Phase Transitions and the Inflationary Cosmological Models

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Abstract

In this paper, we review how the idea of a delayed first-order phase transition was used in the inflationary cosmological model and how it solves, or at least relieves, the problems from the standard cosmological model that are contradicted by the CMB observation. We also discuss the problems of the original inflationary model, and how it was improved by other inflationary models.

Contents

1 Introduction

Our universe, unlike any other physical systems, can neither be studied by observing another universe and comparing the results nor by watching it evolve from the beginning again. Therefore, the study of cosmology can only be done by gathering a vast amount of observational data and building models out of it. With general relativity as its ground base, modern cosmology has been rapidly developed since the early 20th century, during which the standard cosmology model was established. This model successfully described the evolution of our universe and was supported by the early observations.[1] However, as more and more results that cannot be explained by the standard model showed up, during the last several decades the standard model has been continuously modified and improved. One important modification is the insertion of a period in the very early universe in which the universe expanded rapidly, called inflation.

The idea of inflation was originally introduced by Guth in 1980 [1]. The mechanism of the original inflationary model is a delayed first-order phase transition that comes from the spontaneous symmetry breaking of a grand unification theory. As an effect of this phase transition, inflation can solve several initial condition problems in the standard model, such as the horizon problem and the flatness problem, whose answers in the standard model have to be some unnatural initial conditions of the very early universe. However, the original inflationary model also creates its own problems and has been replaced by other inflationary models [2–4]. Although the main stream today are the models that are without a phase transition, it is still pedagogically meaningful to discover how the concept of inflation was conceived in order to overcome the difficulties in cosmology.

We will give a brief summary of the standard model and the observational facts we need as preliminaries in section 2, after which we introduce three typical problems in the standard model that are solved by the inflationary model: the horizon, flatness, and monopole problems. In section 3 we first discuss the phase transition process and how it causes the inflation, and show that they can solve the problems in varying degrees. Then, we will also talk about the upgrade versions of inflation include the "new inflationary model" and the "chaotic inflationary model", the latter of which is considered as the most reliable inflationary model till today [5]. We will use the system of geometrized unit, i.e. $c = G = \hbar = 1$, unless indicated otherwise.

2 Standard Cosmological Model and Its Problems

2.1 Preliminaries

The fundamental assumption of the standard cosmological model is the cosmological principle, which states that our universe is spatially homogeneous and isotropic on a large scale. Based on this principle, the spacetime geometry can be described by the Robertson-Walker

[1] Later on, we will refer to the standard cosmological model as "the standard model" for short when there is no confusion with the Standard Model of particle physics.

(RW) metric [6] :

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{1}$$

where $k = 0, \pm 1$ is the curvature constant, and $a(t)$ is called the scale factor. For the case $k = 1$, the universe is closed, and we can get the volume of whole space (the 3-sphere) as $V = 2\pi^2 a^3$. Under the RW metric, the distance $D_{AB}(t)$ of two galaxies A and B with spatial coordinates (r_A, θ, ϕ) and (r_B, θ, ϕ) is

$$D_{AB}(t) = a(t)\int_{r_A}^{r_B} \frac{dr}{\sqrt{1-kr^2}}. \tag{2}$$

By considering the contents of the universe as perfect fluids, the evolution of $a(t)$ can be derived from the Einstein equation as

$$3\frac{\dot{a}^2 + k}{a^2} = 8\pi\rho, \qquad 2\frac{\ddot{a}}{a} + \frac{\dot{a}^2 + k}{a^2} = -8\pi p, \tag{3}$$

where ρ and p are the density and pressure of the contents, respectively. These are called the Friedmann equations, they can be equivalently expressed as

$$3\frac{\dot{a}^2 + k}{a^2} = 8\pi\rho, \qquad \dot{\rho} + 3H(\rho + p) = 0, \tag{4}$$

where $H \equiv \dot{a}(t)/a(t)$ is called the Hubble parameter. Defining the relative velocity of two galaxies as $u(t) := dD(t)/dt$, we can easily see from (2) that $u(t) = H(t)D(t)$, which means that at a given time t, the recessional velocity between two galaxies is proportional to the distance between them. This result for the present t was given by Hubble in 1929 from observation, and thus it is called the Hubble's law [7].

