Towards an Emergent Macroeconomics

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Abstract

Despite substantial interest after the financial crisis, interest in deriving macroeconomic phenomena as emergent quantities has waned. I will argue that the existence of robust scaling laws and fluctuations on the order of economic quantities themselves are the kinds of things that should be explained by an emergent framework. To make this plausible I will show how emergent frameworks in biology and population modelling are able to explain similar phenomena.

Contents

1 Introduction

Deriving usable emergent macroeconomic predictions from a description of the agents which constitute the economy is the economic equivalent or deriving a testable theory of gravity from a quantum field theory. Like quantum gravity, the theoretical problem is hard. Moreover, an emergent macroeconomics faces the challenge of generating currently testable predictions that cannot be derived from the current paradigm. Unlike quantum gravity, the regime which cannot be explained by the conventional paradigm, and hence the regime which is the target of the emergent theory, is not entirely clear.

The purpose of this essay is to argue that appealing places to look are:

1. *Where we see order emerging from heterogeneity*:
   The examples I will focus on are: scaling law distributions in the economic data of cities [1] and wealth distribution [2].

2. *Where fluctuations are as important as the steady state dynamics*:
   Here I focus on the the departure from Gaussian behaviour in economic data [3] and how agent based models can provide insight into these departures - [4].

There is no consensus on the right theoretical framework to explain these data. The purpose of this essay is to make plausible the view that the kind of theory we need is one where the macroeconomic data emerge as emergent properties. The strategy here will be to argue that the compelling observations above have analogues in other fields that have been explained by emergence based frameworks. In particular I will argue that:

1. Metabolic scaling laws in biology [5] illustrate how models of emergence can be used to derive scaling laws that are present despite enormous variation amongst the sample population.

2. Agent based population models are able to generate fluctuations from *within* the model [6], [7] that lead to behaviour that is very difficult to reproduce in the current macroeconomic paradigm of Dynamic Stochastic General Equilibrium (DSGE) modelling [3].

2 What economic facts can we hope to explain?

2.1 Order from heterogeneity

One of the hallmarks of emergent theories is *universality* - the existence of a relationship, usually for statistical quantities, that holds despite large variations in the sample population. As pointed out in West's book [8], the economic data of cities provide some of the most striking scaling relationships in economics - fig. 1. Income, GDP and patent production all appear to scale

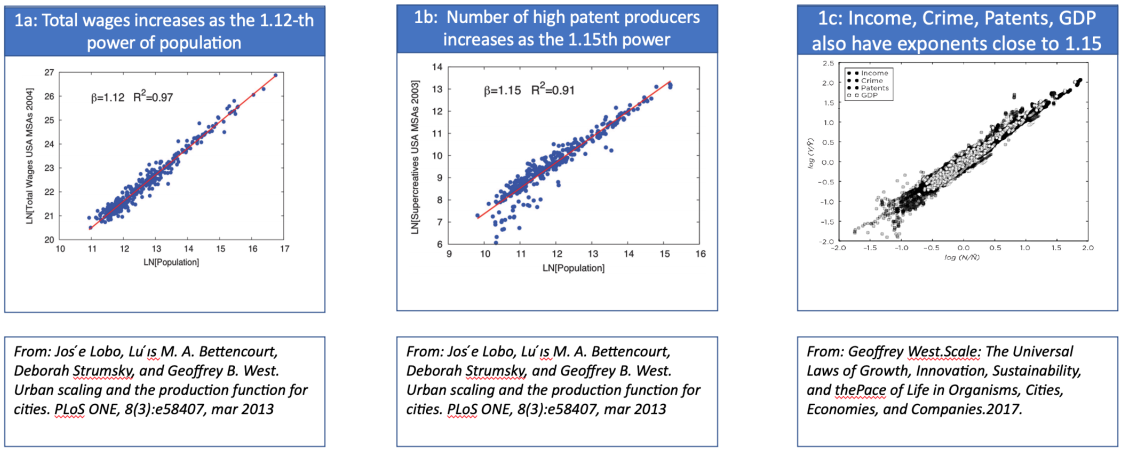

Figure 1: Scaling laws of total income (1a), number of high patent producers (1b) and other economic variables (1c) all are approximately power law distributed with an exponent of ~ 1.15. Figures 1a. and 1b. are taken from [9] and figure 1c. is taken from [8].

as a power-law with an exponent that is *super-linear*. This suggests that it is a regularity of the *interactions* of the increasing population that are responsible for the fact that economic activity reliably increases by 15% more than the population in a city. Interestingly, the density of physical infrastructure scales with an exponent of 0.85 - fig. 2

