Formation and Behavior of Traffic Jams

Tim Miller

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Abstract

This essay will discuss how traffic jams can emerge from free traffic on highways and how these traffic jams behave. This discussion will include an introduction to a few models of traffic flow as well as comparison with experimental data.

1 Introduction

Traffic jams occur almost daily on highways accross the world, extending the duration of certain trips by hours in many cases. The irony of this situation is that (usually) nobody in traffic wants to be slowed to a stop. With all of the lost productivity and environmental impact, one would expect that preventing and mitigating traffic would be in everyone's interest, so why exactly does this slowdown occur?

To understand how traffic behaves, three traffic models will be presented: the first to be presented will be the Lighthill and Whitman model, which is a macroscopic model that tries to understand traffic at the top-level and ultimately gives a description of a compressible fluid; the second will be a rather simple discrete-space microscopic model which is rather simple, but generates reasonable traffic flow and traffic jams; and the last will be a microscopic model in which a vehicle only reacts to the vehicle in front of it, which can be shown to give rise to an almost identical macroscopic equation of traffic. Beyond this, description of real traffic jam formation and behavior will be presented followed by some discussion of the results.

2 Traffic Models

2.1 Macroscopic Models

A major class of traffic models treat traffic flow concerns itself with just the macroscopic properties of traffic, with a notable example being Lighthill and Whitman's model (1955), which is still used in some modified form today. [1] Lighthill and Whitman's model assumes that the number of vehicles travelling past a certain point (the flow, $Q(x,t)$) is purely a function of the number density of vehicles at that point ($\rho(x,t)$), which translates to the equation

$$Q(x,t) = V(\rho(x,t))\rho(x,t),$$

where $V(\rho(x,t))$ is the average velocity of a vehicle, which is a function of the vehicle destiny.[2] The general shape of the Q vs. ρ is given in figure 1. It should be noted that the velocity is decreasing, so that more dense regions move more slowly in general. Now, we should note that in the absence of off-ramps/on-ramps, the number of cars is conserved; that is, $\partial_t\rho + \partial_x Q = 0$, or we may use our relation that Q depends only on ρ:

$$d_t\rho + \partial_x Q = 0 \qquad \rightarrow \qquad d_t\rho + \partial_x(V\rho) = d_t\rho + (\rho\partial_\rho V + V)\partial_x\rho = 0.$$

We see that the last piece describes a travelling density wave with speed given by $\rho\partial_\rho V + V$, which we will call $C(\rho)$. We note that $\partial_\rho V < 0$ which gives $C(\rho) < V$ or that the wave travels slower than a car at the point (which has speed $V(\rho)$), meaning this wave travels slower than a given car (so one will likely escape congested traffic at some point). This propegation of the wave is non-linear (it looks a bit like an inviscid Berger's Equation) as the propegation speed depends on ρ and will lead to the rise of shock waves, or sharp jumps in vehicle density over a short distance. [1] Typically, a diffusion term might be added onto

