Dark Matter as an Emergent Phenomenon of Entanglement

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Abstract

Velocity profile of the outer regions of galaxies tend to stay constant instead of following the expected Kepplerian one over square root law. Modified versions of Newtonian gravity, while being in agreement with the observational results, fail to ascribe a self-consistent microscopic origin to this bizarre phenomenon. Another explanation for the observed profile is Dark Matter, existence of an 'unseen' additional mass that could provide an anchor for the motion of the stellar mass to give rise to the constant velocity behavior. In this essay, we will describe Verlinde's proposition that dark matter or modification to Newtonian gravity comes from a competition between area and volume-law variation of entanglement entropy in de Sitter Space, where non-Newtonian behavior manifests as an elastic response due to entropy displacement. We will also explore the strengths and shortcomings of Verlinde's theory, its regime of validity, and how it matches the observational evidence.

1 Basics of Galaxy Rotation Curves, MOND and Dark Matter

Most of what we know about luminous matter from different galaxies comes from the photons that we observe in our detectors. These photons reveal information regarding two fundamental observables in physics, which are position and velocity respectively. The position to a distant stellar object is measured using *apparent luminosity* (in units of W/m^2) of light that varies by the inverse square law, which allows one to determine the relative location of the object of interest with respect to something whose inherent luminosity (in Watts) is already well known. These are usually distant supernovae referred to as *standard candles.* On the other hand, the velocity is measured by utilizing the *Relativistic Doppler Shift.* This happens due to galaxy either receding from or advancing towards the Earth, which introduces a shift on wavelength observed. The powerful experimental tool used is the discrete lines observed from the emission spectrum of stars, which is unique like our finger prints for each element. Doppler effect introduces a translational shift on this data by $\Delta\lambda$:

$$\frac{\Delta\lambda}{\lambda} = 1 - \sqrt{\frac{1 - v/c}{1 + v/c}} \approx \frac{v}{c} \tag{1}$$

By equating $\frac{\Delta\lambda}{\lambda}$ and $\frac{v}{c}$, one can accurately determine the velocities of stars and galaxies. It is this data on velocity of stars located at different radii from the core of the galaxies that seemingly violate both Newtonian and Einsteinian gravity and induced a several decades long puzzle that is 'What is the missing ingredient?' Below, we demonstrate the velocity profile of a spiral galaxy and comment on the discrepancy with laws of gravity:

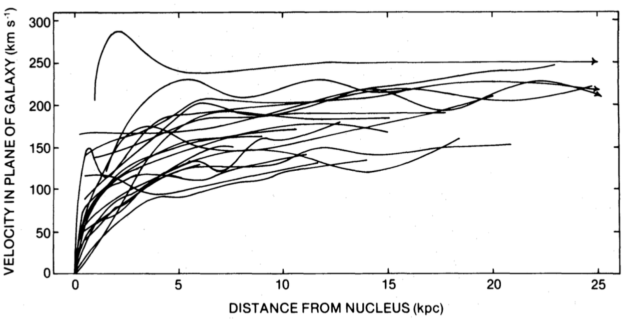

Figure 1: Spiral Galaxy Rotation Curve taken from *Lectures on Dark Matter Physics* by M. Lisanti, originally from Rubin *et al.* At outer rims of the galaxy, curve flattens

From an elemental Newtonian calculation, it is not hard to see why the behavior of these curves is troubling. We demonstrate this below along with modifications required. The expected velocity profile of stars at a given radius from the center of a spiral galaxy can be given by Newton's 2nd Law for centripetal acceleration. M represents the total enclosed mass within the radius of choice, while m is the mass of the star in the profile.