Using the Hubble parameter, we can write the first equation in (3) in SI as $H^2 = 8\pi G\rho/3 - kc^2/a^2$. If we define a critical density $\rho_c := 3H^2/8\pi G$, then we have

$$\rho = \rho_c + \frac{3kc2}{8\pi G a^2}. \tag{5}$$

We can see the ρ_c corresponds to $k = 0$, i.e. the universe is spatially flat. $\rho > \rho_c$ corresponds to an open universe and $\rho < \rho_c$ corresponds to a closed universe. We can also define a density parameter

$$\Omega := \rho/\rho_c = \frac{8\pi G\rho}{3H^2}, \tag{6}$$

and when $\Omega > 1$ the universe is closed. Hence, whether our universe is open or closed can be determined by the detecting the present value of the Hubble parameter H_0 and the mass density ρ_0. According to the observation of the type Ia supernova, $\Omega \simeq 1$, which means our universe is very close to flat.

In the early universe, the universe was dominated by radiation, whose energy density ρ_r satisfies $\rho_r a^4 = \text{constant}$. We can rewrite the first equation in (3) as

$$\dot{a}^2(t) = B^2 a^{-2} - k\,, \tag{7}$$

where $B^2 \equiv 8\pi\rho a^2/3$ is a constant. Since k is negligible when a is small, we have a solution $a = (2Bt)^{1/2}$. Remind that $\rho_r \propto T^4$, and thus $T \propto t^{-1/2}$; this relation can be written as

$$T = \left(\frac{45}{\pi^3 N_{\text{eff}}}\right)^{1/4} \left(\frac{M_p}{4t}\right)^{1/2} = 10^{10} t^{-1/2}\,(\text{SI})\,, \tag{8}$$

where N_{eff} is the effectively massless particle number, $M_p \simeq 2\times 10^{-8}$kg is the Planck mass [8]. Around $t = 10^{11}$s, the contents became matter dominated. At $t \simeq 10^{13}$s, T decreased to about 4000K. Under this temperature, matter began to form neutral atoms from ionized states. As a result, photons can hardly interact with charged particles, so they were decoupled from the matter field and became a background photon gas. After decoupling, the photons were no longer in thermal equilibrium with massive particles, and its temperature will decay as $T_\gamma \propto a^{-1}$. Estimation shows that the temperature of the decoupled photon system is about 3K at present, and the radiation energy is mainly concentrated in the microwave region; thus, it is called the cosmic microwave background radiation (CMB). This isotropic radiation was first detected accidentally by Penzias and Wilson in 1965, although they only observed one point ($\lambda = 7.35$ cm) on the back body spectrum. In the early 90s, Cosmic Background Explorer satellite (COBE) accurately detected the black body spectrum of the CMB, finding that $T_\gamma \simeq 2.735$K, and verified that the CMB radiation is highly isotropic [9].

2.2 Difficulties of the Standard Model

Although the standard cosmological model has solved many issues successfully, when it comes to the very early universe, namely $t \ll 1$s after the big bang, several problems arose. Here, we introduce three of them.

2.2.1 The Horizon Problem

At a given spacetime point p, the 2-dimensional boundary of the region that can be observed at t_p is called the particle horizon of p [6]. The distance D_H between p and any point on the particle horizon can be derived using (2) and the geodesic equation of a photon as

$$D_H(t_p) = a(t_p)\int_0^{t_p} \frac{dt}{a(t)}\,. \tag{9}$$

For $k = 0$, the radiation universe and the matter universe satisfy $a(t) \propto t^{1/2}$ and $a(t) \propto t^{2/3}$, respectively. Hence, in SI we have

$$D_H(t) = 2ct\ (\text{radiation universe})\,, \qquad D_H(t) = 3ct\ (\text{matter universe})\,. \tag{10}$$

