# The Emergence of an Arrow of Time from Microscopic Principles

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**Abstract**

The arrow of time, or an asymmetry in the allowed direction of travel through time, is a fundamental property that differentiates space from time; however, its mechanism of existence is poorly understood. This work will detail explanations for the arrow of time arising from quantum/statistical mechanics and thermodynamics, identifying their pitfalls and describing experimental tests.

## Contents

## 1 Introduction

The arrow of time is a time-reversal asymmetry exhibited by physical phenomena at both the classical and quantum level. Though the laws of physics are assumed to be equivalent under the reversal of time (save for some rare weak nuclear force interactions [1]), there exist physical observables that universally distinguish between early and late times [2]. A physical example arises from classical physics: suppose one records a game of pool on a frictionless table, beginning with the break, and assume that the balls exhibit perfectly elastic collisions. When played forwards, the video displays the break, followed by a chaotic sequence of collisions between the pool balls [3]. When viewed in reverse, the balls begin scattered, exhibit a number of collisions, and finally coalesce into the triangle exhibited prior to the break. Hypothetically, this scenario is perfectly valid both forwards and backwards, as elastic collisions are time-reversal invariant. However, the conclusion of the reversed video provides a distinct marker: while it is physically possible for an apparently randomly-distributed collection of particles to create a perfectly motionless triangle at some point in time, it is exceedingly unlikely. Thus, a statistical arrow of time appears to emerge from a set of laws that is entirely time-reversal invariant [3].

The observable most commonly used to distinguish between early and late times is entropy [4]. As entropy will increase over time at large scales, entropy defines an arrow of time on classical scales. In the pool analogy described previously, the early-time behavior (in which the pool balls are motionless and in a triangle) corresponds with a low-entropy state, while the late-time behavior (in which the pool balls are moving randomly) corresponds with a high-entropy state [3]. Calculating the entropy of the two states rigorously quantifies the statistical unlikelihood of the events described in the reversed video.

However, at the quantum level, it becomes more difficult to use entropy as an indicator of the passage of time due to quantum and thermal fluctuations [2]. These inhomogeneities substantially affect the entropy in small regions, sometimes leading to the appearance of local decreases in entropy over time. Treating the local set of states as a system, this violates the second law of thermodynamics, demonstrating that entropy may not provide an adequate metric of time progression at the microscopic level [2]. Regardless of the effects of quantum and thermal fluctuations, however, it is not surprising that an arrow of time may arise at a macroscopic level: as quantum and thermal fluctuations are uncorrelated with one another and affect the overall energy distribution very little on large scales, they may "wash out." Hence, in macroscopic systems, the dominant behavior of the system is consistent with the second law of thermodynamics, allowing the emergence of a time-reversal asymmetry [4].

However, as the laws of physics are time-reversal invariant, it is remarkable that the dominant behavior at large scales is not time-reversal invariant [2]. This paradox, known as Loschchmidt's paradox, draws into question the physical mechanism of the second law of thermodynamics, and hence, the definition for the arrow of time [2]. A resolution of Loschmidt's paradox would have deep consequences on our understanding of spontaneous symmetry breaking and the emergence of broken symmetries.

Due to the presence of quantum/thermal fluctuations, Loschmidt's paradox, and several other inconsistencies, the fundamental theory describing the emergence of the arrow of time from microscopic scales is incomplete. In this work, I will discuss different phenomena that lead to the emergence of an arrow of time, along with their ability to explain time at a microscopic level. In addition, I will present experimental results related to the emergence of an arrow of time and discuss their compatibility with theoretical descriptions. An understanding of the emergence of an arrow of time would have profound consequences on our understanding of the fundamental nature of time. A fundamental differentiator between general relativity and quantum mechanics/quantum field theory lies in their description of time, so this revelation may provide insight into merging these disparate theories.

Note that, though there exists a substantial body of work that attributes the emergence of time-reversal asymmetry to currently unknown time-asymmetric processes in quantum mechanics and/or classical mechanics; however, these are not explored in this work.

## 2 Emergence from Entropy: The Boltzmann H-Theorem

Unless otherwise noted, all information in ths section was found in [2].

As entropy increases with time, entropy provides a marker for ordering events. This relationship is quantified through the Boltzmann H-Theorem.

