Long Period Phase Oscillations Emerge from Connectivity Symmetry Breaking in the Brain

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1 Introduction and Background

The approach to understanding complex systems from the top-down method discussed in class has made a clear suggestion that solutions to complicated systems do not themselves have to be complicated. The brain is arguably one of the most complex systems humans know of, and this is reflected in our limited understanding of its functional properties. Here we'll report on some results that help to strengthen a reformulation of our understanding of functional properties of the brain.

The authors in [8] set out to aid in developing a systematic description of behavioral functions, such as those found in the brain. The main hypothesis of the groups work is that breaking the symmetry in the connectivity of a network of coupled spiking neurons leads to an emergent lower dimensional dynamics, called structured flows on manifolds (SFM) [3], that have much slower timescales than the symmetric dynamics.

Further, they hope to shed light on the idea that the dynamics, and not the states themselves, of these neuron networks contain the important information necessary to understand the functional behavior of the brain. Attention was brought to previous approaches to studying these dynamics, namely that other groups have used cumbersome models to try and get the complexity expected of behavioral dynamics. Here their method is attractive because of its use of a much simpler model that can still capture the complex dynamics expected. This is because emergent SFM's themselves have complex dynamics, albeit restricted to a much lower dimensional manifold.

In a complementary paper [2], the authors aim to experimentally measure signatures of such emergent dynamics in human brains. The fluctuations of the Kuramoto order parameter [5] should be detectable in fMRI BOLD signals, implying the existence of the mentioned SFM's. They use a novel technique whereby experimental data is used to constrain a network model that is investigated computationally. They explore the functional connectivity dynamics (FCD) to measure metastability in the brain state dynamics in hopes of verifying that it is maximized.

Ultimately, these results help to strengthen and validate an alternate approach to understanding functional behaviors of the brain, one that uses as it's constituent elements these emergent SFM's [6].

2 Methods

Using a spiking theta neuron model [10], the authors describe an individual neuron by it's spiking and rate state variables, θ and ω respectively. Their dynamics are described by the two coupled first order differential equations known to produce Excitator dynamics [4]:

$$\dot{\theta} = \big(\omega + I(t)\big) sin\theta + 1 \tag{1}$$

$$\tau_\omega \dot{\omega} = -\omega + k\delta(\theta) \tag{2}$$

This spiking variable θ physically describes the firing of the neuron and takes on values in $[-\pi, \pi]$. When $\theta = \pi$, eq. (2) describes the firing of the neuron through a Dirac delta impulse function.

Further, you can represent a model of the brain by coupling N of these theta neurons into a network, where each neuron i is connected to the N-1 remaining neurons j, through the coupling matrix $G = g_{ij}$:

$$\dot{\theta}_i = \big(\omega_i + \sum_j^N g_{ij}\omega_j + I_i(t)\big) sin\theta_i + 1 \tag{3}$$

$$\tau_\omega \dot{\omega}_i = -\omega_i + k\delta(\theta_i) \tag{4}$$

The authors now examine the systems dynamics in two regimes – a "rate reduced" description, that is described by one equation of only the rate variable ω

$$\tau_\omega \dot{\omega}_i = -\omega_i + \sqrt{1 - \big(\sum_j^N g_{ij}\omega_j + I_j\big)^2} \tag{5}$$

and a "phase reduced" description, also described by only one variable θ

$$\dot{\theta}_i = \big(\omega + g\bar{r}\delta(\bar{\phi})\big) sin\theta_i + 1 \tag{6}$$

where the Kuramoto order parameter has been introduced to treat this problem in the spirit of mean-field theory

$$z = \frac{1}{N}\sum_k^N e^{i\theta_k} = \bar{r}e^{i\bar{\phi}} \tag{7}$$

2.1 Firing rate dynamics

Setting the coupling strengths g_{ij} to a constant, g_0, the authors look at the dynamics of the rate variable in the symmetric regime i.e. $g_{ij} = g_{ji}$, followed by a look at the changes to the dynamics by introducing asymmetry in the coupling matrix G, thereby breaking the connectivity symmetry of the network.

2.2 Spike timing dynamics

Here the authors study the fluctuations about the synchronized state, $\theta_i = \theta_j \; \forall$ i,j, again comparing the dynamics with a symmetric coupling scheme to those of the symmetry broken scheme.

