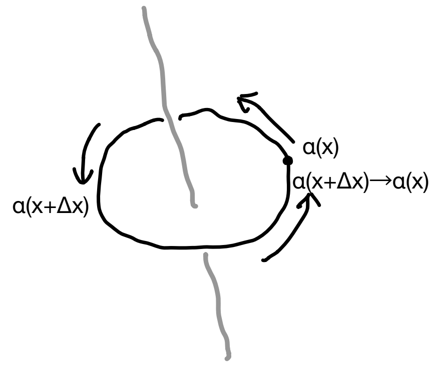


Figure 2: The phase returns to itself plus $N \cdot 2\pi$ as the trajectory moves along a circle and goes back to the start point.

Now suppose the universe is at the ground state. Because the phase factor $\alpha(x)$ is a function of spacetime coordinates, it continuously varies along trajectories of the three dimensional space. After moving along a loop, the phase factor should return to its original value plus $N \cdot 2\pi$, because the complex scalar field $\phi(x)$ needs to be single-valued at each point (See Fig.2.). When N is zero, the situation is trivial.

Things become interesting when N is non-zero. For example, if $N = 1$, the phase factor gains a 2π as moving along the loop. Since the phase factor varies continuously, we can make the area enclosed by the path smaller and smaller while the phase change along a loop remains unchanged. However, we cannot not continuously make the loop reduce to a point, because a point structure changes the topology of the loop and destroys the phase change in one circle. To solve this issue, some point in the region enclosed by the loop should give $\phi = 0$ with the phase factor not defined. Such a point serves as a topological defect.

Because the space is three-dimensional, we expect the topological defects have a linear structure, which are usually called Cosmic Strings.

When did the phase transition occur? This question may not be easy to answer. Instead, we can ask the inverse question: was the broken gauge symmetry restored earlier in the universe? The answer is yes by the current physics theory.

The complex scalar field we have discussed above is usually thought to be the Higgs field. At high temperature, we can add a temperature-dependent term into the effective potential $V(\phi)$ such that, (see section 9 of [1])

$$V_T(\phi) = AT^2|\phi|^2 + \frac{1}{2}\lambda(|\phi|^2 - \eta^2)^2, \tag{3}$$

of which the effective mass of ϕ is

$$m^2(T) = AT^2 - \lambda\eta^2. \tag{4}$$

At high temperature, the effective mass is positive to make the vacuum stay at $|\phi|= 0$, where the gauge symmetry is restored. As the temperature decreases, the mass term turns negative and the symmetry breaks.

Simply, we can understand the temperature-dependent term as a thermal energy, the interacting energy among particles and been scaled by T^2. But rigorously, such temperature-dependent terms are added due to higher order quantum corrections to the classical field potential $V(\phi)$. Cosmologically, the phase transition happens when the higgs field rolls down the potential hill to break the $SU(2)_L \times U(1)_Y$ gauge symmetry and reheats the universe. It happens at the energy scale about $300GeV$. [2] Current inflationary cosmology holds the view that the structure of the universe is seeded during the inflation, hence cosmic strings may also contain the structure information of the universe at the inflationary age. In addition, some recent works (for example see [3]) intend to connects the idea of cosmic strings to super string theory, which shows the potentially significance of cosmic strings in high energy theory.

Till now we have seen the mechanism that phase transition in the early universe may generate cosmic strings. Because we are confident on the symmetry breaking and phase transition theory, we might also believe the existence of cosmic strings. However, the existence of cosmic strings has not been confirmed. It is also difficult to answer the question that if the topological defects are more energetically favored than the trivial case where the universe do not have topological defects (even after a phase transition). Yet, it is commonly thought that cosmic strings are stable (for example, see section 1 of [1]).

To detect cosmic strings in reality, we need to study their gravitational properties first.