At the present time t_0, the spacetime subset that contains all of the particle world lines within the particle horizon of an observer in the Milky Way is called the presently observable universe. Its spatial radius is a function of t, denoted by $D_{POU}(t)$. Note that D_{POU} not is the radius of the observable universe at t, and only at $t = t_0$ we have $D_{POU} = D_H(t_0)$. Since $t_0 \simeq 3 \times 10^{17}$s, this value can be estimated using (10) as

$$D_H(t_0) = D_{POU}(t_0) = 3ct \simeq 3 \times 10^{26}\text{m}\,.$$

Since both D_H and D_{POU} are expanding over time, let us consider the time when the photons were decoupled, i.e. $t_{\gamma d} \simeq 10^{13}$s. From the first equation in (10), we have

$$D_H(t_{\gamma d}) = 2ct_{\gamma d} \simeq 6 \times 10^{21}\text{m}\,.$$

On the other hand, since D_{POU} can be regarded as the spatial distance between a Milky Way observer and an observe at the edge of the presently observable universe, we can know from (2) that $D_{POU} \propto a(t)$. According to statistical mechanics, the temperature of the background radiation satisfies $T_\gamma \propto a^{-1}$, and thus $D_{POU} \propto T_\gamma^{-1}$. Note that $T_\gamma(t_{\gamma d}) \simeq 4000$K and $T_\gamma(t_0) \simeq 2.7$K, we have

$$D_{POU}(t_{\gamma d}) = D_{POU}(t_{\gamma d})\frac{T_\gamma(t_0)}{T_\gamma(t_{\gamma d})} \simeq 2 \times 10^{23}\text{m}\,.$$

Therefore, the $D_{POU}(t_{\gamma d})$ is about 33 times larger than $D_H(t_{\gamma d})$. From (10) we roughly have $D_{POU} \propto t^{1/2}$ or $D_{POU} \propto t^{2/3}$, and hence this multiple is even larger in earlier times. For instance, at the Planck time $t_p = 10^{-43}$s, the time at which classical theory breaks down, $D_{POU} \simeq 10^{-5}\text{m} \gg D_H \simeq 10^{-34}$m, which makes the ration become 10^{29} [8]. However, due to the existence of the particle horizon, two particles with a spatial distance larger than D_H could not have had any interaction before, which means the homogenous and isotropic result from the CMB observation can not be naively explained as the "self blending" from the interaction. This is called the "horizon problem" of the standard model, and is also known as the "homogenous problem".

The horizon problem can be alternately expressed as follows [1, 8]: when detecting the isotropy of the CMB, the temperature measured by two back-to-back antennas are the same to high precision. The photons received by these two antennas came from two sources S_1 and S_2 with a spatial distance $D_{12}(t_{\gamma d})$ when the photons were decoupled. Thus, at $t = t_{\gamma d}$ our universe should be spatially homogenous at least in the scale of $D_{12}(t_{\gamma d})$. However, according to the estimation [10], $D_{12}(t_{\gamma d})$ is at least 90 times lager than $D_H(t_{\gamma d})$, which implies that there could not be any thermo-interaction between S_1 and S_2, and this long range homogeneity is a complete mystery in the standard model.

2.2.2 The Flatness Problem

According to the standard model, the deviation of Ω from 1 will grow rapidly over time. Use $\epsilon(t) \equiv |1 - \Omega^{-1}(t)|$ to represent the deviation. From (5) we have

$$\epsilon(t) = |\frac{\rho - \rho_c}{\rho}| = \frac{3|k|}{8\pi\rho(t)a^2(t)} \propto \begin{cases} 0, & k = 0, \\ (\rho a^4)^{-1}a^2 \propto a^2, & k \neq 0,\ \text{radiation universe}, \\ (\rho a^3)^{-1}a \propto a, & k \neq 0,\ \text{matter universe}. \end{cases} \tag{11}$$

Again, we consider the Planck time $t_p = 10^{-43}$s. Remembering that the early universe satisfies (8), we have $T(t_p) \simeq 3 \times 10^{31}$K. Since a is inversely proportional to T, $a(t_0) \simeq 10^{31} a(t_p)$, and thus from the equation above we can see that $\epsilon(t_0)$ is $10^{31} \sim 10^{62}$ times larger than $\epsilon(t_p)$. (A more precise estimation gives 10^{60}.) Since observation have shown that $\Omega(t_0) \sim 1 > 0.1$, $\epsilon(t_0) < 10$; hence, $\epsilon(t_p) \simeq 10^{59}$, which means $\Omega(t_p) \simeq (1 \pm 10^{-59})$. This implies that the very early universe has to be extremely flat. In addition, Ω at 10^{-43}s has to be so close to 1 that the non-zero numbers only show up at after the 59th place of decimals. Any tiny deviation would imply that the universe either has already became a "big crunch" singularity or expand too fast to form any star or galaxy. Ω has to be fine-tuned to this special value to make the universe look like what it looks like today; therefore, the flatness problem is also called the "fine-tuning problem".