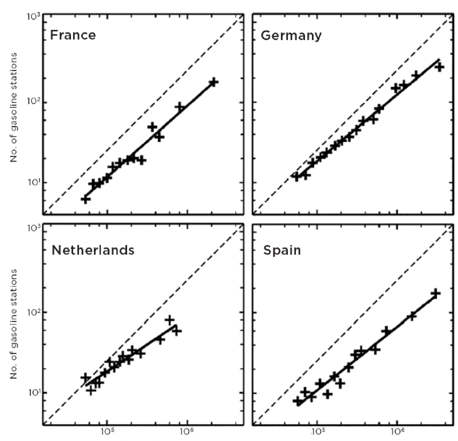

Figure 2: Scaling laws for the number of petrol stations with population in different European countries taken from [8]. The exponent in each case is close to 0.85. This illustrates how physical infrastructure becomes more efficient as a city grows.

Scaling laws also apply to income and wealth distributions, where the empirical data suggests a power law distribution for the top 10% of the population, and an exponential distribution for the remaining 90% [2]. Proposing a framework in which to understand this scaling was one of the first success of the new field of 'econophysics' and the data for US income and wealth are reproduced from the review [2] in fig. 3.

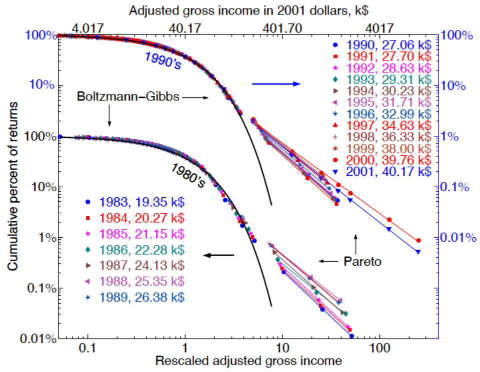

Figure 3: The distribution of income as reported on tax returns (x-axis) with cumulative share of the population (y-axis). The graph shows the transition from an exponential to a power law distribution for the top decile of earners. The data is taken from [2] and similar data was obtained for wealth and for the UK.

There is an appealing intuitive explanation for the exponential part of the distribution. If we assume that money is conserved, then we can draw an analogy with classical statistical mechanics. In a system whose sole constraint is the total energy U and the number of N equivalent particles, the distribution of energy which maximises the entropy amongst those particles is exponential in the limit $N \to \infty$. In the economic case, it is appealing to argue that a fixed quantity of money is shared between the N agents of the economy, and so the distribution of money should also be exponentially distributed. Clearly, this argument breaks down if we consider the economy to be in a non-equilibrium state. However, the exponential distribution is found across time periods and economies for 90% of the population, suggesting that there may be a way to think about income as being close to equilibrium.

2.2 Systemic Fluctuations

The other piece of economic data that is natural to explain with an emergent framework is the *generation of fluctuations*. Economic output and financial assets exhibit fluctuations that are comparable in size to the level of the variable fluctuating. This is in contrast to what we would usually expect from an aggregated system since the central limit theorem implies that the fluctuations should scale as the square root of the output or asset value. Tests of macroeconomic data repeatedly find fluctuations larger than that implied by the central limit theorem [3]. In financial markets, just looking at a time series of a representative index shows that the fluctuations are comparable to the quantity of interest - fig. 4, and the departures from the central limit theorem are well documented.

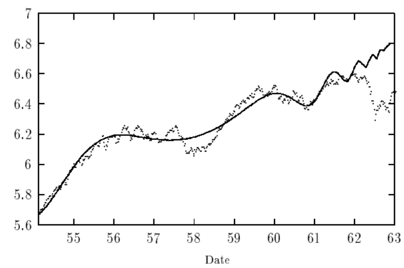

Figure 4: Financial indices exhibit fluctuations that are comparable to the value of the index. The data here is for the market crash of 1962 to illustrate the scale of fluctuations both in 'normal' periods and during a crisis. Data taken from [10].

3 Order from heterogeneity - the paradigm of metabolic scaling

Now that we know the kind of economic data that we could hope to explain, we can ask what theoretical tools are available to us. A compelling precedent for the application of emergent models in economics is the explanation of the metabolic scaling given by [5]. Here I will give a brief review of this work, and end by indicating the attempted extensions to economic phenomena.