3 Gravitational properties of cosmic strings

3.1 The mass parameter

Firstly, let us define the **linear mass density** of cosmic strings. Suppose there is just one infinite straight string in the space. We use cylindrical coordinate to describe the space, where the z-axis lies on the string. Then the mass of the string per length is defined as

$$\mu = \int \mathscr{H} r dr d\theta, \tag{5}$$

where $\mathscr{H}$ is the Hamiltonian density. In the model given by the Lagrangian (1), the mass density is

$$\mu = \int_0^\infty \int_0^{2\pi} r dr d\theta \Big(|\frac{\partial\phi}{\partial r}|^2 + |\frac{1}{r}\frac{\partial\phi}{\partial\theta} - ieA_\theta\phi|^2 + V(|\phi|) + \frac{B^2}{2} \Big), \tag{6}$$

with $\vec{B} = \vec{\nabla} \times \vec{A}$.[2] However, there is no explicit form of the linear mass density, and a precise definition of the linear mass density may require a full treatment of particle physics Lagrangian. To make measurements simple and effective, the linear mass density is usually regarded as an unknown parameter to be determined.

Conventionally, the mass of cosmic string is specified by a dimensionless parameter

$$\frac{G\mu}{c^2} = G\mu, \tag{7}$$

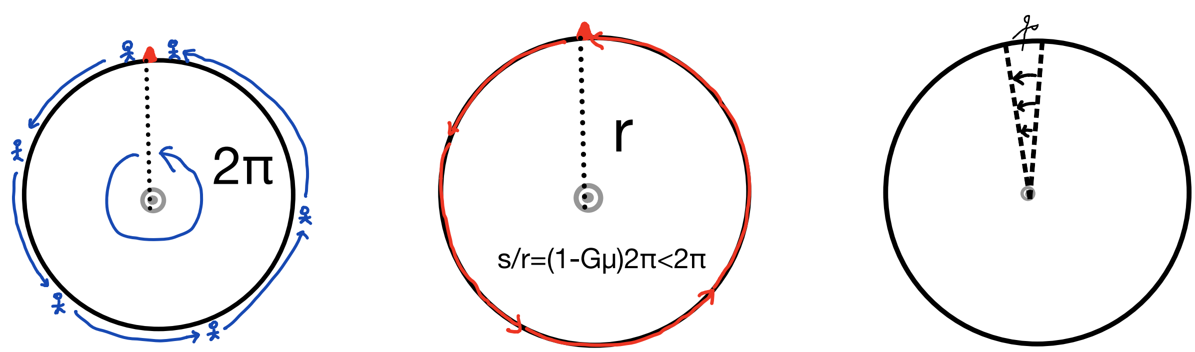
where μ is the linear mass density defined above and G, c being the Newton's gravitational constant and vacuum light speed respectively. Usually we set $c = 1$. Since the tension of a string is quantified by the same parameter, $G\mu$ is also called cosmic string tension.

3.2 The conical space

The definition of μ in (6) reveals that the mass density of cosmic strings is positive. Hence, we expect a gravitationally attracting effect from the cosmic string. In fact, in the weak field approximation $G\mu \ll 1$, the metric of the spacetime with one infinite straight string is found to be [4]

$$ds^2 = dt^2 - dz^2 - dr^2 - r^2(1 - 4G\mu)^2 d\phi^2. \tag{8}$$

The metric is similar to a flat Minkowski spacetime except that the azimuthal angle is slightly stretched. Such a spacetime is call conical.



(a) Circular motion observed by a distance observer. The angle per circle is 2π.

(b) Circular motion measured by the moving person. The angle per circle is given by the quotient of the distance travelled divided by the radius, which is smaller than 2π.

(c) "Conical space"

Figure 3: Demonstration of the conical structure along a cosmic string.

To understand the conical space, imagine a traveller moves along a circle centered at a cosmic string, depicted in Fig.3. As this traveller performs a circular motion and returns back to the original point (labelled by a red tower), a distant (maybe living in a higher dimension) observer who construct the z, r, ϕ coordinates find that the total angle travelled along a circle is 2π. Meanwhile, the traveller is able to measure the distance walked through. After the traveller returns to the original point (red tower), the angle travelled can also be found by dividing the total distance along the trajectory and the radius of the circle. However, the metric (8) tells us that the angle measured by the traveller is a fraction of $4G\mu$ smaller than 2π, which means some angle is removed from the local space outside of the string, and the space is similar to a "conical structure".