$$m\frac{v^2}{r} = \frac{GMm}{r^2} \tag{2}$$

$$v = \sqrt{\frac{GM}{r}} \tag{3}$$

The expected Kepplerian decay of velocity in the outer regions of the galaxy contradicts with the observed behavior that v= constant. Most prevalent theory that was proposed to resolve this issue is the existence of Dark Matter. Previously, we said that most of our data come from the photons from the luminous matter. However, this is not true for matter that does not interact through electromagnetic force. Presence of the possibility of such matter can be tested through indirect observations of gravitational lensing of the nearby luminous masses, which would account for their existence. Similarly, simplest solution to the problem of flat rotation curve can be thought as the existence of this proposed Dark Matter that could provide an anchor for the nearby masses affecting their velocities. By a simple calculation, one can see how the density of dark matter should vary to yield a flat curve:

$$
\begin{aligned}
v &= \sqrt{\frac{4\pi G r^2 \rho(r)}{3}} \\
\rho(r) &\propto \frac{1}{r^2}
\end{aligned}
\tag{4}
$$

An alternative theory to the Dark Matter proposition is that Newton's Laws require a modification in the outer regions of spiral galaxies (MOND). Force of gravity would be described by a piece-wise function in this case. Up to a certain radius $r < r_0$, we would have $F_{in}(r) = \frac{GMm}{r^2}$, whereas for $r > r_0$, one would expect $F_{out}(r) = \frac{GMm}{r_0 r}$, which yields constant velocity. Dark matter relies on the existence of a yet unobserved particle in Standard Model, while MOND describes an alteration to Newtonian dynamics without a microscopic justification. We now aim to describe Verlinde's proposition which requires neither of the above and ascribes an entropic origin to the observed flatness.

2 Verlinde's Proposal for the Observed Flatness

Baryonic content of the universe, stuff that makes us and we have a good understanding of only makes up 4 percent of the observable universe, while the remaining 96 percent is comprised of stuff that we do not have a good understanding of, dark matter and energy respectively. This promotes Verlinde to rethink our understanding of space-time and consider its fundamental building blocks from a quantum information and entropic perspective. Verlinde's main motivation for taking up this line of reasoning comes from an empirical evidence, which mathematically can be recast as an entropic equation. Verlinde considers the radii bound that one must exceed for flat scaling of velocity to occur in spiral galaxies, which is given by the following expression:

$$
\frac{M}{A(r)} < \frac{a_0}{8\pi G}
\tag{5}
$$

In the above expression, the l.h.s. represents the surface mass density of the spiral galaxy. When this falls below a universal value (r.h.s.), which is dictated by cosmological acceleration scale a_0, one expects flatness in rotation curves. It is very peculiar that this observed behavior for one galaxy is related to cosmological acceleration. The cosmological acceleration a_0 can be written in terms of the Hubble constant:

$$
a_0 = cH_0
\tag{6}
$$

where H_0 is the Hubble constant with the dimensions of s^{-1}, describing a parameter that relates recession velocity of distant galaxies with their observed distances. Verlinde recasts this relationship as the following:

$$
a_0 = \frac{c^2}{L}
\tag{7}
$$

where the acceleration parameter is now expressed in terms of the Hubble horizon, which we will talk about in section 4. Local dynamics of galaxies are somehow affected and possess information about the large-scale non-local behavior of our universe. One can think about this phenomenon as a possible hint of large scale entanglement. To substantiate this possible relationship, first we need to recast this empirical inequality in terms of entropy carried in a spherical region:

$$
S_M = \frac{2\pi M}{\hbar a_0} < \frac{A(r)}{4G\hbar}
\tag{8}
$$

In this expression the l.h.s represents the entropy increase associated with addition of mass M in de Sitter universe, which is the space-time that describes the universe we live in, while the r.h.s. is the entropy of a black hole that could be fitted inside a region of area A(r). Verlinde's main purpose is to show how observed change in Newtonian dynamics could emerge if one satisfies the above entropic inequality. To understand this, we first need to have a better intuition about how geometry is related to entropy and entanglement.