3.1 The model for metabolic scaling

As noted by West, Enquist, and Brown [5], the metabolic rate of organisms scales as the 3/4-th power of their mass over a remarkable twenty one orders of magnitude. Just like for cities the scaling law holds despite enormous variations in organisms. To explain this [5] assumed:

1. That the supply network of essential metabolic resources (i.e the circulatory network in animals or the vascular system in plants) has an *invariant end-unit*. This means that the size of the last branch of the network is the same for every organism.
2. That the supply network is *space-filling* - that is, it reaches all of the nodes of the network.
3. That the network *minimises the energy* expended in the flow of resources.

Since the network does not loop round, we can consider the network as a series of branchings. If we let each tube at level k branch into n_k other tubes and note that every branch before the terminal unit splits into the next level $k+1$, the total number of branches at the second level is just $n = n_k n_{k+1}$. Extending this argument to the total number of branchings N_k at level k:

$$N_k = n_0 n_1 ... n_k \tag{3.1}$$

If we model the branches as spherical, and let the velocity through the branch at level k be u_k and its radius r_k. Then we can use the fact that the fluid of resource is conserved in the system to relate the flow at any branch level, Q_k, to the flow at the invariant terminal unit:

$$\dot{Q}_0 = N_k \dot{Q}_k = N_k \pi r_k^2 u_k = N_T \pi r_T^2 u_T \tag{3.2}$$

Where T is the level of the terminal unit. Since resources cannot build up in the end-points we must have the resources being used at the same rate as they are delivered so we have that the metabolic rate $B \propto \dot{(Q)}$. Thus, we have related the total number of branches to the metabolic rate.

We now need to characterise how the network branches. Let β_k be the ratio of radii between successive levels, $\beta_k \equiv \dfrac{r_{k+1}}{r_k}$ and γ_k be the ratio of lengths: $\gamma_k \equiv \dfrac{l_{k+1}}{l_k}$. Then the volume of fluid in the network is equal to the sum of the number of branches at each level times their volume:

$$V = \sum_{k=0}^{T} N_k V_k = \sum_{k=0}^{T} \pi r_k^2 l_k N_k \tag{3.3}$$

In the case of plants, the vascular system is area preserving just by nature of its structure since each level must be formed from the previous one. In that case the network preserves the total area at each branch level and so we have:

$$n_k \pi r_k^2 = N_{k+1} \pi r_{k+1}^2 = \pi N_{k+1} \beta_k^2 r_{k+1}^2 \implies \beta = \left(\frac{N_{k+1}}{N_k}\right)^{-1/2} \tag{3.4}$$

We then consider the volume supplied by the network. If we consider the bundle of tubes at level k of length l_k then for long tubes $l_k >> r^k$ the volume supplied by the network can be thought of as the volume spanned by the tubes emanating from a point. These approximately form a sphere of length roughly the average length of the tubes at a given point in the volume $l_k/2$ so that:

$$V_k \sim \frac{4\pi}{3}\left(\frac{l_k}{2}\right)^3 N_k \tag{3.5}$$

is the *total volume* of the network, for every level. This means that we have:

$$\frac{4\pi}{3}\left(\frac{l_k}{2}\right)^3 N_k = \frac{4\pi}{3}\left(\frac{l_k+1}{2}\right)^3 N_{k+1} \implies \gamma_k \equiv \frac{l_{k+1}}{l_k} = \left(\frac{N_{k+1}}{N_k}\right)^{-\frac{1}{3}} \tag{3.6}$$

It was proved in [5] that the network structure which minimises the energy output required for resource to flow is a self-similar fractal. This structure has the same branching relations at each branching: $\beta_{k+1} = \beta_k = \beta$, $\gamma_k = \gamma$, $n_k = n$. Hence, we have that $\dfrac{N_{k+1}}{N_k} = n$ and so we derive that $\beta = n^{-1/2}$ and $\gamma = n^{-1/3}$. Our previous formula for the volume then becomes:

$$V = \sum_{k=0}^{T} \pi r_k^2 l_k N_k = \sum_{k=0}^{T} (\pi r_0^2 l_0 n_0)(\beta^2\gamma)^{-k} n^{-k} \tag{3.7}$$

Which is just a geometric sequence and so:

$$V = \pi r_0^2 l_0 n_0 \left(\frac{(n\beta^2\gamma)^{-(T+1)} - 1}{(n\beta^2\gamma)^{-1} - 1} \right) n^T V_T \tag{3.8}$$