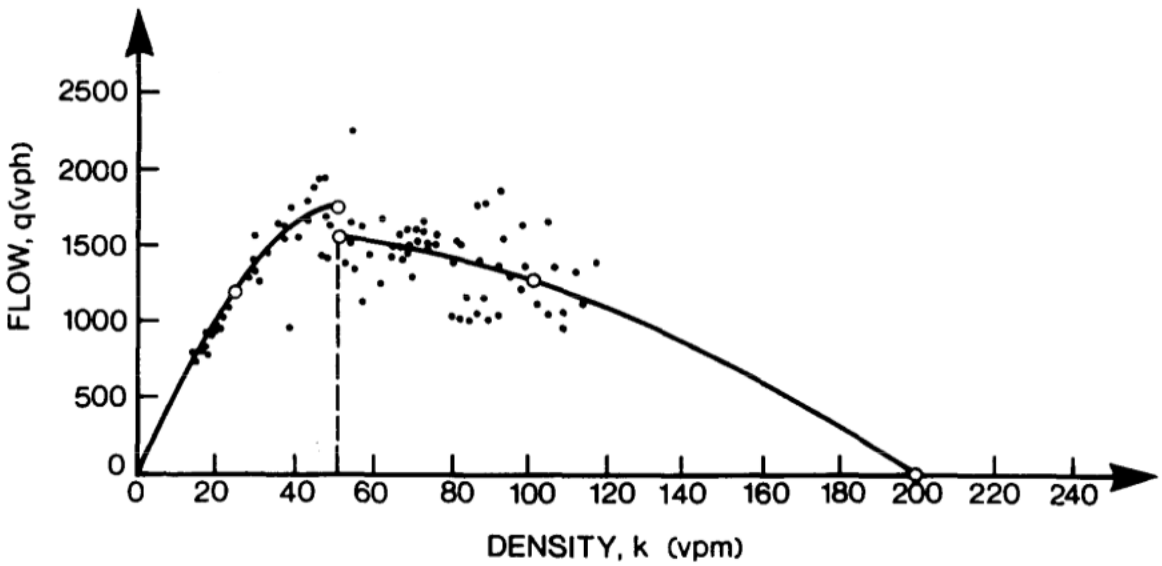

Figure 1: The general shape of the Flow Q vs. ρ, the density. We note that there is a maximum value for the flow indicating the capacity of the road ,Q_c at some medium density, which is a characteristic of the piece of road being considered (it may vary along the road). The traffic at this point of maximum flow may move more slowly than free traffic. When one pushes beyond this point, the density increase causes a more significant slowdown in velocity or even a jam. This graph is from [3].

this to ensure that the shock width does not go to 0, and the equation will be similar to the equation of motion for a compressible fluid:

$$\rho\frac{\partial V}{\partial t} + \rho V\frac{\partial V}{\partial x} = \frac{\rho}{\tau}\left[\tilde{v}(\rho) - V\right] - c_0^2\frac{\partial \rho}{\partial t} + \mu\frac{\partial^2 V}{\partial x^2},$$

where c_0,τ, and μ are constants to be determined from data and $\tilde{v}(\rho)$ is a function describing the steady state velocity of the system at a given density, which one might call the optimal velocity. [5]

Now, the understanding of this leads to some analysis of why congestion occurs: Lighthill and Whitman assume that a bottleneck on a road (a point on the road with a capacity, Q_c, that is smaller than upstream and downstream) will limit traffic to the maximum flow of the bottleneck, and any waves with flow greater than this maximum capacity, will reflect backwards (leading to greater congestion propegating upstream). This implies that beyond the bottleneck, the flow is not congested and that the velocity will increase significantly and that traffic congestion will have a fixed downstream end and will travel upstream from a bottleneck. [2]

2.2 Microscopic: Cellular Automata

Now, we will look at the Nagel-Schreckenberg model, which is a somewhat less detailed version of what are called "follow-the-leader" models.[1] The Nagel-Schreckenberg modal is a Cellular Automata with discrete time and discrete space with each space having two states: occupied and unoccupied. Each car occupies a single space and has a velocity associated with its movement, which may take on integer value starting at 0 up to some specified $v_{\max}$. The update scheme for each car is done in parallel as follows:

1. If $v_{\text{car}} < v_{\max}$ and next car is at least $v_{\text{car}} + 1$ spaces away, then $v_{\text{car}} \to v_{\text{car}} + 1$.
2. If the car directly ahead currently is j spaces downstream and $v_{\text{car}} > j$, then $v_{\text{car}} \to j - 1$.
3. With probability p, set $v_{\text{car}} \to v_{\text{car}} - 1$.
4. Now advance the car by v_{car} spaces.

The first step essentially states that a driver will choose to speed up if possible up to some optimal speed. The second step is there to prevent collisions. The third step is there to account for some variation in driving behavior, where we assume that not everyone accelerates as quickly as possible (with probability p, a car that can accelerate will not accelerate) and that not everyone drives at a constant speed away from traffic. The last step now lets the car advance to the next time step. With these simple rules, traffic jams and many other features of real traffic are reproduced. [7] As it turns out, the parameter p that controls randomly decreasing velocity or not accelerating actually determines whether or not jams will appear, as $p \to 0$ just yields a pattern that just shifts to the left by 1-space every iteration. There are many small variations that have been tested on this model to make the model reproduce the constants observed in real traffic (a comparison of the dynamics of this model and real traffic is provided in figures 2 and 3), yet the overall picture is still that this probabilistic aspect controls everything about the congestion and jamming of traffic. [1] This suggests that delays in responding to the leading car may be responsible for traffic jams.