Consider the quantity

$$H(t) = \int_0^\infty f(\mathbf{r}, \mathbf{v}, t) \ln f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r} d\mathbf{v},$$

where $E$ is the energy of a particle and $f$ is the energy distribution function at a time $t$. Using the Boltzmann equation,

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = Q,$$

and assuming that the particles exhibit uncorrelated positions and velocities, it may be shown that $\frac{dH}{dt} < 0$ for all $f$. Note that while the position and velocity assumptions have been historically controversial, they are supported by experimental results such as [5]. As $H$ may be identified with the negative of the Shannon entropy, i.e. $S(t) = -H(t)$, this indicates that entropy increases over time for all distribution functions $f$. Thus, for an arbitrary system of particles, the entropy will increase over time, providing a method for assigning temporal order in the absence of a clock by associating later times with higher entropy.

However, problems arise from the use of Boltzmann's H-theory. As argued by Loschmidt, Boltzmann's construction was created entirely from classical mechanics, which is time reversal invariant. Thus, questions are raised as to how an irreversible process may arise from timer reversal invariant laws. In addition, the theorem does not take into account quantum and thermal fluctuations, which may have great effect for small numbers of particles.

## 3 Emergence from Newtonian Mechanics

Unless otherwise noted, all information in this section was fonud in [6]

A simpler identification of an arrow of time, albeit in a more restrictive case, may be found using purely Newtonian mechanics. Consider the combination of two states of an ideal gas with probability density functions $\rho_1$ and $\rho_2$. Assuming that the velocities and positions of the particles are uncorrelated with one another, the equation of state obeyed by particles in a state $s$ is

$$\frac{\partial \rho(s)}{\partial t} = \text{gains} - \text{losses},$$

where "gains" and "losses" refer to the proportion of particles attaining or leaving state $s$, respectively. Here,

$$\begin{aligned} \text{gains} &\propto \rho(\neg s) \\ \text{losses} &\propto \rho(s). \end{aligned} \tag{1}$$

If there are more particles in state $s$ (that is, if $\rho(s)$ is large), there will be a greater number of particles that can transition to state $\neg s$; hence, the second equation holds. Similarly, if there are more particles not in state $s$ (that is, if $\rho(\neg s)$ is large), there will be a greater number of particles that can transition to state $s$; hence, the first equation holds. However, note that this is not entirely general: for example, in systems that exhibit a "snowball effect," where an accumulation in the density of one state leads to increased gains, this construction does not apply. Alternatively, a more complicated equation may related "gains" and "losses" with $\rho$. However, assuming that an increase in the concentration of one state will lead to some sort of increase in losses from that state, similar results will arise, as the system will still approach a stable equilibrium.

Define $\rho_1 \equiv \rho(s_1)$ and $\rho_2 \equiv \rho(s_2)$, where $s_1$ and $s_2$ are two separate states of an ideal gas. Making the assumptions in (1), the equation of state is

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} &= K(\rho_2 - \rho_1) \\ \frac{\partial \rho_2}{\partial t} &= K(\rho_1 - \rho_2), \end{aligned}$$

where $K$ is a constant of proportionality. Solving these equations yields

$$\begin{aligned} \rho_1(t) &= C_1 e^{-Kt} \cosh(Kt) + C_2 e^{-Kt} \sinh(Kt) \\ \rho_2(t) &= C_1 e^{-Kt} \sinh(Kt) + C_2 e^{-Kt} \cosh(Kt) \end{aligned}$$

These solutions both converge to an equilibrium density function $\rho_{eq} = \rho_1(t \gg 0) = \rho_2(t \gg 0) = C_1 + C_2$, which is defined by boundary conditions.

One may identify the flow of time with the degree of separation between the densities (where a larger separation corresponds with a system that is further in the past). In this way, an arrow of time emerges in this system without initially utilizing entropy or other thermodynamic observables. Through a more complicated equation of state, this system may be generalized to describe systems with a larger number of different states. In addition, it will approximate some systems that are not ideal.

However, it may be shown that this construction is equivalent to the emergence from entropic principles. Define the Shannon entropy as $S = -\sum_i^2 \rho_i \ln \rho_i$; then

$$\frac{dS}{dt} = K(\rho_1 - \rho_2) \ln\left(\frac{\rho_1}{\rho_2}\right).$$

Assuming that $K > 0$ and $\rho_1 > \rho_2$, $\frac{\partial S}{\partial t} \geq 0$. Hence, the entropy will increase monotonically over time. This indicates that the construction will suffer the same flaws as Boltzmann's H-theorem (the construction does not take into account quantum/thermal fluctuations and fails to explain Loschmidt's paradox). However, it does explain why entropy increases over time, providing an alternate viewpoint for the source of an arrow of time.

It would be of interest to repeat this analysis using an equation of state that does not assume that the rate of change of $\rho(s)$ is proportional to the difference between gains and losses.