3 Entropy Laws and Bekenstein Bound

Bekenstein Bound places an upper limit on information contained on the event horizon of a black hole. This bound puts a limitation on the amount of information one can put into a spherical region in a gravitational theory. It is given by the following expression:

$$S_{BH} = \frac{k_B c^3}{4\pi G} A \tag{9}$$

This bound can be derived and understood from considering entropy contribution from adding a small mass inside a black hole. One would expect the variation of entropy to be proportional to the mass that we lose to the black hole:

$$dS = MdM \tag{10}$$
$$S \sim M^2$$

On the other hand, we also can express the radius of a black hole beyond which classically light is not allowed to escape, and this upper distance is given by the Schwarzchild radius. Using this, we have an intuition of what the area law looks like in the presence of a horizon:

$$r_s = \frac{2GM}{c^2} \tag{11}$$
$$S \leq (constant)A$$

Area-law plays a key role in condensed matter systems as well, since ground states of gapped systems obey it. In this case, entropy is due to the entanglement between the degrees of freedom that live on either side of the boundary of two subregions of the whole system. However, in the absence of gravity and in an ordinary thermodynamic system of homogeneously distributed particles, one expects entropy to be extensive, with number of indistinguishable configurations of particles to increase with respect to volume and to be bounded by the dimension of the Hilbert Space. In this case, the entropy follows a volume-law:

$$S \sim \log[dimH] = Volume \tag{12}$$

Verlinde cites these as examples to demonstrate how geometry of space-time is related to entanglement. Due to evidence from Black Hole Thermodynamics and Holographic Correspondence, it is believed that information in black holes is encoded on the event horizon in a non-local way. This is the main motivation for the idea of emergent gravity, which basically states that entanglement between the degrees of freedom in space-time is essential to describe gravity.

Motivated by this, Verlinde claims that similarly, the entropy carried by the de Sitter horizon, which is the space-time that describes our universe could be encoded in the form of delocalized excitations. He claims while ordinary matter is formed due to localized excitations, dark matter manifests due to delocalized ones. Having described the Bekenstein bound for black holes, we now discuss its extension to de Sitter space as it is the relevant space time for cosmology and present Verlinde's postulates about entanglement in de Sitter space.

4 de Sitter Space and Entropy Content

Verlinde makes two postulates about the nature of entanglement in de Sitter space. Verlinde's first postulate states that each region of space-time can be thought as a tensor factor of Hilbert Space. When these factors describing subregions are traced out, the complementary region satisfies an area-law of entropy, which is given by the sum of contributions of the short range correlations that live across each side of the boundary between the two regions. This claim is motivated by Bekenstein entropy bound that we talk about in the previous chapter and Holographic Correspondence, where entanglement entropy of states that live in the low-energy sector of Conformal Field Theories can be calculated by the area of the geodesics that describe a gravitational theory. The second postulate, on the other hand, states that de Sitter space is built not only by short-range correlations, but also by long range correlations at the Hubble scale, which is the scale for size of the universe. This accounts for a volume law contribution to entropy in addition to area law, for which quantum information can be thought as being divided homogeneously on each region of space-time. In the absence of matter, the entropy counted by the horizon of de Sitter space is distributed uniformly over the space-time which is carried by the mysterious 'dark energy'. Verlinde claims that this entropy would be extensive being proportional to the volume of the region considered. On the other hand, inclusion of mass reduces this entropy, a claim that we will show in section 5, giving rise to a non-Newtonian force caused by displaced entropy content. If the mass inclusion removes entropy that is less than the available de-Sitter entropy distributed over the same volume, Verlinde claims that volume law variation of entropy will manifest in the form of a non-Newtonian force. To substantiate these claims, we first look at de Sitter space in the absence of mass.

de Sitter space is a solution to Einstein's equations with a positive cosmological constant. Since our universe is expanding with an accelerated rate, which is attributed to dark energy/cosmological constant, this space-time provides the ideal description for the current epoch of our universe. As it plays an important role in Verlinde's argument, we begin by outlining the metric:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \tag{13}$$

where $f(r) = 1 - \frac{r^2}{L^2}$

One of the most important features of de Sitter space is the presence of cosmological horizons, which are associated with entropy content. One can see this by considering the value of r for which $f(r)$ vanishes, this is given by r=L.