As depicted in Fig.3(c), a conical surface can be made by cutting off a small sector of a circle and glue the cuts. By an analogy to the conical surface, the space outside of the string has the same internal geometry, hence it is called conical. For a cosmic string, the cut off fraction is $4G\mu$.

3.3 Geodesics

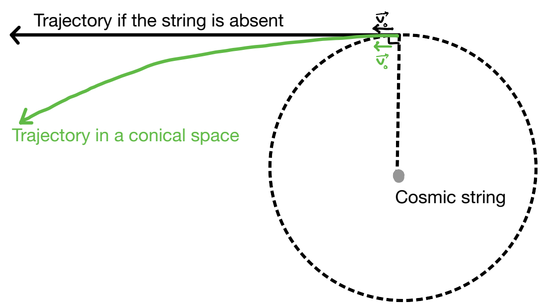


Figure 4: Deflection of the trajectory along geodesics due to the presence of cosmic string.

Now in the conical spacetime, we study a motion along a geodesic. Consider an object has an initial velocity tangent to the circle, as depicted in Fig.4. In cylindrical coordinate, the initial velocity is $\vec{v}_0 = v_\phi \hat{e}_\phi$. Calculate the acceleration in the radial direction by using the metric given by (8)

$$\frac{d^2r}{d\tau^2} = \Gamma^r_{\phi\phi} v_\phi^2 = r(1-4G\mu)^2 v_\phi^2. \tag{9}$$

The radial acceleration is positive, which is consistence with our observation because the radial velocity needs to increase from zero to enable the object move away from the center. However, the result (9) also shows non-zero $G\mu$ makes the radial acceleration smaller, resulting in a deflected path. In Fig.4, the black curve is the trajectory of a flat space ($\mu = 0$) while the green curve is the deflected path due to the presence of the cosmic string.

Because the trajectory curves in the $-r$ direction, we expect the cosmic strings to serve as a source of gravitational lensing. Due to the linear geometry of the string, a gravitational lens induced by cosmic strings has a linear structure.

The lensing effect of cosmic strings makes several interesting consequences, which are later used to detect cosmic strings.

4 The detection of cosmic strings

Till now we have studied one infinite cosmic string as a toy model to derive its gravitational properties. It should be noted that during the phase transition of the early universe, the number of cosmic strings generated is not just one, but many. The large amount of cosmic strings form a gas and evolve with the expansion of the universe. Hence, the astrophysical effects caused by cosmic strings may be found at each region of the large scale structure, while most cosmological surveys are about doing statistics over the large scale structure to screen specific structures predicted by theories.

4.1 Double Images

A direct effect of the gravitational lensing is the double images of a star or galaxy, when a cosmic string is in the middle of the observer and the light source. For an infinite string, because of the 1-dimensional geometry, there are two images of a star as the light from the star passes by both sides of the string and reaches the observer.

The double images of galaxy serve as an important and a direct detection of cosmic strings. Occasionally, some seemingly double images of a galaxy are found by astronomical observation. However, it is difficult to tell if the double images are from one distinct galaxy or a pair of galaxies living in the vicinity of each other. See Fig.5 for an example.