## 4 Measurement of Entropy

Unless otherwise noted, all information in this section came from [4]

The previous methods for assigning an arrow of time at the quantum level require the measurement of the entropy or local density of individual particles. As quantum and thermal fluctuations are large at microscopic scales, it is important to take them into account when measuring these quantities. In this work, the authors perform measurements of the entropy of individual particles in non-equilibrium systems. It has previously been demonstrated [7] that the Kullback-Leibler divergence acts as a measure of entropy for non-equilibrium systems; this divergence is defined as [8]

$$D_{KL}(P||Q) = \sum_i^n P(i) \ln \left( \frac{P(i)}{Q(i)} \right).$$

The Kullback-Leibler divergence, which is always non-negative, effectively measures the difference in amount of information between the probability distributions $P$ and $Q$. It may also be thought of as being the information "distance" between the states $p$ and $q$; however, note that it may not be formally interpreted as a distance because the expression for $D_{KL}(P||Q)$ is not symmetric under exchange of $P$ and $Q$. The entropy difference between states $\rho_t^F$ and $\rho_{\tau-t}^B$ is defined as

$$\langle S \rangle = \mathrm{tr} \left[ \rho_t^F \left( \ln \rho_t^F - \ln \rho_{\tau-t}^B \right) \right]. \quad (2)$$

The authors aim to experimentally verify this result by evaluating both sides of Eqn. (2) independently. As the right-hand side of Eqn. (2) is the Kullback-Leibler divergence between a state and its time-reversed counterpart, a successful measurement will experimentally verify the emergence of an arrow of time from thermodynamics due to spontaneous symmetry breaking from a "preferred" state progression direction. To do so, a qubit was constructed from a $^{13}$C nucleus in a chloroform liquid bath. A radio-frequency transverse magnetic field and longitudinal field-gradient pulses were applied to the qubit; a diagram of this setup may be seen in Figure 1. The pulses had a frequency $k_BT/h = 1.56 \pm 0.07$ kHz, corresponding to a temperature of $T = 75 \pm 3$ nK, which was the $^{13}$C nucleus temperature. The system was driven from equilibrium using a fast quench of its Hamiltonian $\hat{H}_t^F$. A quantum quench occurs when the Hamiltonian is changed suddenly, while "fast" indicates that

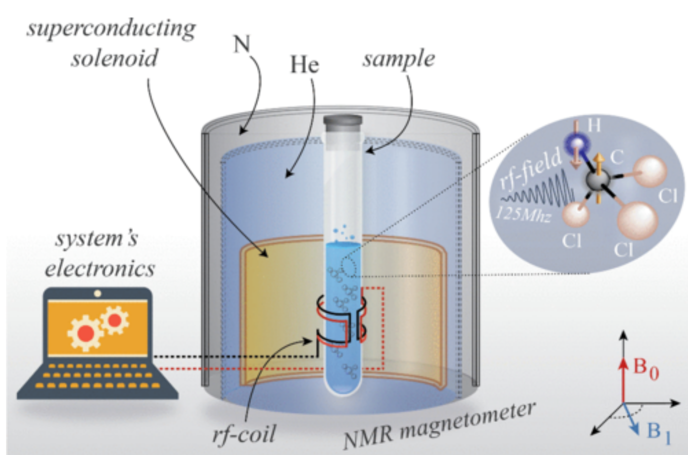

**Figure 1:** The experimental setup used to measure the entropy of individual particles. After performing a fast quench using the magnetic field, the entropies of the particles in the chloroform bath were measured [4].

the alteration to the Hamiltonian was time-limited. In this case, the fast quench altered the unperturbed Hamiltonian by performing the transformation $\hat{H} \rightarrow \hat{H} + \hat{H}_t^F$, where $\hat{H}_t^F = 2\pi\hbar\nu(t)\left[\sigma_x^C \cos\phi(t) + \sigma_y^C \sin\phi(t)\right]$. Here, $\sigma$ are the Pauli spin operators, $\phi = \frac{\pi t}{2\pi}$, and $\nu(t) = \nu_0\left(1 - \frac{t}{\tau}\right) + \upsilon_\tau \frac{t}{\tau}$, the linear modulation over the radio-frequency field during a time $\tau$. The fast quench lasted for a time $\tau = 10^{-4}$ s. In addition, $\nu_0 = 1.0$ kHz and $\nu_\tau = 1.8$ kHz. The fast quench will cause the spins of the nuclei to rotate, which requires work and alters the system's entropy.