The entropy of de Sitter universe is related to the horizon area and given by the following, which is reminiscent of the Bekenstein bound:

$$S(L) = \frac{A(L)}{4G\hbar} \tag{14}$$

where area is given by d-2 dimensional sphere with radius L:

$$A(L) = \Omega_{d-2} L^{d-2} \tag{15}$$

Verlinde conjectures that this entropy due to event horizon is evenly evenly distributed over the entire volume and microscopic degrees of freedom. Hence, entropy of a spherical region can be expressed as the following:

$$S(r) = \frac{V(r)}{V_0} \tag{16}$$

where $V(r)$ represents the d-1 dimensional volume and V_0 functions as the normalization on the entropy of the given region:

$$V(r) = \frac{\Omega_{d-2}}{r^{d-2}} d - 1 \tag{17}$$

One can write the entropy of a spherical region in a more familiar form:

$$S(r) = \frac{r}{L} \frac{A(r)}{4G\hbar} \tag{18}$$

$$A(r) = \Omega_{d-2} r^{d-2}$$

Metric for de Sitter space can be represented in different slicing. One such representation, which is widely used in cosmological scenarios is the following:

$$ds^2 = dt^2 - e^{Ht}(dr^2 + r^2 d\Omega^2) \tag{19}$$

Here, the coefficient e^{Ht} captures the accelerating expansion of the universe. One can see the presence of the horizons by considering null trajectories that start at $x(t=0) = 0$:

$$\frac{dx}{dt} = e^{-Ht} \tag{20}$$

which yields the following trajectory:

$$x(t) = H^{-1}(e^{-Ht_0} - e^{-Ht}) \tag{21}$$

Hence, the horizon scale is given by,

$$x(t \to \infty) = H^{-1} \tag{22}$$

Having described de Sitter space, presence of horizon and conjectured distribution of entropy over the volume, we now consider the case in which one adds matter to the universe as it is the behavior of matter we are interested in to explain the flatness in rotation curves.

5 Adding Mass to de Sitter Universe

In this section, we demonstrate how addition of mass affects the location of de Sitter horizon. This plays a key role in understanding the entropy displacement force that could be responsible for the observed flatness of rotation curves. We consider the two sided perspective on de Sitter space, where one can think of horizon as separating the degrees of freedom of two complementary regions denoted by Hilbert Spaces $H_A \otimes H_{\bar{A}}$. While one region can be thought as the observable universe, the complementary region can be thought as the inaccessible region and area of the horizon quantifying this inaccessibility and lack of knowledge. The mass added on one side is accompanied by the mass lost in the complementary region in this doubled space-time picture. In the presence of a mass M, one has the spherically symmetric Sitter-Schwarzschild solution in d +1 dimensions with the following form:

$$ds^2 = -\left(1 - \frac{r^2}{L^2} + 2\Phi(r)\right) dt^2 + \left(1 - \frac{r^2}{L^2} + 2\Phi(r)\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{23}$$

where $\Phi(r)$ is the mass dependent Schwarzschild term:

$$\Phi(r) = -\frac{8\pi GM}{(d-2)\Omega_{d-2} r^{d-3}} \tag{24}$$

In the absence of the mass, the horizon is located at r=L for which $f(r) = 0$ as expected. In the presence of the horizon there is a shift. In the approximation that $\Phi(L) \ll 1$, which would correspond to adding a small mass. One has the following expression for the shift:

$$L \to L + u(L) \tag{25}$$
$$u(L) = \Phi(L)L$$

Since the sign of $\Phi(r)$ is negative, addition of mass decreases the de Sitter entropy. This decrease is given by the following expression:

$$S_M(L) = u(L)\frac{d}{dL}\left(\frac{A(L)}{4G\hbar}\right) = -\frac{2\pi ML}{\hbar} \tag{26}$$

To generalize this to arbitrary length-scales, we want to calculate the change in the area as a function of the geodesic distance in cases with mass and without mass. We consider the two spheres with the same area in both geometries. In the absence of mass, the geodesic distance is $dr = ds$, while in the presence of the mass, we have $dr = (1 + \Phi(r)ds$. We express the difference of the variation of entropy of a spherical region with respect to geodesic distance for geometries with and without mass. This is given by the following expression:

$$\frac{d}{ds}\left(\frac{A(r)}{4G\hbar}\right)_M - \frac{d}{ds}\left(\frac{A_{M=0}(r)}{4G\hbar}\right)_{M=0} = \Phi(r)\frac{d}{dr}\left(\frac{A(r)}{4G\hbar}\right) = -\frac{2\pi M}{\hbar} \tag{27}$$