2.3 Experimental methods

The group uses a local node neural mass model [11] that is fit to experimental MRI data, in order to gain access to the fluctuations and their structure in time, refereed to in this paper as the functional connectivity dynamics (FCD)

3 Results and Discussion

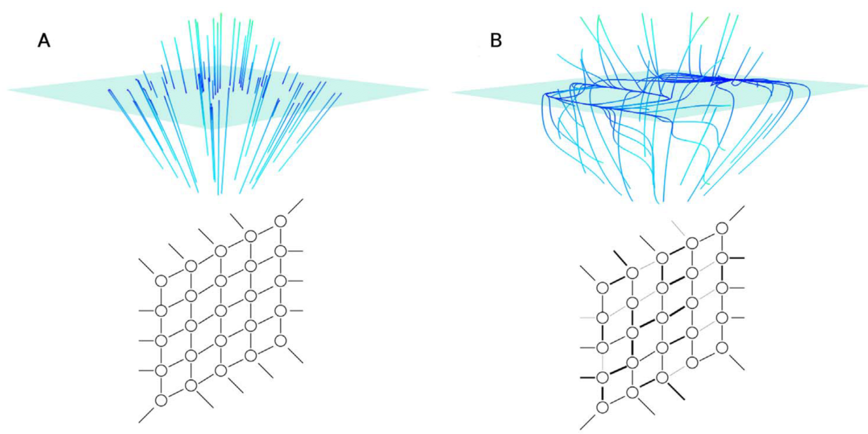

Figure 1: Visual representation of the low dimensional manifolds and the SFM's that emerge on these manifolds upon breaking the symmetry of the coupling matrix

3.1 Results for ω

A steady state solution to (5) is found, that describes all N of the neurons (mean-field)

$$\omega_0 = \frac{g_0 N I_0 \pm \sqrt{g_0^2 N^2 - I_0^2 + 1}}{g_0^2 N^2 + 1} \tag{8}$$

Importantly, the authors treat this fixed point solution as a functional mode that describes the whole network as a function of position and time

$$r(x,t) = s(x)A(t) \tag{9}$$

It is here that the authors make the following claim: In the presence of a symmetric network coupling, the system is described by dynamics that are "fast" in the sense that they flow to the fix point of the system (again described here by the mode $r(x,t)$).

In actual networks, there will likely be multiple modes $r_i(x,t)$ that describe the symmetric system, from which one can construct a lower dimensional space. It is in this lower dimensional space that "slow" dynamics (SFM's) emerge as a consequence of symmetry breaking in G. These lower dimensional dynamics are referred to as phase flow, and they represent the slow modulations of an order parameter after the breaking of a symmetry. Table 1 shows the four coupling schemes investigated, here for a network of just two neurons. Here we see the symmetric coupling matrix

$$G = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{10}$$

leads to fixed point dynamics, pictured in Fig 1. A. However, when the symmetry is broken

$$G = \begin{pmatrix} 0.2 & -5.8 \\ -0.7 & -0.7 \end{pmatrix} \tag{11}$$

the dynamics switch to that of a limit cycle attractor, pictured in Fig 1. B.

Attractor type	$G = \begin{pmatrix} g_a(1+\mu_a) & g_x(1+\mu_x) \\ g_x(1-\mu_x) & g_a(1-\mu_a) \end{pmatrix}$	$G^* = \begin{pmatrix} g_x & g_a \\ \mu_x & \mu_a \end{pmatrix}$
Fixed point	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 0. & 0. \end{pmatrix}$
Limit cycle	$\begin{pmatrix} 0.2 & -5.8 \\ -0.7 & -0.7 \end{pmatrix}$	$\begin{pmatrix} -3.3 & -0.3 \\ 0.8 & -1.71 \end{pmatrix}$
Bistability	$\begin{pmatrix} 0 & -1.4 \\ -1.4 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1.4 \\ 0 & 0 \end{pmatrix}$
Monostability	$\begin{pmatrix} 0.5 & -5.8 \\ -0.7 & -1.1 \end{pmatrix}$	$\begin{pmatrix} -3.3 & -0.3 \\ 0.8 & -2.5 \end{pmatrix}$

Table 1: Symmetric and Anti-Symmetric coupling used to produce the corresponding dynamics

3.2 Results for θ

For the spike timing dynamics, recall the authors imposed a uniform steady state solution with $\theta_i = \theta_j \,\forall$ i,j, representing a network of fully synchronized oscillators. It is about this steady state that fluctuations in θ are examined. With symmetric coupling, as they found in the ω dynamics,a type of mode formation occurs in terms of the relative phases of the N oscillators. Therefore here, when the symmetry is broken, the phase flows emerge in the form of relative phase oscillations.