To determine the work and entropy production of the $^{13}$C nucleus due to the fast quenching of the system, the authors examined a $^{1}$H atom in the chloroform bath. As $^{1}$H couples to $^{13}$C, a natural interferometer was created, allowing the statistical analysis of processes of time duration less than the characteristic time of the spin lattice, which was found to be much greater than $10^{-4}$ s. This provides justification for choosing $\tau = 10^{-4}$ s.

Performing these steps will provide information on the work and entropy production of the time-forward Hamiltonian. To measure the time-reversed properties, the same steps were performed using a time-reversed Hamiltonian $\hat{H}_{\tau-t}^F$, beginning from the final state of the time-forward process and ending at the initial state.

To measure the right-hand side of Eqn. (2), the spin state of the $^{13}$C nucleus was tracked as a function of time $t$ during the fast quench using Quantum-State Tomography (QST) techniques. This will allow the reconstruction of the work performed by and entropy of the $^{13}$C nucleus as a function of time. These measurements were performed for the time-forward and time-reversed Hamiltonians. To measure the left-hand side of Eqn. (2), the Tasaki-Crooks fluctuation relation [9] was used:

$$\frac{P^F(W)}{P^B(-W)} = e^{\beta(W-\Delta F)},$$

where $P^F$ and $P^B$ are the probability distributions of the forward and backward states, respectively, $W$ is the work done by the system, and $\Delta F$ characterizes the magnitude of quantum and thermal fluctuations. The Tasaki-Cooks fluctuation relation is a generalization of the second law of thermodynamics that takes into account quantum fluctuations; on average, it implies that $\langle S \rangle = \beta\left(\langle W \rangle - \Delta F\right) \geq 0$. The probability distribution $P(S)$ for the irreversible work performed was measured using Nuclear Magnetic Resonance (NMR) spectroscopy, extracting $\beta$, $W$, and $\Delta F$, allowing the reconstruction of $\langle S \rangle$. A plot of the resulting probability distribution is displayed in Figure 1.

The mean entropy as measured by the process is displayed as a red line in Figure 2(a). Here, it can be seen to be non-negative, demonstrating that the second law of thermodynamics holds for quantum systems on average (while individual measurements may have exhibited a negative change in entropy over time, the average change was net positive, indicating that the negative results are likely due to the probabilistic nature of the measurements).

The mean entropies of the forward and backward states are displayed in Figure 2. The two sides of Eqn. (2), represented by the dashed lines (left-hand side) and dotted lines (right-hand side), are in agreement within the bounds of experimental error, indicating that Eqn. (2) is supported. This directly verifies the existence of a thermodynamic arrow of time at the quantum level by indicating that, even in microscopic systems, entropy increases in time on average. Therefore, for quantum systems that are driven and uncorrelated/weakly-

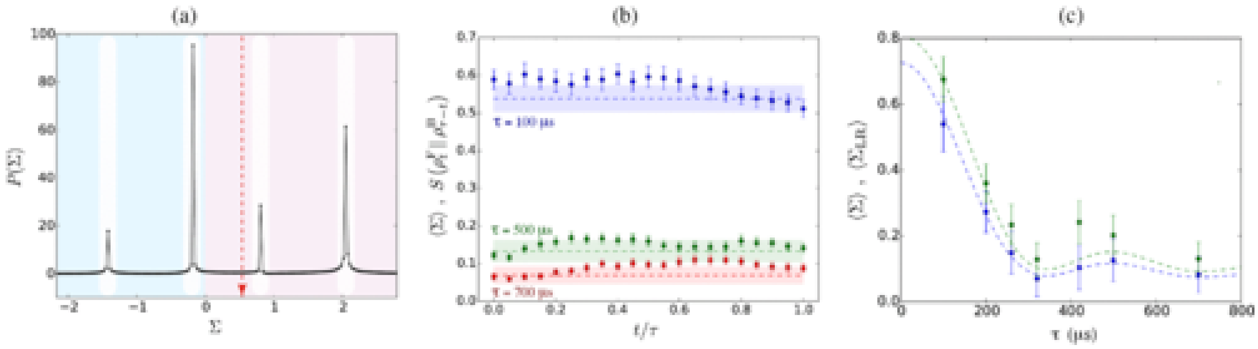


**Figure 2:** Plots of the data collected in [4]. (a) shows the probability density function of the system for the net entropy change; note that the number of samples taken was not reported. The red dashed line indicates the average entropy change, which is non-negative, indicating that the second law of thermodynamics was upheld on average. (b) shows the average entropy production (dashed lines) and Kullback-Leibler divergences (dots) between the forward and backward time states; the colors represent different quench durations. The blue, green, and red markers represent $\tau = 100\,\mu$s, $\tau = 500\,\mu$s, and $\tau = 700\,\mu$s, respectively. Most Kullback-Leibler divergence data points were within the error bounds of the average entropy production, strongly indicating the existence of an arrow of time.

correlated, an entropic arrow of time does emerge at the microscopic level. However, note that this is not general: in many everyday systems, particles exhibit non-negligible correlations, so this does not construct a generally applicable arrow of time.