Equivalently, the flatness problem can also be regarded as the "entropy problem". So long as the local thermo-equilibrium of the contents is maintained in the universe, it can be proved from the first and the second law of thermodynamics [11] that the entropy S in any co-moving volume does not change with time. We have showed that the volume of the whole space for $k = 1$ at t is $V = 2\pi^2 a^3(t)$. The entropy density in this volume is defined as $s \equiv S/2\pi^2 a^3$, it is also given by [11] that

$$s = \frac{2\pi^2}{45} N_{\text{eff}} T^3 \,. \tag{12}$$

Its present value s_0 can be calculated as $s_0 \simeq 3 \times 10^9 \text{m}^{-3}$ [8, 11]. From (6) we obtain that

$$1 - \frac{1}{\Omega} = \frac{3kc^2}{8\pi G \rho a^2} \,,$$

and hence the entropy S can be expressed in SI as

$$S = 2\pi^2 a^3 s = 2\pi^2 \left(\frac{2kc^2 \Omega}{8\pi G \rho (\Omega - 1)} \right)^{3/2} s = 2\pi^2 \left(\frac{kc^2}{H^2 (\Omega - 1)} \right)^{3/2} s \,.$$

Plug in $H_0 \simeq 10^{-18}\text{s}^{-1}$ and consider that $k/(\Omega(t_0) - 1) > 1$, we have $S > 10^{87}$. There is no physical explanation for where this incredibly large entropy of the universe comes from. To interpret this is essentially the same as to understand why Ω is so close to zero.

2.2.3 The Monopole Problem

Another difficulty of the standard cosmological model is in the context of applying particle physics to the early universe. According to the grand unification theories (GUTs), there exists a symmetry group, e.g. $SU(5)$, that is in charge of the unification of electromagnetic, weak, and strong interactions. At $T_c \simeq 10^{16}$GeV$\simeq 10^{27}$K, this symmetry group is spontaneously broken into $SU(3) \times SU(2) \times U(1)$, i.e. the symmetry group of the Standard Model of particle physics [12]. In this phase transition process, magnetic monopoles would show up as topological defects. The number density of the magnetic monopoles can be estimated as $n_M \simeq 1/\xi^3$, where ξ is the correlation length of the Higgs field [8]. For causality consideration, this length cannot be larger than the particle horizon, i.e. $\xi < 2ct$. Using (8), we have $t_c \simeq 10^{34}$s. Thus, the mass density at t_c can be expressed as

$$\rho_M = n_M m_M > \frac{m_M}{8c^3 t_c^3}\,, \tag{13}$$

where m_M is the mass of a magnetic monopole. To see the present value of ρ_M, we can divide this by the entropy density can make it independent of the scale factor. From (8), (12) and (13), we get

$$\frac{\rho_M}{s} > 4\pi^2 \left(\frac{\pi N_{\rm eff}}{45}\right)^2 \frac{m_M T_c^3}{M_p^3}\,.$$

By plugging in $s_0 \simeq 310^9 \mathrm{m}^{-3}$, $N_{\rm eff} \simeq 100$, $m_M \simeq 10^-11$kg and $M_p \simeq 2 \times 10^{-8}$kg, we can see that the present value of ρ_M satisfies $\rho_M > 10^{-15}\mathrm{kg/m}^3$. Therefore, the contribution of the magnetic monopoles to Ω is

$$\Omega_M = \frac{\rho_M}{\rho_c} > 3 \times 10^{11}\,. \tag{14}$$

This implies a tremendous overproduction of magnetic monopole in the early universe, which is far from the fact that we have not detected a single magnetic monopole.

3 Inflationary Models

3.1 The Original Inflationary Model

As the first step of solving the above puzzles in the standard model, Guth [1] suggested that a phase transition process may drive the universe to expand rapidly in a very short time. This can solve the horizon problem as well as the flatness problem completely, and at least relieve the monopole problem to some extent.