2.3 Microscopic: Opimal Velocity and Micro-Macro Link

Somewhat similar to the Nagel and Schreckenberg model, is the continuous space "follow-the-leader" model called the *optimal velocity* model which was proposed by Newell in 1961. [6] The statement of this model is that a car's velocity will generally be determined by the car ahead of it; that is,

$$\frac{\mathrm{d}x_j(t+\tau)}{\mathrm{d}t} = V(\Delta x_j(t)),$$

which essentially states that the velocity of a car depends on the distance between that car and the car ahead of it, with a delay given by τ to account for actual delay in achieving the optimal velocity in real situations. Without considering lane-changes, this is generally how a driver has to react in order to drive safely on the highway and to avoid accidents. We may taylor expand this to get the differential form:

$$\frac{\mathrm{d}^2 x_j(t)}{\mathrm{d}t^2} = \frac{1}{\tau}\left(V(\Delta x_j(t)) - \frac{\mathrm{d}x_j(t)}{\mathrm{d}t}\right).$$

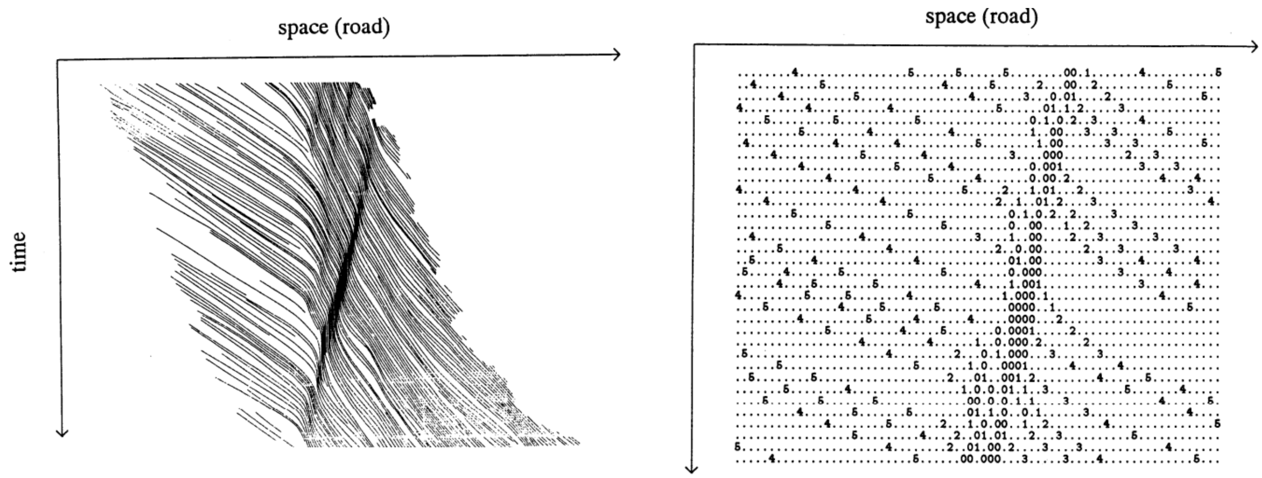

Figure 2: Above is a comparison of the Nagel and Schreckenberg Cellular Automata model and real traffic, with both situations showing a jam propegating backwards. The simulation denotes a space by a '.' and a car with velocity v by 'v' (velocity ranges from 0 to 5) and is done with periodic boundary conditions. The real traffic data was obtained through aerial photography and denotes a car's position down the road by a line. Both graphs were obtained from [7]