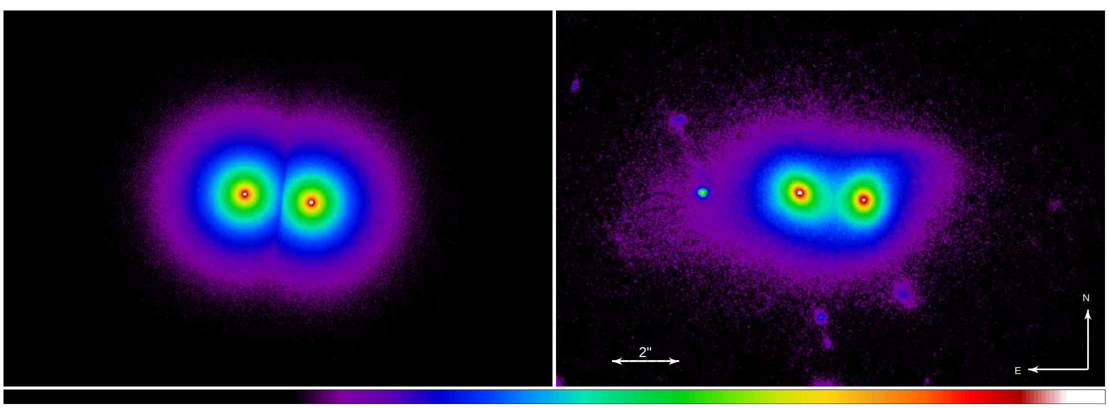
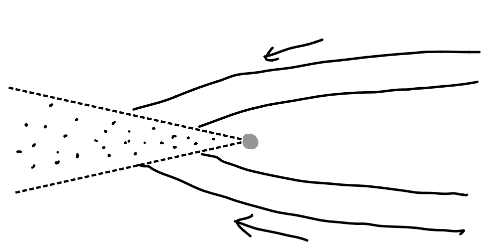


Figure 5: (This is Fig.1 of [5]) A set of double images of CSL-1 galaxy. The left figure is the numerically simulated image while the right one is from the observation of Hubble Space Telescope. Colors in the two figures label the regions with different light intensities. By carefully comparing the contours of the simulated figure and the observed one, researchers argue that the CSL-1 is not a set of double images caused by a cosmic string because the two images observed are not in a similar shape as predicted by the simulation.[5]

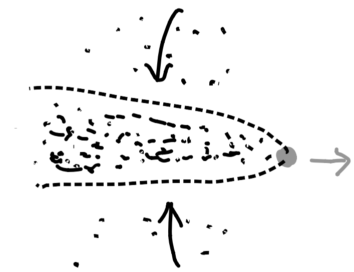
If we put aside the problem whether two images are from the same galaxy or not, it is possible to do a full sky survey and study the correlations of galaxy distribution. A such work gives an upper bound on the cosmic string mass: $G\mu < 3.1 \times 10^{-7}$. [6]

4.2 Matter Accretion

When the lensing effect performs on matter particles, there will be matter accretion around a cosmic string. Consider a relative motion between a string and background particles. In the reference frame of the string, particles passing by will be deflected into a wedge-shaped region behind the string, illustrated in Fig.6(a). Now in the background reference frame, the string is moving and the background particles behave to be attracted and gather along the path of the string's motion, creating a wedge-shaped "tail" to be detected, depicted in Fig.6(b). The wedge-shaped structure induced by motion strings is called **cosmic string wakes**.



(a) Accretion happens behind a moving infinite string, viewed in the frame of the string.



(b) When viewed in the background reference frame, a moving string displays a wedge-shaped "tail".

Figure 6: Illustration of the formation of a wake behind a moving string.

The work [7] has carried out a numerical study on the wake formation. In this work, researchers have simulated the dark matter particle density distribution in two conditions: there is a moving cosmic string; the cosmic string is absent. By doing a so-called "3D ridgelet transform", they transform a spatial density distribution into a wavelet function space. Similar to Fourier transformation, it is able to catch out a plane overdensity structure in the spatial particle distribution by comparing the "Ridgelet coefficients" R_ρ in the two conditions. Part of the result is given in Fig.7.

Since cosmic strings are generated in the inflationary age, the detection of cosmic string wakes is an effort to decode the density perturbation at high redshift (early in the universe history) before the formation of galaxies. A useful tool to detect the light element density distribution is the hydrogen 21-centimeter spectrum. It is expected that the 21-centimeter signals will also be helpful to find the cosmic string wakes in the high redshift universe. This topic is been heavily studied in recent years. For example, [8] gives a calculation of the power spectrum of the 21cm radiation sourced by cosmic string wakes.