3.1.1 Phase Transition and Inflation

The Higgs field ϕ in a GUT can be expanded as a fluctuation around its vacuum expectation value, which is the minimum value of the effective potential $V(\phi)$ at zero temperature.[2] This is called the true vacuum, denoted by ϕ_T, and $V(\phi_T)$ can be viewed as the vacuum energy density, which can also represent the cosmological constant as $\Lambda = 8\pi G V(\phi_T)$. Although this value is fairly large according to the observation, the estimation from the Planck energy indicates that it is only 10^{-120} times the vacuum energy density of the vary early universe. Thus, we can say that $V(\phi_T) \simeq 0$ for $T \simeq 0$ [1]. There also exists a secondary local minimum for ϕ called the false vacuum, denoted by ϕ_F. This is a very special state of matter whose

[2]To simplify the discussion, we assume the Higgs field has only one component even if it is actually a multicomponent field.

energy density ρ_F is proportional to the fourth power of the critical energy and can be estimated as

$$\rho_F = V(\phi_F) \simeq 10^{76}\text{kg/m}^3 \,, \tag{15}$$

(this is basically the density of a massive star when compressed into the size of a proton). The false vacuum can also be treated as a perfect fluid with a negative pressure whose equation of state is $p_F = -\rho_F$. When T approaches T_c from zero, $V(\phi_T)$ will gradually increase and reach $V(\phi_F)$ when $T = T_c$. When $T > T_c$, the potential can only have one minimum, so we can choose the parameter of the theory and set it as $\phi = 0$.

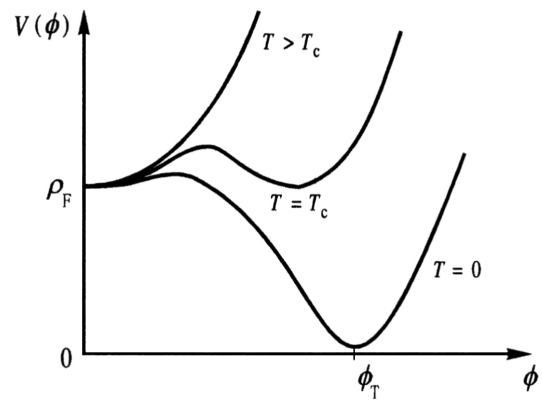

Figure 1: The effective potential V_ϕ of the Higgs field.

The inflationary model assumes that there were some expanding small region in the early universe whose temperature was higher than T_c. The Higgs field in these region is in the vacuum state $\phi = 0$. As the temperature decrease with the expansion gradually to T_c, the minimum of $V(\phi)$ splits into $V(\phi_T)$ and $V(\phi_F)$. As shown in figure 1, ϕ would become the false vacuum state at T_c. However, as the false vacuum is a metastable state since it has a higher energy, ϕ would eventually cross the barrier via tunneling and turn into a real vacuum state. This process corresponds to a first order phase transition. Due to the choice of parameter in the GUT, ϕ would stay in the false vacuum for a while and slowly turn into the true vacuum, which is similar to the supercooling phenomenon. One can even choose the parameter and make the supercooled state hold when T is close to zero. In this situation, the false vacuum density ρ_F is much larger than the vacuum density of the radiation contents ρ_r. Now we look at the effect of this delayed phase transition on the scale factor. Suppose the region is homogenous and isotropic.[3] Since k is negligible when a is small, and $\rho \simeq \rho_F$, the first equation of (4) gives

$$\frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho_F}{3}} \equiv H \,, \tag{16}$$

where $H \simeq 10^{34}\text{s}^{-1}$ is the Hubble parameter for the supercooled state. The solution can be easily seen to be

$$\alpha(t) \propto e^{Ht} \,. \tag{17}$$

[3]For the general discussion without these assumptions, see [8].

Therefore, a increases sharply during the time in the supercooled state, which makes a way larger than the value given by the standard model at the time corresponds to the end of the phase transition in the inflationary model, and that is where the name "inflation" comes from. In addition, since $\rho \simeq \rho_F$ and $p = -\rho$ during the phase transition, we can see that the second Friedmann equation in (4) is satisfied automatically.