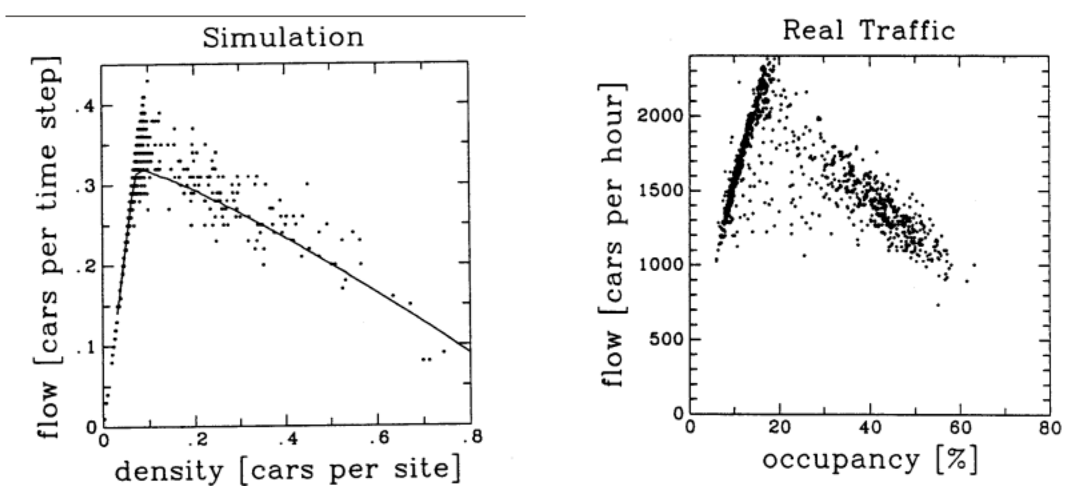

Figure 3: Above is a comparison of the Nagel and Schreckenberg Cellular Automata model and real traffic with respect to the relationship of the density of cars and the flux of cars. For the graph of the simulation, the dots represent an average over 100 time steps and the line represents the average over 100,000 time steps. Both graphs have the λ-shaped curve that is expected. Both graphs were obtained from [7]

This model allows a steady state solution where each car is spread out with the same separation L/N, where L is the length of the road and N is the number of cars on the road. It turns out that this steady state solution is unstable for long-wavelength fluctuations when

$$\tau > \frac{1}{2V'(L/N)},$$

where τ is the reaction time and $V'(L/N)$ is the derivative of the optimal velocity. Beyond this point, fluctutuations grow and homogenous traffic will be destroyed through waves in the density along the road. [5]

Following the work of Lee *et al* [8], we will show that if we coarse-grain, we will actually end up with the same equation as the one from the modified Lighthill and Whitman model. First, we introduce microscopic fields for destiny and flow $\hat{\rho}(x,t)$ and $\hat{q}(x,t)$ respectively:

$$\begin{aligned}\hat{\rho} &= \sum_n \delta(y_n(t) - x) \\ \hat{q} &= \sum_n \dot{y}_n(t)\delta(y_n(t) - x),\end{aligned}$$

where y_n is the coordinate of the n-th car. Now, we introduce a coarse-graining function $\phi(x,t)$ which is non-negative, normalized, and centered on $(x,t) = (0,0)$, which we use to define our coarse-grained destiny and flow $\rho(x,t)$ and $q(x,t)$:

$$\begin{aligned}\rho(x,t) &= \int \mathrm{d}x'\,\mathrm{d}t'\phi(x - x', t - t')\hat{\rho}(x',t') \\ q(x,t) &= \int \mathrm{d}x'\,\mathrm{d}t'\phi(x - x', t - t')\hat{q}(x',t').\end{aligned}$$

Essentially, we are finding the average flux and density of our microscopic model about some point. As we defined $\hat{\rho}$ and $\hat{q}$, we see that $\partial_x \hat{q} = -\partial_t \hat{\rho}$, and we may show (using integration by parts) that

$$\partial_x q(x,t) = -\partial_t \rho(x,t).$$

With more algebra, we can show

$$\partial_t q(x,t) = \rho(x,t)\langle \ddot{y}_n\rangle_\phi(t) - \partial_x\left[\rho(x,t)\langle \dot{y}_n^2\rangle_\phi(t)\right],$$

where the notation $\langle A_n\rangle_\phi(x,t)$ meaning

$$\langle A_n\rangle_\phi(x,t) = \frac{1}{\rho(x,t)}\int \mathrm{d}x'\,\mathrm{d}t'\;\phi(x - x', t - t')\sum_n A_n(x',t')\delta(y_n(t') - x').$$