Where we have used that the total volume is preserved. Since $n\beta^2\gamma < 1$ and $N >> 1$ we can approximate this series by its value as $N \to \infty$:

$$V \approx n^T V_T \frac{1}{1 - n\gamma\beta^2} = \frac{V_0}{1 - n\gamma\beta^2} = \frac{V_T(\gamma\beta^2)^{-T}}{1 - n\gamma\beta^2} \tag{3.9}$$

Here we can again use the assumption that the terminal units are invariant to infer that the volume of the supply network for an organism scales as $(\gamma\beta^2)^{-T}$ with an overall constant $\dfrac{V_c}{1 - n\gamma\beta^2}$. It remains to see how the number of layers in the supply network T relates to the volume (which scales as the mass). We have that $N_T = n^T \implies T = \dfrac{\ln(N_T)}{\ln(n)}$. We can then use this condition along with our earlier derived result that the number of terminal units is proportional to the metabolic rate $N_T \propto B \propto M^a$. We can write the constant of proportionality as $\dfrac{1}{M_0^a}$ to write $N_T = \left(\dfrac{M}{M_0}\right)^a$. This implies:

$$N_T = n^T = \left(\frac{M}{M_0}\right)^a \implies T = a\frac{\ln\left(\frac{M}{M_0}\right)}{\ln(n)} \tag{3.10}$$

We can then combine these two results. Since $V \propto (\gamma\beta^2)^{-T}$, $V \propto M$ we infer $M \propto (\gamma\beta^2)^{-T} \implies \dfrac{M}{M_0} = (\gamma\beta^2)^{-T}$. Hence

$$T = a\frac{\ln\left(\frac{M}{M_0}\right)}{\ln(n)} = a\frac{\ln\left((\gamma\beta^2)^{-T}\right)}{\ln(n)} \implies -1 = a\frac{\ln(\gamma\beta^2)}{\ln(n)} \implies a = -\frac{\ln(n)}{\ln(\gamma\beta^2)} \tag{3.11}$$

Then substituting in our values of $\beta = n^{-1/2}$, $\gamma = n^{1/3}$ we get:

$$a = \frac{3}{4} \tag{3.12}$$

Which is the well-documented metabolic scaling law.

3.2 Can this paradigm be applied to cities?

The approach in section 3 can be summarized as follows:

1. Impose constraints that the system must satisfy (space-filling and invariant terminal unit requirements in the metabolic model).

2. Use an optimization condition (energy minimization requirement) to choose from the remaining space of possible solutions.

The data in section 2.1 suggests that a fruitful area for future research is to try to extend a similar line of argument to derive the economic scaling relationships in cities. [11] were able to derive an exponent of 1.2 on the basis of a hierarchical network model of social interactions. Unfortunately, this falls short of what is required, since social networks are known to deviate from a tree-like structure. Thus, the question of whether and how a similar paradigm can be extended to explain the scaling relationships in cities remains open.

4 Systemic fluctuations - mean-field vs. birth-death population models

We now turn to the theoretical tools that an emergent macroeconomics could provide to explain our second set of phenomena - the presence of large fluctuations in economic systems. The need for such a theory was acutely felt after the financial crisis. A common quip in the aftermath was reported by Delong: when the then US secretary of state was asked whether macroeconomics could "*name where to turn to understand what was going on in 2008, [he] cited three dead men, a book written 33 years ago, and another written the century before last*" [12]. In this section I will argue that this deficiency is *not an accident but rather a design feature of the conventional macroeconomic framework* which treats fluctuations (or 'shocks') as generated exogenously from the model of the macroeconomy. I will illustrate the deficiency in conventional macroeconomic modelling with an analogy to two very simple population models and use work by [3] and [12]to suggest that the emergence framework of "agent based modelling" is a natural way to solve this deficiency.

4.1 Two population models

Any population model is a specification of how a birth rate and a death rate interact to change the number of people in the population. The simplest possible population model simply subtracts the average death rate ν from the birth rate λ to form a differential equation for the average population:

$$\frac{d\bar{n}(t)}{dt} = (\lambda - \nu)\bar{n}(t) \implies \bar{n}(t) = Ae^{(\lambda-\nu)t} \tag{4.1}$$

This is effectively a *mean field model*. We have modelled the average behaviour of the population as simply the average of a single member of the population. In economics, this would be similar to the 'representative agent' paradigm.