In addition to the wakes, the **formation of galaxies** may also be credited to the accretion effect of cosmic strings. A loop cosmic string may be a start point of the formation of a galaxy. As the background matter particles accreted by the loop string, a spherical planer overdensity region emerges, and finally grows into a galaxy, suggested in [9].

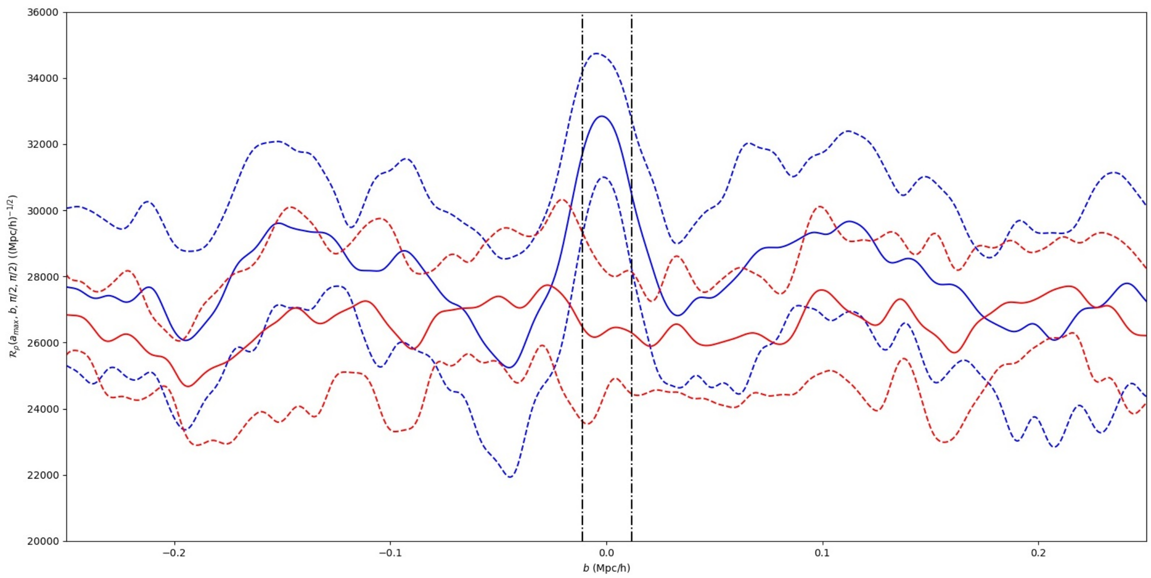


Figure 7: (This is Fig.13 of [7]) The "Ridgelet coefficients" R_ρ in the conditions with a moving cosmic stringc(blue curve) and the condition without a string (red curve). The dashed curves show the standard deviation of the solid curves. The horizontal axis b represents a spatial parameter. We can see that in the case with a moving string, the is a signal of overdensity in a wake form at $b = 0$.[7]

4.3 Cosmic Microwave Background anisotropy

Cosmic Microwave Background (CMB) is the map of photons at the last scattering surface. Since the universe is assumed to be homogenous and isotropic at the large scale, the CMB map shows a nearly isotropically distributed black-body radiation at 2.17K. However, some astrophysical process such as the CMB photons scattering off galaxies may cause non-isotropy on the CMB map, which is named CMB anisotropy. Moving cosmic strings also serve as a source of CMB anisotropy.

The mechanism of CMB anisotropy sourced by the moving cosmic string is suggested by [11]. Similar to the double-image mechanism, photons from the same region of the last scattering surface will also have two images when they passing through both sides of a string and reach an observer. However, when the string lens is moving perpendicularly to the line of sight, the two signals get redshift or blueshift differently. Just as in Fig.8, in the reference frame of the string, the source (S) of photons and the observer (O) are moving in the same direction. If the space is flat, then the distance between S and O is invariant. However, since the space is conical, the leftward parallel motion of S and O decreases their distance on the left hand side and increases their distance on the right hand side, making different Doppler effects on the photons travelling along the two trajectories. Hence, on the CMB map, a temperature discontinuity happens as the two images of photons has different redshifts, which constitutes the anisotropy.