We next define $v(x,t) = \langle \dot{y}_n\rangle_\phi(x,t) = \frac{q(x,t)}{\rho(x,t)}$, which we put in our earlier expression to eliminate q and we see something that looks a little familiar:

$$\rho\left(\partial_t v + v\partial_x v\right) = \rho\langle \ddot{y}_n\rangle_\phi(t) - \partial_x(\rho\theta),$$

where $\theta(x,t) = \langle \dot{y}_n^2 \rangle_\phi(t) - v(x,t)^2$. We notice our standard advective derivative on the left-hand side, which means that the right-hand side might be understood as forces. Now we will start assembling the right-hand side of the equation from our optimal velocity model (one can use other microscopic models as well). We notice $\ddot{y}_n(t)$ inside of the our expectation value is the same acceleration from the optimal velocity model, which is given by $\frac{1}{\tau}\left(V(\Delta x_j(t)) - \dot{y}_n(t)\right)$. Now, derivation of exactly how the force terms are acquired from the expectation values will be omitted here, but is available in the article by Lee *et al* for those interested [8]. The treatment used in the article essentially expands the expectation values as a Taylor series and goes to linear order to obtain:

$$\rho\left(\partial_t v + v\partial_x v\right) = \frac{1}{\tau}[V(\rho^{-1}) - v] - \frac{V'(\rho^{-1})}{2\tau\rho^3}\partial_x \rho + \frac{1}{6\tau\rho^2}(\partial_x)^2 v,$$

which is very close to our modified form the equation for the Lighthill and Whitman model, with the constants replaced by the constants of the optimal velocity model, suggesting that these follow the leader models do bring about similar macroscopic traffic behavior.

3 Real Traffic

At this point, three models of traffic have been presented, each suggesting a similar behavior for traffic, but some consideration must be given to what occurs in real traffic. First, we should define some of the standard terms used to describe traffic, where there is free traffic, where the flow Q increases linearly with the density ρ; synchronized traffic, which generally occurs at densities greater than free traffic,with Q lower than the maximum flow in free traffic and with velocities typically decreased, where spacing between cars is generally fixed and velocities do not appreciably differ between lanes; and jams, which is a high density and low flow region where cars may be entirely stopped for either a moment or some longer duration. Synchronized traffic can occur with just a travelling upstream front, with the downstream front fixed on a bottleneck, or with both fronts moving together along the road with a characteristic speed.[11] Now, a jam can occur from either free or synchronized traffic as a shock built up from some density perturbation as predicted in the Lighthill and Whitman model, with the shock propegating upstream, with an example of the transition from free traffic to a jam shown in figure 4.

More interesting, however, is the growth of wide-moving jams from synchronized traffic, which can be observed in figure 5. This transition of synchronized flow to jam is observed much more frequently than the free flow to jam transition. Initially, a small array of narrow jams or even a single narrow jam is formed. These small jams propegate upstream (at a speed slightly faster than larger jams) and will grow into wide jams with some regular spacing or may catch up to wide jams and join them. It is observed that a narrow jams may be damped by the existence of nearby jams (narrow or wide) and that there is a minimum distance between wide jams that is greater than the mean distance between narrow jams. Once these wide jams are formed, their velocities are observed to be the same so that these jams may will remain some fixed distance until they are destroyed. These narrow jams are formed at a region near a bottleneck within the traffic such as an off-ramp or an on-ramp, where synchronized traffic will persist over an extended period of time, which is known as the ''pinch region'', which starts at this bottleneck and extends downstream to where wide-jams begin to form. This means that a jam formed through this will generally not be situated at the bottleneck, but some distance upstream.[10]

It should be noted that speed of the front of the jam's shock travels at a constant velocity, and we may plot, then, this line on the flow vs density plot to note the behavior of jams, which has been done in figure 6. The narrow jams are only created in more compressed synchronized flow, where the state lives above the line J shown in the third plot in figure 6, which is called the ''pinch effect'' in synchronized flow. These more compressed states might be considered metastable, where there is a strong possibility of a transition to a jammed state. [10]

The transition from free flow to synchronized is rather tricky, as it can occur over a wide range of flows and densities, not just when the capacity of the road is exceeded. Not only that, the backwards transition generally occurs at flows considerably lower than the forward transition, suggesting some sort of hysteresis in the system, suggesting that the transitions are not as simple as the models make them out to be. [4]