Another way to interpret inflation is that in the standard model the expansion of the universe would be decelerating, since from the two equations in (3) we have $\ddot{a} = -4\pi a(\rho + 3p)/3$ with $\rho + 3p > 0$ in the standard model, which means $\ddot{a} < 0$. This is essentially because of the positive value p and ρ of a normal perfect fluid. However, for the false vacuum $pF = -\rho_F < 0$, which leads to $\ddot{a} = 8\pi\rho_F/3 > 0$. Hence, the inflationary model makes the fact that the expansion of the universe to accelerate. This effect is because the repulsion comes from the negative pressure is stronger than the attraction arising from the positive energy density.

The process of inflation started at the critical point $t_c \simeq 10^{-34}$s and lasted for about 10^{-32}s; the symmetry breaking happened at end of this period. After this process, each of those regions in the universe which used to be in the false vacuum state turned into a region with a real vacuum state, called a bubble; they are similar to the droplets that form in supercooled water vapor. The huge energy difference between the true and false vacuum are released as latent heat, which leads to a reheating process in that region, making the temperature raise up close to T_c again, and then continue the expansion at the same speed as the radiation dominated case of the standard model. One of the advantages of this original inflationary model is that it is much less sensitive to the initial condition of the universe than the standard model, so the universe can be easily look like what it looks like today. Also note that inflationary model is not to replace the standard model completely; rather, it inserts a period in the very early universe. Now let us look at how can it solve the puzzles in the standard model.

3.1.2 Solving the Problems

To see how the inflation can solve the horizon problem, note that the presently observable universe is much larger than the particle horizon in the very early universe, we look at the $t_p = 10^{-43}$s again. Since D_H has not been affected by inflation yet, we still have $D_H \simeq 10^{-34}$. However, D_{POU} before inflation will be much smaller than we estimated in the standard model. Seeing that inflation continued for $\Delta t \simeq 10^{-32}$, we know from (17) that a was expanded by a factor of $e^{H\Delta t} \simeq 10^{43} \gg 10^{29}$. In this case, even if D_{POU} is many times larger than D_H after the inflation it was just a tiny region before, and hence particles could interact with each other as we wish, becoming homogenous and isotropic. It is not hard to believe that the homogeneity and isotropy would be maintained during inflation and the standard model expansion after that. Therefore, the horizon problem is solved.

The other expression of the horizon problem can been solved as follows: from (9) we can see that $D_H(t)$ would expand exponentially with the growth of $a(t)$. After that, although $D_{POU}(t)$ also has expanded exponentially during inflation, since we just found that $D_{POU}(t) \ll D_H(t)$ before the inflation, $D_H(t)$ would be larger than D_{POU} all the way to $t = t_0$. Therefore the confusing estimation we got from the standard model that $D_{12}(t_{\gamma d})/D_H(t_{\gamma d}) > 75$ is not true anymore (see figure 2).

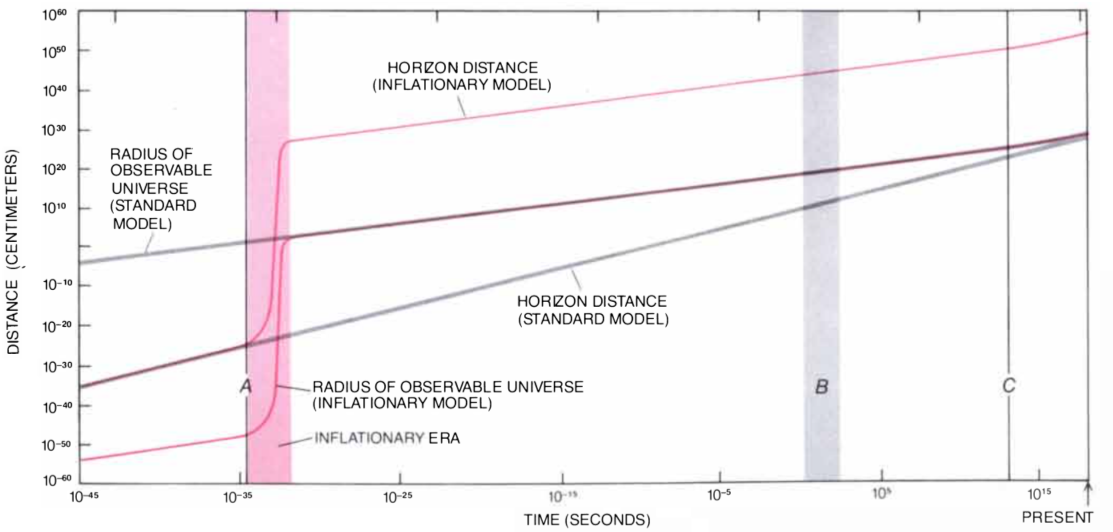

Figure 2: The evolution of D_H and D_{POU}, where A represents the start of the inflation, B represents the primordial nucleosynthesis, C represents the photon decouple. [13]