This mechanism indicates that the anisotropy sourced by the cosmic strings is in small

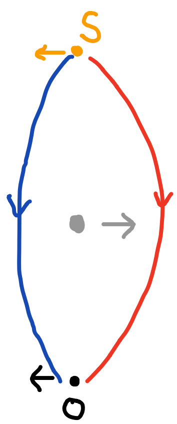


Figure 8: A demonstration of temperature discontinuity caused by a moving string lens. The light travels on the left get blue-shifted as the the distance between source (S) and observer (O) decreases in the conical space. On the other hand, photons travel along the red path get red-shifted.

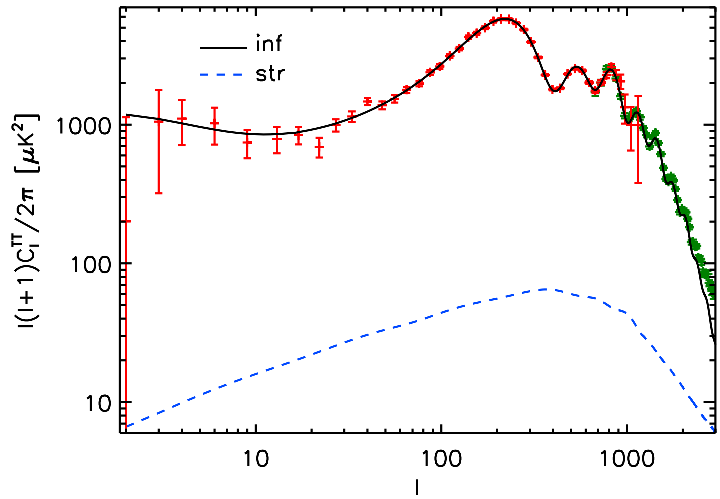


Figure 9: (This is Fig.1 of [10].) The multipole coefficients C_l of CMB data provided by WMAP7 (the upper solid curve) versus the multipole coefficients sourced by the string model (the blue dashed curve) computed at about $G\mu \sim 10^{-7}$. We can see that the anisotropy sourced by strings is weak compared the anisotropy observed. Also, it is peaked about $l \sim 300$ in this model.

scale. In the work [10], researchers has made a parameter space study on the possibility of finding anisotropy sourced by string in the WAMP data. They compare the string template function with the CMB multi-pole coefficients (Fig.9) in different sets of cosmological parameters. After doing a likelihood analysis, the researchers put an upper bound of the mass parameter: $G\mu < 1.7 \times 10^{-7}$.

5 Summary

In this paper, we have talked about the mechanism of cosmic string generating, the basic properties about the cosmic strings and the astrophysical effects of the cosmic strings.

The nature of cosmic strings are topological defects generated at the phase transition in the early universe. Making a careful and comprehensive study on the cosmic strings requires the phase transition theory and high energy theories as well as gauge theories at high temperature and maybe topology. Hence the cosmic strings are an interesting theoretical topic in a broad range of physics.

Cosmic strings also have important implications on the structure formation of the universe. They leave signals in various astronomical detection scopes including 21cm spectrum and Cosmic Microwave Background, enabling the detection of the structure in the early universe, high redshift universe and the universe after the galaxies formed. Thus, cosmic strings provide possibilities to probe the universe at different ages.

The current observational upper bound of the mass parameter is $G\mu < 10^{-7}$ and there is yet no sound evidence to prove the existence of cosmic strings. The future of the detection of cosmic strings remains uncertain. After all, we can still keep cosmic strings as a possibility and expect the 21cm observation provides more evidence to prove or falsify cosmic strings in the future.

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