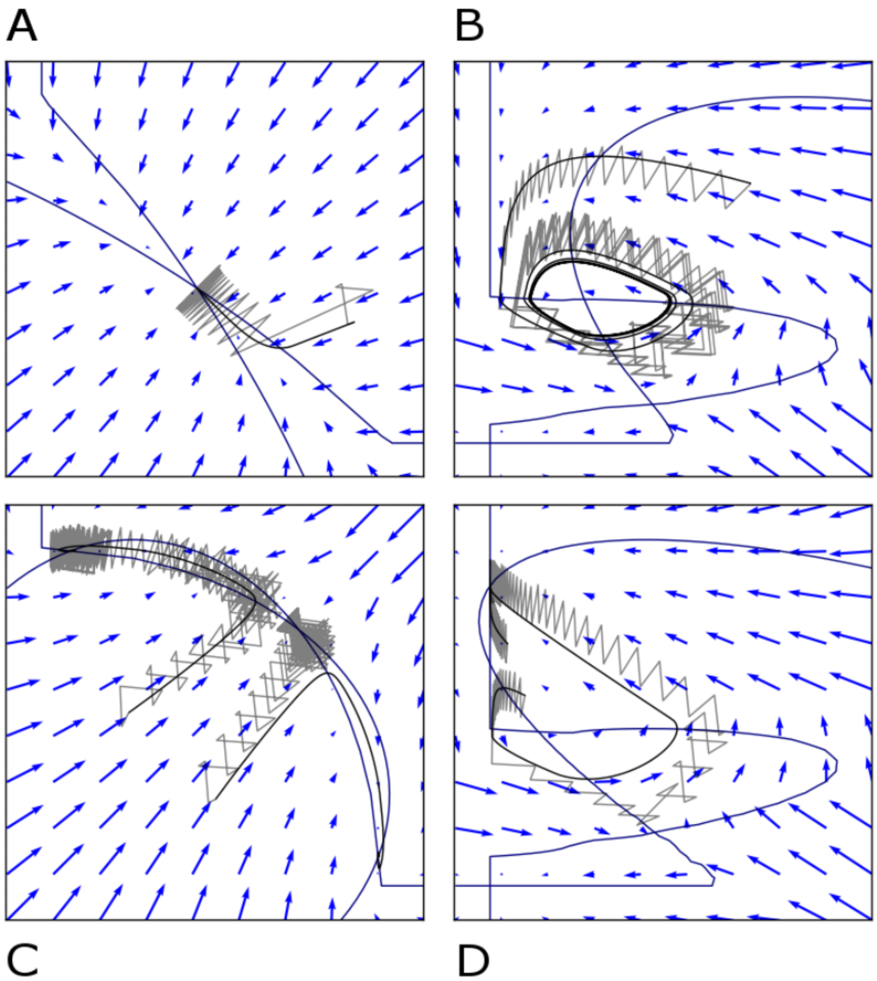

Figure 2: Phase flows from the 4 coupling schemes investigated: A) Fixed point B) Limit cylce C) Bistability D) Monostablity

It is relevant here to note that in the symmetric coupled synchronous state, the relative phase (again for two neurons) $\phi(i,j) = 0$, only switching to a nonzero value after asymmetry in G is introduced. This resembles the order parameter used to study phase transitions in ESM. As in the ω dynamics, these oscillations are slow compared the dynamics of the rest of the system, which itself is reminiscent of the long-wavelength properties of the emergent Goldstone modes studied in ESM. Fig 2. column C shows both the modulus of the Karumoto order parameter and the relative spike timing $\phi(i,j)$ for the symmetric fixed point state and the broken symmetry limit cycle state. This nicely shows, on the one hand, the phase difference approaching some constant value in the symmetric coupling (fixed point) state. While on the other hand, the oscillation of $\phi(i,j)$ on a visibly slower time scale than $\|z\|$ in the broken symmetry state (Column C).