3.1 Discussion

At this point, three models of traffic have been presented as well as some characteristics of how jams form and propegate. The remaining piece is to see really how the models compare. It has been shown that by coarse-graining the *optimal velocity* model, which proposes a reasonable explanation of how drivers base adjust their speed, yeilds the exact macroscopic equation found in the macroscopic model by Lighthill and Whitman. With the instability of the *optimal velocity* model about the steady state, the idea that the observed narrow jams can occur is not unimaginable, and the non-linear macroscopic behavior certainly explains how jams propegate, but fail to explain the characteristics of the transitions between free and synchronized traffic, suggesting that there is more at play than what the models mention. The issue with the transition from free to synchronized traffic not occuring at a single point is sort of solved by treating saying there are a range of value at which it can occur and that there is a probability of transition when the free flow is between some minimum and maximum flow in the system [4], which might be explained that it is inherently due to some random behavior like driver preference or behavior, although I am not so sure about that.

One might point out that the real data includes on-ramps and off-ramps as well as multiple lanes, suggesting that there are a number of factors unaccounted for in the models that are present in real situation. To begin with, the fact that there are multiple lanes is rather insignificant in synchronized flow, where each lane moves roughly together. That's not to say that people don't switch lanes in this traffic, but it offers no clear advantage to do so in the absence of outside factors. There are, however, corrections that must be added to correct for lane-changes at low densities of traffic in the macroscopic models, which have been studied, but will not be presented here. [1] In the case of on-ramps and off-ramps, it should be noted that this might be considered as a sharp jump in the density of traffic in the