The problem with this population model is that it does not incorporate the stochasticity in the birth rate and the death rate - and in particular it does not incorporate the possibility of them conspiring to produce *internal fluctuations*. The simplest process that does take this into account is the linear birth death process. Following [7] Here we can write down a master equation for the probability to have a population $P_n(t)$ at time t:

$$\frac{dP_n(t)}{dt} = \mu(n+1)P_{n+1}(t) + \lambda(n-1)P_{n-1}(t) - (\mu+\lambda)P_n(t) \tag{4.2}$$

Since the probability of being at a population n can grow by a death in the $n+1$ state, or a birth in the $(n-1)$ state, and either process in that state will reduce the probability. This equation is simple enough to be solved exactly by introducing the generating functional for the $P_n(t)$:

$$G(t) = \sum_{k=0}^{\infty} s^k P_k(t) \tag{4.3}$$

If we multiply eq. (4.2) by $s^n k$ and sum over n we end up with a partial differential equation for the generating function:

$$\sum_n s^n \frac{dP_n(t)}{dt} = \left(\frac{\partial G}{\partial t}\right)_s = \lambda s^2 \underbrace{\sum_n s^{n-2}(n-1)P_{n-1}}_{\left(\frac{\partial G}{\partial s}\right)_t} + \mu \underbrace{\sum_n (n+1)s^n P_{n+1}}_{\left(\frac{\partial G}{\partial s}\right)_t} - (\lambda+\mu) \underbrace{\sum_n s^{n-1} P_n}_{\left(\frac{\partial G}{\partial s}\right)_t} \tag{4.4}$$

We have written the terms in this way so that we can identify all the summations over n as $\left(\frac{\partial G}{\partial s}\right)_t$. And so we have:

$$\left(\frac{\partial G}{\partial t}\right)_s = (\lambda s - \mu)(s-1)\left(\frac{\partial G}{\partial s}\right)_t \tag{4.5}$$

If the initial value of the population is a (with $P = 1$) then we have an initial condition $G(t = 0) = s^a$. If s is a function of t which we then set equal to a constant s_0, we get the relation:

$$\left(\frac{\partial G}{\partial t}\right) = \left(\frac{\partial G}{\partial s}\right)\left(\frac{\partial s}{\partial t}\right)_{=} 0 \tag{4.6}$$

Which allows us to write down the ODE:

$$\frac{ds}{dt} = (\lambda s - \mu)(1 - s) \tag{4.7}$$

Which can be solved by simple seperation of variables:

$$\frac{1}{-\lambda}\int ds' \left(\left(s - \frac{\lambda+\mu}{2\lambda}\right)^2 - \left(\lambda\frac{\lambda-\mu}{2\lambda}\right)^2\right)^{-1} = t \tag{4.8}$$

Which is just the standard arctan integral:

$$= -\frac{1}{\lambda}\frac{1}{2}\frac{2\lambda}{\lambda-\mu}\ln\left(\frac{1+\frac{s\lambda}{\lambda-\mu}}{1-\frac{s\lambda}{\lambda-\mu}}\right) = t + C \tag{4.9}$$

Which when combined with the boundary condition $s = s_0$ gives:

$$s_0 = \frac{\mu(s-1) + (\lambda s - \mu)e^{(-(\lambda-\mu)t}}{\lambda(s-1) + (\lambda s - \mu)e^{-(\lambda-\mu)t}} \tag{4.10}$$

This solves the equation for the generating function G since we have $G(t = 0) = s^a$ and our earlier Lagrange multiplier captured all the t dependence in S and so we can straightforwardly write down:

$$G(t) = \left(\frac{\mu(s-1) + (\lambda s - \mu)e^{(-(\lambda-\mu)t}}{\lambda(s-1) + (\lambda s - \mu)e^{-(\lambda-\mu)t}}\right)^a \tag{4.11}$$

We can now analyze the resulting population dynamics by calculating the moments of the probability density function by using the generating function property that the $m-th$ cumulant is given by: $\kappa_m = \left(\frac{\partial^m G}{\partial s^m}\right)_t\Big|_{s=0}$. We find that the mean is given by ([6]):

$$\bar{n} \equiv \kappa_1 = ae^{(\lambda-\mu)} \tag{4.12}$$

Which is in agreement with the mean-field model, as we would hope. To assess the potential of this model for generating fluctuations, we are interested in the ratio of the mean to the variance:

$$\frac{(\Delta n)^2}{\bar{n}} = \frac{\lambda+\mu}{\lambda-\mu}\left(e^{(\lambda-\mu)t} - 1\right) \tag{4.13}$$