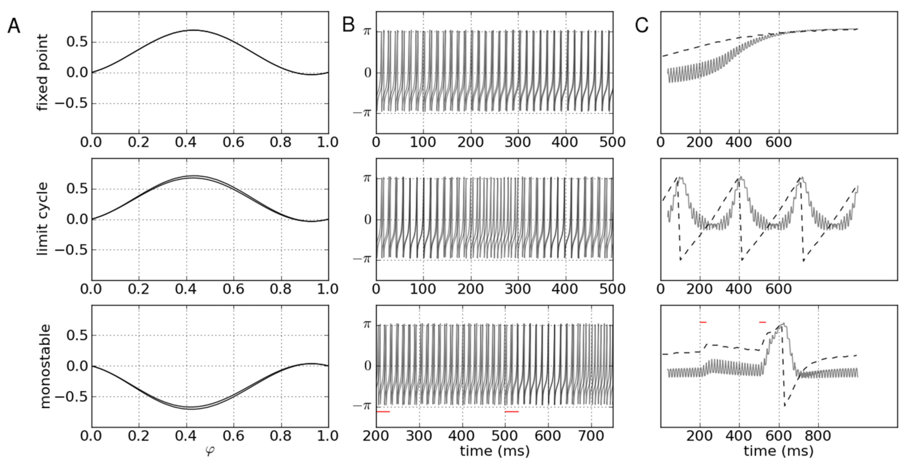

Figure 3: Spike timing dynamics for symmetric and asymmetric couplings. A) Two neuron phase response curves. B) Time series of the two neuron spike variable θ. C) The Karumoto order parameter $\|z\|$ and the phase difference $\phi(i,j)$, shown with grey and dashed black lines respectively

3.3 Implications

SFM based functional architectures predict timescale hierarchies [6] and systematic dependencies between spike timing and rate. The phase space of the rate state variable has a fixed point synchronous attractor that destabilizes when the "firing rate ratio" is non-integer. This produces transient synchronization which may be responsible for coherent long range communication between largely separated neurons. These synchronizations should help us in understanding the functional structure of behavior dynamics and also may themselves be the fluctuations of the rate state variable.

3.4 Experimental validation

The underlying dynamics of these neuron networks is relevant in understanding the dynamics of real neuron networks such as the brain. Indeed, fMRI experiments have succeeded in capturing these phase fluctuations in the BOLD signal. The authors describe this state through is metastability, which they quantify as the standard deviation of the phase uniformity order parameter R(t) [1]. After introducing the parameter a, which they call the global bifurcation parameter, they find an optimal parameter setting, namely a=0, G=2.85, where the signal displays maximal metastability Fig 4 A. Indeed, these authors attribute this metastability to the difference in phase between different neurons. This resembles the emergence of SFM's manifested in the relative phase fluctuations from the theory paper.

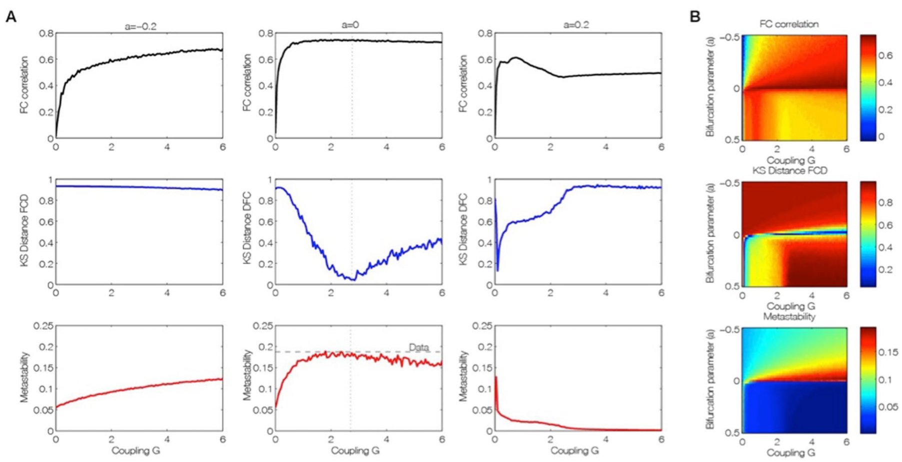

Figure 4: metastability of fMRI signals in the brain

It seems likely that continuing to look at the brain from this perspective will be rewarding. The low energy modes emerging from spontaneous symmetry breaking may be traversing large sections of the brain, allowing communication to far neighbors that otherwise would not be connected. In this respect, it might be reasonable to think more interesting phenomena are arising from these long range correlations. One example can be seen in [9] in which the authors experimentally verify the onset of long range correlations between separated regions of the brain while listening to music. They exploit a novel measure called the phase lag index (PLI) [7], to accurately capture the correlations between 64 electrodes corresponding to an EEG signal. Could the power of musics ability to evoke emotions lie in the emergent structure of our brains dynamics? It would be interesting to try and follow this papers spirit to functionally describe music and the emotions it can evoked, possibly through its interaction with these lower dimensional phase flows.

4 References

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