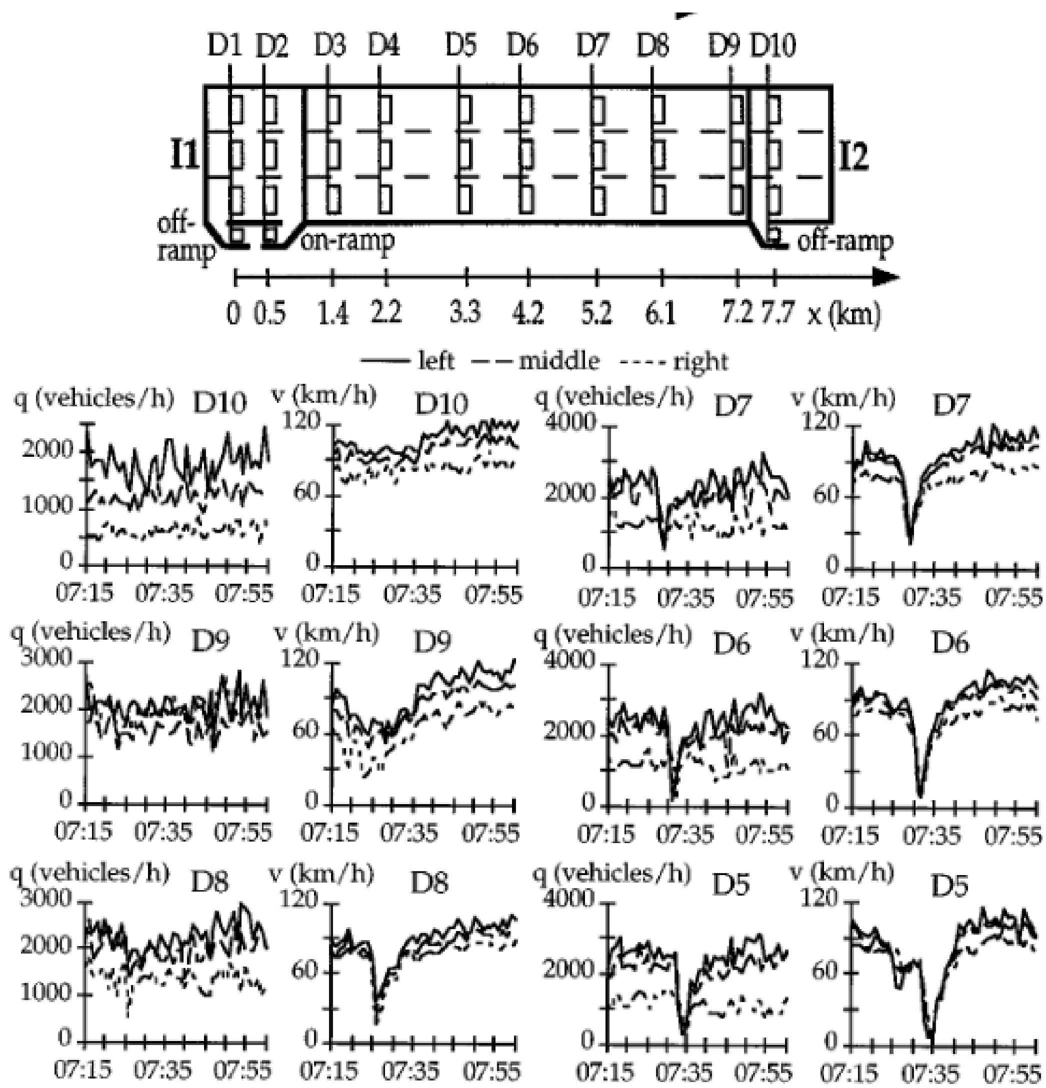

Figure 4: Above is an example of a jam forming from free traffic on the German Highway A5 between Frankfurt and Bad Homburg. The top diagram shows the setup of the induction-loop detectors measuring the traffic, where the vehicles travel to the right, and each graph shows the measurements from that detector labelled just above the graph as a function of time (each line represents a different lane as shown just below the diagram). Outside of the clear drop in flux and velocity, one sees that the velocities and flux are quite high, suggesting free flow. If one compares the time of the onset of the flux drop at each snapshot, it is observed that the jam is upstream through traffic. The components graphs of this figure are from [9].