In the long-time limit, as we would expect, in the limit where deaths are higher than births $\mu > \lambda$ we have fluctuations which vanish as $n \to \infty$. But in the limit where births are higher than deaths we have:

$$\frac{(\Delta n)^2}{\bar{n}} \to \frac{\lambda+\mu}{\lambda-\mu}e^{(\lambda-\mu)t} = \frac{\lambda+\mu}{\lambda-\mu}\bar{n} \implies (\Delta n)^2 = \sqrt{\frac{\lambda+\mu}{\lambda-\mu}}\bar{n} \tag{4.14}$$

Unlike the mean-field theory based on a representative agent, the presence of interacting birth and death process in this model leads to *fluctuations on the scale of the mean population.* The most striking consequence of this is that the probability of extinction for this model is non-zero and is given by:

$$P_0(t) = \frac{\mu}{\lambda}\frac{\bar{n}-1}{\bar{n}-\frac{\mu}{\lambda}} \tag{4.15}$$

In the limit of long times $P_0 \to 1$ for $\mu < \lambda$ (since $\bar{n} \to 0$). But for a an on average growing population $\lambda > \mu$ in the long-time limit the probability of extinction tends to a constant $P_0(t) = \frac{\mu}{\lambda}$.

4.2 Application to economics

We are now in a position to say what this has to do with economics. Once we include interactions between agents, as in the birth-death population model, we are able to recover fluctuations that are comparable to the system being modelled. This leads to the results that avoiding extinction and optimizing the growth rate are *not*the same, although in this simple model they are similar. The policy maker who was content with the mean-field model would have lost the ability to protect themselves against extinction!

Moreover, the fluctuations are generated endogenously in the model, as opposed to being supplied externally. These are precisely the shortcomings of conventional DSGE models revealed by the financial crisis. It can still be asked - could conventional models be modified to incorporate this feature? As [12] has argued, this is unlikely to be the case without a significant reworking of the models. Ascari et al. [3] showed first that macroeconomic data show fluctuations that depart significantly from the normal distribution (i.e $\sqrt{n}$ fluctuations); and second that conventional models, even when supplied *exogenously* with non-Gaussian shocks, simply pass through the externally supplied behaviour shock. This means that for the model to predict the departures from normal distributions in a crisis we would need to supply the distribution the crisis would force.
The 'beyond mean field theory' analog of the birth-death process that has been currently pursued is agent based modelling. A classic early example was the 'fundamentalists and chartists' financial markets model of [4]. In this model there are three types of agents: fundamental investors, chartists, and market makers. Each have different behaviours - fundamental investors' demand for assets is driven by their estimate of their value, whereas chartists update their demand based on the recent history of the asset's price. The interaction of the two, mediated by the market maker, is able to reproduce endogenous fluctuations on the scale of the mean asset values and switch between regimes of rising and falling markets - fig. 5.

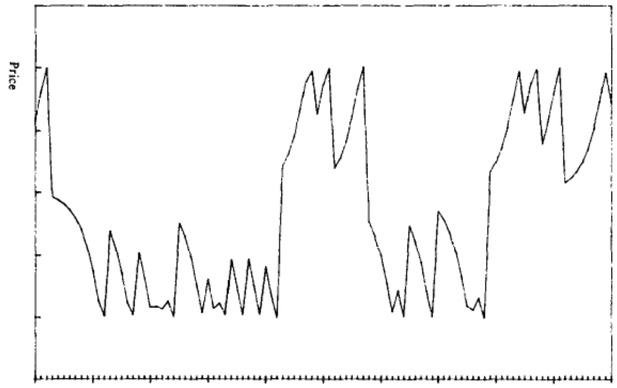

Figure 5: A typical asset value trajectory taken from [4]. Notice that the fluctuations are on the order of the asset value and are generated within the model.

5 Conclusion

The purpose of this essay has been to motivate emergent models in economics. From our discussion of scaling laws, we see that one approach to developing these emergent models is to focus on the conserved quantities (fluid flow in the case of metabolic scaling, money in the case of income distributions) and find a solution following an optimization principle (energy minimization for metabolic scaling, entropy maximisation for income distributions). This is a phenomenological approach, somewhat like the Landau theory. In our discussion of agent based modelling, we saw that a second approach was to generate interactions between different types of processes (births and deaths in the population model, different types of of market participants in the financial market model).

So far both of these approach have the drawback that they are largely concerned with *explanation* of currently known relationships rather than the *prediction* of new relationships. If the analogies presented in this paper are found plausible, however, then there is reason to hope to that a theory that can explain the known observations can also point the way to as yet unknown ones.

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