In the above equation, despite the fact that areas are equal for both geometries, the derivative acts differently due to difference in line elements. This equation can be understood as the amount of entanglement entropy removed from a spherical region of space by a mass M. However, the equation relies on utilizing two geometries. It would be nice to have a single equation that relates entropy to mass without referencing any geometry. This is done replacing $\frac{A(r)}{4G\hbar}$ by $S_M(r)$ and assuming weak gravity regime for which the geodesic distances are approximately equal. Doing this we obtain the following equation and its integrated version:

$$\frac{dS_M(r)}{dr} = -\frac{2\pi M}{\hbar} \tag{28}$$

$$S_M(r) = -\frac{2\pi M r}{\hbar}$$

For de Sitter space with horizon length L, inclusion of a small spherical mass of radius r is expected to take away a fraction $\frac{r}{L}$ of the de Sitter entropy from the nearby space-time region. Having expressed the entropy content of de Sitter space removed by the inclusion of mass, we now relate entropy of a spherical mass of radius r and empty de Sitter vacuum (dark energy if you like) of the same size. We know that the entropy of a region of de Sitter space is given by the following. We will denote this by S_D:

$$S_D = \frac{r}{L}\frac{A(r)}{4G\hbar} \tag{29}$$

In relation to matter entropy this quantity will be either greater or less than the matter entropy:

$$S_M(r) \lesseqgtr S_D(r) \tag{30}$$
$$\frac{2\pi M r}{\hbar} \lesseqgtr \frac{r}{L}\frac{A(r)}{4G\hbar}$$

The entropy removed by the spherical mass can be redefined in terms of the volume that has the same entropy as the one removed by the mass. This is given by the following expression:

$$S_M(r) = \frac{V(r)}{V_0} \tag{31}$$
$$V_0 = \frac{4G\hbar L}{d-1}$$

One can check that V_0 has the dimensions of volume in d dimensions. This can be seen by checking r.h.s of bottom equation (20). A(r)r has dimensions of volume, hence the denominator should have the same dimensions to keep entropy dimensionless. Intuitively, one can think about V_0 quantity as follows. If $\frac{1}{4G\hbar L}$ represents the entropy associated with unit volume. V_0 is proportional to the reciprocal of quantity, which is the volume necessary to have an entropy of 1, functioning as a normalization factor. Placing everything on one side the relationship between the two entropies can be expressed by the following:

$$\frac{V_M(r)}{V(r)} \lesseqgtr 1 \tag{32}$$

Verlinde interprets this equation to be equivalent to the empirical relationship described in (5). Depending on whether the mass removes all or part of de Sitter entropy, one observes flattened rotation curves in spiral galaxies. To make this relationship more evident, one can replace the Hubble scale by the acceleration parameter and solve for $V_M(r)$:

$$V_M(r) = \frac{8\pi G}{a_0}\frac{Mr}{(d-1)} \tag{33}$$

Verlinde defines the quantity $\epsilon_M(r) \equiv \frac{V_M(r)}{V(r)}$. For the case $\epsilon_M(r) < 1$, one is in the low acceleration regime, where one expects flattening effect to be prevalent. Verlinde refers to this as the 'dark gravity' regime. Below, we aim to look at how the displacement of entropy content by matter can manifest itself as a force, which could explain the modified dynamics.