The flatness problem comes from the fact that ρa^3 or ρa^4 is a constant in the standard model, which makes $\epsilon(t) = |1 - \Omega^{-1}(t)|$ grow rapidly in the order of a or a^2. However, in the inflationary model, the energy density $\rho \sim \rho_F$ is almost a constant. In this situation, one can see from (11) that $\epsilon(t)$ would actually shrink in the order of a^{-2} during the inflation, which can be much larger than the increasing of it later in the normal expanding case. Therefore, we do not need the requirement that $\Omega(t_p)$ is extremely close to 1 any more. Moreover, as long as $\epsilon(t_p)$ is not too large, its present value would become close to zero, and thus the present value of Ω would be close to 1. This can not only avoid the fine-tuning problem in the very early universe, but also support that the universe today is spatially close to flat, although we still cannot tell it is open or closed.

On the other hand, the reheating process caused by the inflation can explain the other expression of the flatness problem, i.e. the entropy problem. After inflation, the temperature in the bubble region is reheated to close T_c again, which makes the entropy density s close to its value before the inflation. Since the volume of the universe has increased by a factor a^3, the entropy of the universe has increased $e^{3H\Delta t} \simeq 10^{129}$ times. This explains where the huge amount of the entropy in the present universe comes from.

In the monopole problem, we have showed that the number density of the magnetic monopole $n_M \simeq 1/\xi^3$ has a lower bound $n_M < 1/D_H^3$. Since the standard model does not have the supercooled state, the phase transition happens at $t_c \simeq 10^{-34}$, while the inflationary model postpones this to $t \simeq 10^{-32}$. During inflation D_H expands exponentially, and this lower bound also decreases sharply by a factor a^{-3}. We do not know the exact number of the magnetic monopoles in the early universe, and therefore we cannot say the monopole problem is solved perfectly like the problems above. But we do know that, since the lower bound of has been reduced, the universe might not need to overproduce magnetic monopoles, and the monopole problem is greatly relieved.

3.2 Improvements to the Original Model

The original inflation model has helped us find a way out of the difficulties in the standard model. However, it did not solve the problems thoroughly, (such as the monopole problem). Moreover, the original model has a fatal weakness, called the "graceful exit problem". These problems are overcame by the follow-up models of inflation.

3.2.1 The New Inflationary Model

The first improvement to the original inflation model is called the "new inflation model" [2], [3]. In Guth's original model, the bubbles were expected to collide with each other and merge together at last, in order to create the necessary radiation for the reheating process as well as to make it homogenous and isotropic. However, if the supercooled state lasted as long as what we need (10^{-32}s), they would move away from each other and never have the chance to merge. This "graceful exit problem" was avoided by interpreting the phase transition as a "slow-roll" process rather than a tunneling effect in the new inflation model. In this model, the effective potential $V(\phi)$ satisfies (a) a global minimum at $\phi \neq 0$ (true vacuum) and a very flat local maximum at $\phi = 0$ (false vacuum) when $T < T_c$; (b) a global minimum at $\phi = 0$ when $T > T_c$. Before inflation, $\langle\phi\rangle$ is at its minimum $\phi = 0$. As the universe cooled down to T_c, spontaneous symmetry breaking would happen and $\phi = 0$ would become a false vacuum. Since the vicinity of this maximum point is flat and not that unstable, $\langle\phi\rangle$ would start to roll down the potential hill slowly, and inflation would happen in this period due to the same reason as during the supercooled process in the original model. When the slope gradually becomes larger and larger, $\langle\phi\rangle$ would roll down to the true vacuum quickly which leads to the end of the inflation. This rolling behavior is described by the equation of motion $\Box\phi = -\partial V/\partial\phi$ [12]. It can be computed that in this inflation scenario, the scale of a typical bubble would expand by the factor of $e^{H\Delta t}$ and become about 10^{800}cm, which is much larger than D_{POU} [5].