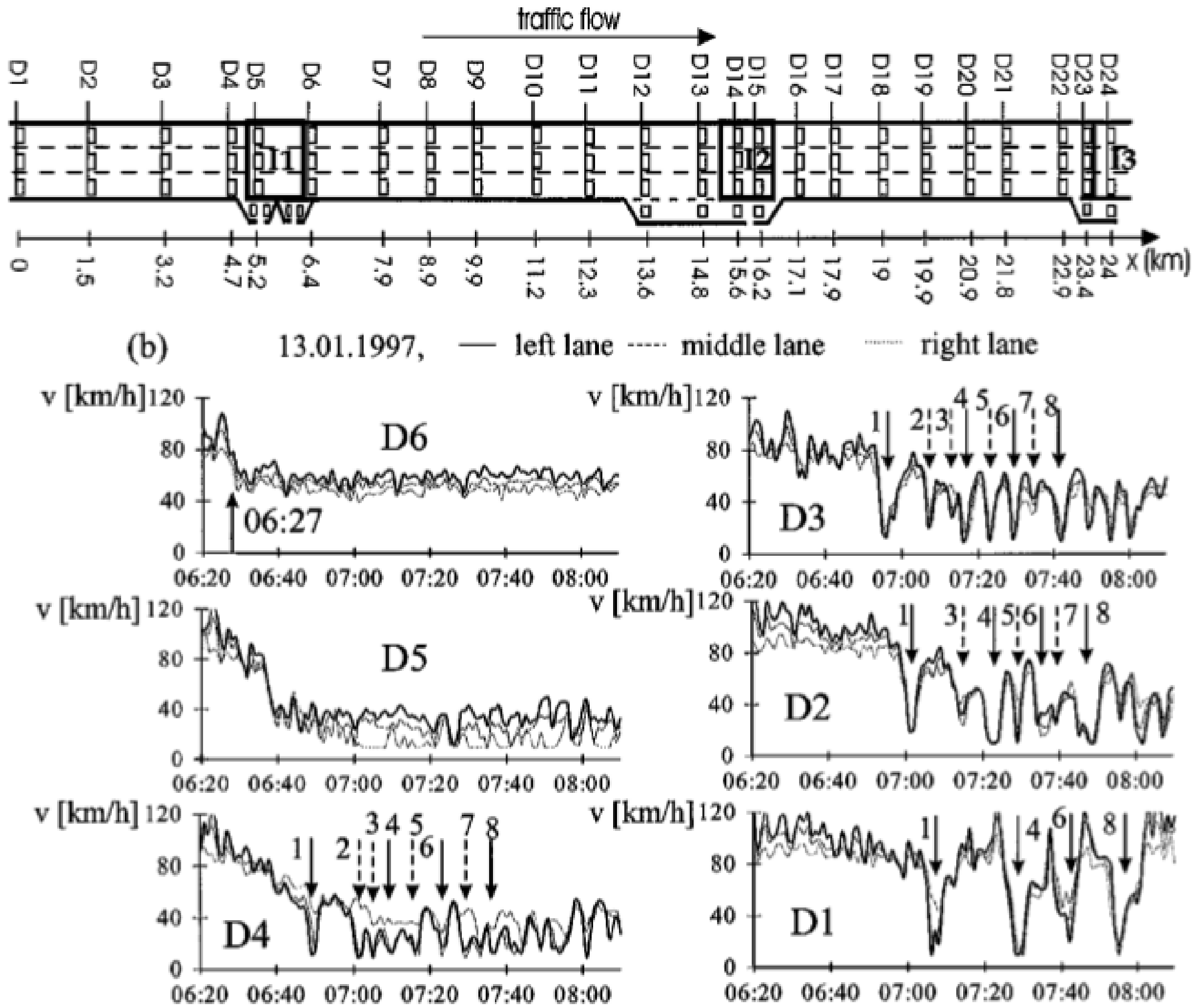

Figure 5: Above is an example of a jam forming from synchronized flow and propegating through traffic, with the setup of the highway and detectors shown on top and graphs showing measurements of the detector denoted above each graph as a function of time. First, many small jams are formed starting at D4, with different propegation speeds upstream. These combine into larger jams as the smaller jams propegate backwards, with the spacing between jams becoming larger. These graphs are from [10].

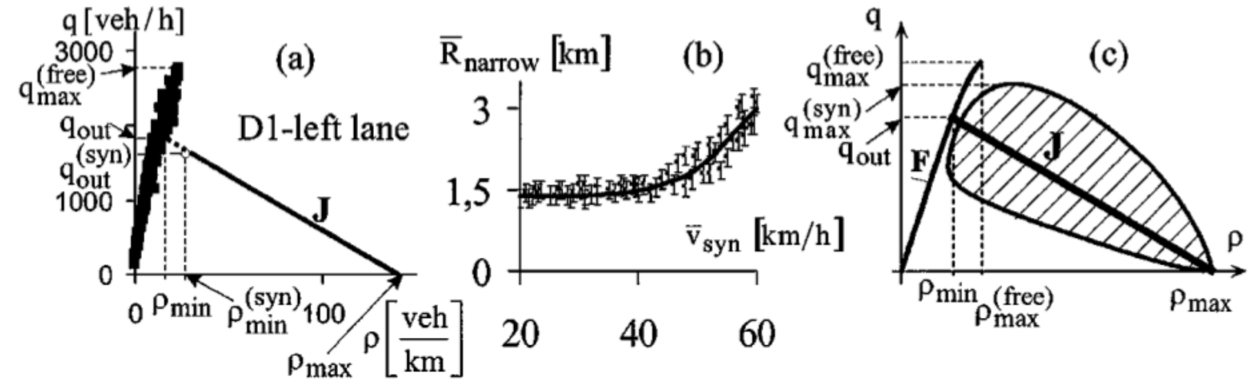

Figure 6: Above is a series of graphs giving some characteristics of jams. The left gives the standard λ-shaped graph of flow Q against ρ, the vehicle density, with the line J denoting the line on which jams live, as the fronts of jam's shock has a constant velocity. The second graph gives the mean spacing of narrow jams as a function of the velocity of synchronized traffic within the pinch region, or the region of synchronized flow near the bottleneck. The third graph shows the general trends of the flow vs density graphs, with the shaded region surrounding J showing where synchronized flow is observed. It is observed that narrow jams are observed in synchronized traffic above the line J. These graphs are from [10].

macroscopic models, although I suspect that at the off-ramp and on-ramp, lane-swapping is significant.

A last piece to note is that there has been some discussion in the literature about "phantom" traffic jams, which supposedly do not occur at a bottleneck and seemingly have no cause.[1] In the models and data considered in this essay, there has always been some assumption of a bottleneck to cause congestion or just some flow above the capacity of a road, but there have been some disagreements about whether these phantom traffic jams actually occur or not. Trieber *et al* suggest that such a phenomena might be due to the distance between the bottleneck which produced the jam and the actual jam itself, which may propegate upstream into free traffic [11], but still others think these "phantom" traffic jams may exist.

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