6 Entropy Displacement

In this section, we will comment on how Verlinde establishes the relationship between properties of de Sitter space and an elastic medium. In the previous section, we have seen that adding and removing a spherical mass altered the location of de Sitter horizon. Analogously, in an elastic medium, one expects a radially symmetric displacement of the medium with the assumption that volume removed is incompressible. The change in the volume due to addition or removal of mass is given by the product of displacement and area at a given radius with $\Delta V = u(r)A(r)$. This can be shown by the figure below:

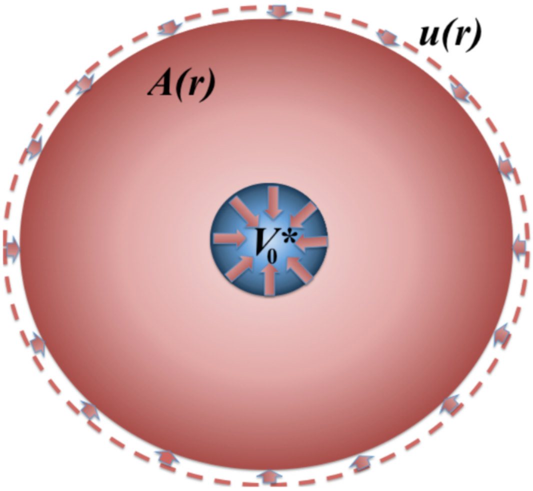

Figure 2: Entropy displacement caused by addition of mass and removal of empty space treated as an elastic medium, figure taken from E. Verlinde's *Emergent Gravity and Dark Universe*

For instance, the de Sitter entropy removed by the mass M would cause an inward radial displacement. The change in entropy due to mass can be expressed by a ratio of volumes, the same as the formalism of previous chapter.

$$S_M(r) = \frac{u(r)A(r)}{{V_0}^*} \tag{34}$$

Here, V_0 is expected to be of the same order of magnitude as ${V_0}^*$. To see how the two volumes are related, we go back to the expression in (15), which relates how the displacement is related to the Newtonian potential $\Phi(r)$ introduced by the inclusion of the mass. The displacement at Hubble scale can be expressed as the following:

$$u(L) = \Phi(L)L = \frac{\Phi(L)}{a_0} \tag{35}$$

Since $S_M(r) = \frac{2\pi M r}{\hbar}$ is linear in r, and A(r) scales as r^{d-2}, looking at equation (24), we see that displacement $u(r)$ must scale as $\frac{1}{r^{d-3}}$, which is the same scaling as the Newtonian potential. Combining this scaling argument with equation (25), one obtains the following expression for arbitrary radii:

$$u(r) = L\Phi(r) \tag{36}$$

From this, one can obtain an expression for the volume ${V_0}^*$.

$$ {V_0}^* = \frac{4G\hbar L}{d-2} \tag{37}$$

$$\frac{{V_0}^*}{V_0} = \frac{d-1}{d-2}$$

In addition, one has the following relations:

$$u(r) = -\frac{{V_M}^*(r)}{A(r)} \tag{38}$$

$${V_M}^*(r) = \frac{8\pi G}{a_0}\frac{Mr}{d-2}$$

Verlinde claims that the relative factor between ${V_M}^*(r)$ and $V(r)$ can be attributed to a transition from area law to volume law of entropy due to matter removing less entropy content than the entropy of de Sitter space distributed evenly over the volume. Displacement field in an elastic medium gives rise to normal strain, which we will denote by $n(r)$. When $\epsilon_M < 1$, which corresponds to the case where only a fraction of the de Sitter entropy is removed by the matter, Verlinde makes the assumption that elastic medium is incompressible with entropy $S_M(r)$ removed from volume $V_M(r)$. Verlinde claims that entropy removed by matter is not perfectly symmetrical but can be thought as a superposition of ball shaped regions, which he describes as 'inclusions'. Verlinde then goes on to propose that the part of the elastic medium with volume $V_M(r)$ can be thought as being composed by a superposition of ball shaped regions B_i, each contributing a volume factor $N_i V_0$. The volume removed by mass inclusion is equal to ${V_M}^*(r)$. From each of these ball shaped regions B_i, one removes an entropy content equal to $N_i {V_0}^*$. After adding the contribution from all these ball shaped regions, the removal of total volume $N{V_0}^*$ corresponds to the following displacement field:

$$u(r) = -\frac{N{V_0}^*}{A(r)} \tag{39}$$

The normal strain is given by the derivative of the displaced volume u(r):

$$u^{'}(r) = n(r) = \frac{NV_0}{V(r)} \tag{40}$$

The reason that the $*$ went away is because $A(r)$ scales by $\frac{1}{r^{d-2}}$, and the factor of d-2 that pops up from the derivative can be absorbed into the volume. Note that this quantity precisely looks like the expression we had in equation (31). Hence, this allows us to interpret that depending on whether the strain is greater than or less than 1, one obtains the observed non-Newtonian behavior. Now, knowing that total entropy removed by mass is equal to $S_M(r)$ we can express the removed volume and total volume in more familiar form.

$$\begin{aligned} NV_0 &\rightarrow V_M(r) \\ N{V_0}^* &\rightarrow {V_M}^*(r) \end{aligned} \tag{41}$$

We arrive at an expression that relates surface mass density to normal strain:

$$n(r) = \frac{a_0}{8\pi G}\frac{M}{A(r)} \tag{42}$$

Having related the two quantities, now we will show, in the regime $\epsilon < 1$, how we can use this fact to come up with the expected modification to Newton's laws. First, we consider the strain in the region outside of spherical region B_0 with $V(r) > NV_0$ denoted by $\bar{B}_0$. This integral over the complementary region is given by the following:

$$\int_{NV_0}^{\infty} n^2(r)A(r)dr = \int_{NV_0}^{\infty} \left(\frac{NV_0}{V}\right)^2 dV = NV_0 \tag{43}$$

This integral represents the elastic energy in the presence of placement of a matter. One can repeat this calculation for all ball shaped regions B_i whose union construct the region with volume $V_M(r)$. Each region contributes $N_i V_0$ to the integral. The sum of the energy contributions of strains are given by the following:

$$\int_{B(r)} n^2(r)dV \approx V_M(r) \tag{44}$$

Here Verlinde makes the assumption that individual energy contributions from strains ${n^2}_i(r)$ add like sum of squares with negligible contribution from cross terms $n_i n_j$. Moreover, main contribution to integrals are assumed to come from the vicinity of B_i due to $\frac{1}{r^{d-3}}$ dependence of the displacement field. Assuming the spherically symmetric case, this expression can be recast as the following:

$$\int_0^r n^2(r^{'})A(r^{'})dr^{'} = V_M(r) \tag{45}$$

We differentiate this expression with respect to r and arrive at the following expression for the strain:

$$n^2(r) = \frac{1}{A(r)}\frac{dV_M}{dr} = \frac{1}{A(r)}\frac{8\pi G}{a_0}\frac{M}{d-1} \tag{46}$$

While $\frac{M}{A(r)}$ can be identified as baryonic surface density, Verlinde identifies $\frac{a_0}{8\pi G}n(r)$ as the dark matter surface density caused by strain. One can see that the surface density of dark matter scales as $\frac{1}{r}$, which means the volumetric density must scale as $\frac{1}{r^2}$. This is the expected behavior consistent with what we showed previously. Moreover, the relationship between surface densities of dark matter and baryonic matter can be expressed in one equation:

$$(\Sigma_{DM}(r))^2 = \frac{a_0}{8\pi G}\frac{\Sigma_B(r)}{d-1} \tag{47}$$

This result represents how the surface density of dark matter profile is related to the baryonic matter profile. This is consistent with what we would expect as it gives us the right density profile for dark matter introduced in section-1. This result is also consistent with Tully-Fisher scaling predicted for the surface densities of baryonic and dark matter.