Since the presently observable universe is already contained in one bubble, no bubble collision is necessary in this model. This not only can solve the graceful exit problem of the original model, but also makes the monopole problem go away. In GUTs, magnetic monopoles only show up at the place where different types of Higgs fields collide, which only happens in the process of bubble collision. Therefore, no monopole was produced in our observable universe at all. One should also notice that unlike the old inflation model, we cannot use the Higgs field as the field ϕ since the vicinity of the maximum point could not be flat enough, which would lead to a huge mass fluctuation. Therefore, the field ϕ in this model is a gauge singlet field introduced only for the purpose of driving the inflation [8].

3.2.2 Chaotic Inflation

Although the new inflation model improved the original model, it still has several problems of its own. One of the most severe one is that it requires some unnatural conditions for the slow-roll process to start. For example, the field ϕ has to be less 10^{-20} times the Planck mass. Soon after that, Linde [4] come up with a model called the "chaotic inflation model" that greatly improved the this issue. In this model there is no supercooling or phase transition, only a massive inflation field ϕ whose value is as large as the Planck mass at some of the regions in the very early universe due to quantum fluctuations. In these regions, inflation can be easily started if the effective potential $V(\phi)$ has the shape shown in figure 3, such as $V(\phi) = m^2\phi^2/2$ [14]. Since ϕ is large before inflation begins, it would take some time for ϕ to roll down to the minimum point. During this process, this field would contribute an energy momentum tensor that has a repulsive effect, which causes inflation and makes the scale factor a expand exponentially in a very short of time. The expansion of the universe would get back to the normal speed after inflation is over. Similar to the new inflation scenario, our presently observable universe is inside one of these inflation regions, and thus the problems of horizon, flatness and monopole all disappear in this model.

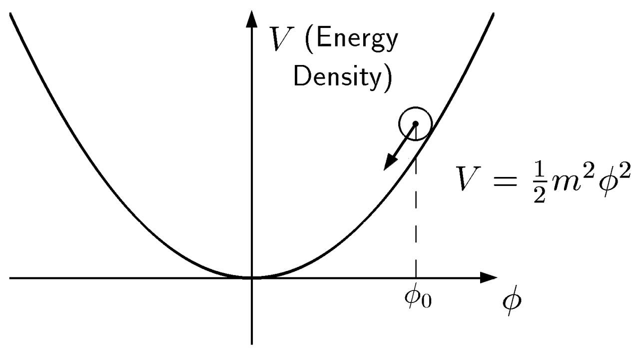

Figure 3: Effective potential in the chaotic inflationary model. [14]

4 Conclusion

We have demonstrated that the idea of inflation, a sharp exponential expansion at the very early universe, can help to improve the standard model. In the original inflation model, this process can be caused by a delayed phase transition, during which the Higgs field turns from the false vacuum into the true vacuum via tunneling effect and makes the scale factor sharply increased. After the phase transition is over, the latent heat released from the supercooled state will reheat the universe to close to T_c and the universe expands normally again. As long as our universe was in one of the inflation regions, i.e. one of the bubbles formed by the supercooled state, it is assured that the presently observable universe is always much smaller than the scale of the particle horizon, and the value of Ω would easily approach 1 after the inflation, and hence the horizon and the flatness problem are solved. However, to make inflation end properly, it can be improved by the new inflation model, in which inflation is not driven by the Higgs field but an inflation field ϕ, and the supercooled process was replaced by a slow-roll process. Since in this scenario there is no bubble collision that creates magnetic monopoles, the monopole problem is solved in this model. After this, the chaotic inflationary model with the slow-roll of ϕ from its quantum fluctuation provided us a better scenario.

When Guth first came up with the inflation model in 1981, astrophysicists did not really take prescription to it since the observed value of Ω is less than 1. However, the observation of the type Ia supernova have shown that the universe is under an accelerated expansion, caused by “dark energy”, also contributes to Ω [12]. In addition, inflation models provide the primordial perturbation spectrum of the ΛCDM model of structure formulation. All these evidence demonstrated that inflation is an essential part of the "new standard cosmological model". On the other hand, although phase transition as the prelude for the inflation story has stepped down from the stage of history, it is still a compelling idea that shows the connection of different physics from the phenomenological perspective.

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