7 What Verlinde's Theory does not Explain

While Verlinde's theory treats space-time and dark energy as an elastic medium to explain the observed deviation from Newtonian dynamics and successfully explains the observed velocity profile for the outer regions of spiral galaxies, the theory does not explain the dark matter distribution towards the center and how the entropic displacement force affects this region of the galaxy. Moreover, only spiral-galaxies have well defined velocity profile at a given radius due to their disk shape attributed to a well defined angular momentum vector. These galaxies are much younger than the elliptic galaxies, whose shapes are not flat like a disk but are rather volumetric . The velocity of stars in an elliptic galaxy is much more random. Unlike the Tully-Fisher relation, which works for describing the spiral galaxies, elliptic galaxies follow a velocity-luminosity dispersion relationship called Faber-Jackson relation. This relation is heavily sensitive to the size of the elliptical galaxy. It might be interesting to extend the notion of entropy displacement to elliptic galaxies or even irregularly shaped galaxies formed as a result of a past collision between two different ones, since Verlinde's theory only applies to spherically symmetric case.

Moreover, Verlinde's theory also does not account for the observed ratio of baryonic matter to dark matter, which comes out to be a roughly $\frac{\rho_B}{\rho_D} = \frac{1}{5}$. This ratio is attributed to primordial density fluctuations in the early universe at which the effective elastic microscopic description might be expected to break down. Furthermore, early universe was dominated by radiation. It might be interesting to think about how one can conceive of dark matter behavior in terms of elastic response when radiation dominated the universe. A point related to this was argued by Tortora *et al.* who claimed that de Sitter space is not the best approximation for Verlinde's theory since the Hubble parameter is a constant and the space-time describes a universe, in which one only has cosmological constant/dark energy and nothing else. However, our current universe is composed of many ingredients and Hubble parameter describing the expansion of our universe is dynamical and is a function of red-shift, which can be understood as a time parameter. The justification is as one looks further into space, one looks further past in time as light rays should have left much earlier to arrive on our telescopes now. Moreover, distant galaxies are receding with greater velocity than closer ones due to uniform expansion of the universe giving rise to a bigger red shift. Without further ado, we show how Hubble parameter scales as redshift denoted by $z = \frac{\Delta\lambda}{\lambda}$.

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \tag{48}$$

where Ω_m is the term that represents the ratio of current matter density to critical density, which is the density needed to have non-expanding universe. Ω_Λ represents a similar ratio for cosmological constant. One can see that Ω_m term has a z dependent cube term. This is due to the fact that matter density is inversely proportional to volume. On the other hand, density of dark energy is constant with respect to the volume of the universe.

Regarding the Hubble parameter and the acceleration scale, the authors claim that these variables can also contain information about or affect the displacement of the elastic medium. They propose in that situation, the strain function should take the following form:

$$\int_0^r n^2(r')A(r')dr' = \nu^2 V_M(r) \tag{49}$$

where the dimensionless parameter ν tunes and corrects Verlinde's proposition that elastic medium has maximum response to the entropy displacement caused by ordinary baryonic matter.

8 Possible Inconsistencies in Verlinde's Proposal

The authors Dai and Stojkovic claim that there is a mistake with regards to how Verlinde derives normal strain n(r) from the displacement field u(r) as a result of mass inclusion. The authors make the simple claim that since the displacement field has the same scaling as the Newtonian potential $\Phi(r)$ which goes by $\frac{1}{r}$ in 3d, the strain n(r) given by the gradient of the displacement field should scale as $n(r) \sim \frac{1}{r^2}$, which would imply the following relation for the dark matter surface density:

$$\Sigma_{DM}(r) \sim \frac{1}{r^2} \tag{50}$$

This is exactly what one would expect if Newton's second law was exactly satisfied without any flattening of rotation curves.

9 Conclusion and Outlook

In this paper, we have shown Verlinde's proposal that dark matter can be explained by the displacement force created in an elastic medium due to addition of matter taking away the de Sitter entropy. Verlinde's proposal was motivated from connections between entanglement and geometry and the empirical fact that cosmological acceleration scale, a macroscopic non-local quantity shows up in condition for observing a flat velocity dispersion relation in spiral galaxies. When entropy lost by matter is less than the de Sitter entropy, ($\epsilon < 1$), one expects a volume-law variation of entropy, which manifests as a violation of Newtonian dynamics. On the other hand, when this bound is saturated ($\epsilon = 1$), one obtains the area-law variation which yields Newtonian gravity. It would be interesting to see whether one can reach the same conclusions for cases considered in section 8.

10 References

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