## 5 Emergence from Resource Theory

Unless otherwise stated, all information from this section was obtained from [1].

An alternative formulation of the arrow of time that sidesteps many of the issues of classical approaches may be derived from information theory. The entropic arrow of time at the macroscopic level arises due to the second law of thermodynamics, which states that entropy increases with time. Because entropy increases due to irreversible processes, an alternative viewpoint states that the arrow of time may be constructed by noting the start and endpoints of irreversible processes. As measurements demonstrating irreversibility of an interaction are easier to perform at the quantum level than those of entropy differences of individual particles, this may provide a mechanism to quantify a universal arrow of time at microscopic scales. In this paper, the authors demonstrate the emergence of an arrow of time from resource theoretical techniques.

In resource theory, some states and operators are designated as “free.” Free operators acting on a free state, a process called a “free transformation,” may only map that state into another free state, and does so by performing no work. In this way, free transformations act as analogues to (or, in some cases, are identical to) reversible processes. Using free transformation, a thermodynamic order may be constructed: if a state $\rho$ may transform to a state $\sigma$ using only free transformations, then $\rho$ occurs before $\sigma$ in thermodynamic order. In addition, it may be proven that an equilibrium state $A$ may only transform to an equilibrium state $B$ if $B$ has a lower free energy than $A$. As states typically evolve toward a state with lower free energy over time, it is natural to use these two aspects to assign a thermodynamic arrow of time that corresponds with the thermodynamic order: a state $\rho$ occurs temporally before a state $\sigma$ if $\rho$ may be transformed into $\sigma$ using only free transformations.

However, problems arise when neither $\rho$ nor $\sigma$ may be transformed into the other using free transformations, yet this transition is predicted by other theories or observed experimentally. This situation occurs most commonly with incoherent states. It has been proven that, for incoherent states $\rho$ and $\sigma$, a free transformation exists from $\rho$ to $\sigma$ if the Rènyi $\alpha$-divergences between $\rho$ and thermal equilibrium is less than the Rènyi $\alpha$-divergences between $\sigma$ and thermal equilibrium for all $\alpha$. The Rènyi $\alpha$-divergences between states $p$ and $q$, $D_\alpha(P||Q)$, are defined as [10]

$$D_\alpha(P||Q) = \frac{1}{\alpha - 1} \ln \left( \sum_i^n \frac{P(i)^\alpha}{Q(i)^{\alpha-1}} \right),$$

where $P(i)$ $(Q(i))$ represents the discrete probability density functions of the state $p$ $(q)$; for a continuous distribution, the summation becomes an integral over all values of of $P$ $(Q)$. The Rènyi $\alpha$-divergences are a generalization of the Kullback-Leibler divergence because $D_{KL}(P||Q) = \lim_{\alpha \to 1} D_\alpha(P||Q)$ [10]. Observing the similarities between the expression for the Rènyi $\alpha$-divergences and for the Kullback-Leibler divergence, the Rènyi $\alpha$-divergences may be interpreted as alternative methods of weighting the Kullback-Leibler divergence.

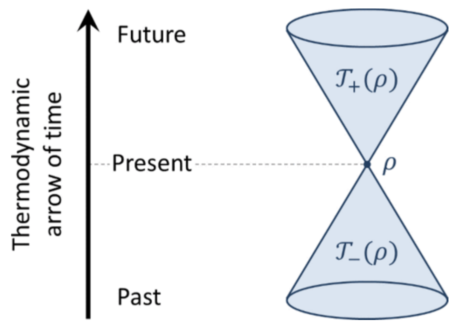


**Figure 3:** An example of a thermal cone diagram for a state $\rho$. $\mathcal{T}_+(\rho)$ represents the future thermal cone, while $\mathcal{T}_-(\rho)$ is the past thermal cone. Note the similarity to light cones from special relativity [1].

Using a lattice, the authors demonstrated the existence of partial ordering of thermodynamic states. It was assumed that the free transformations consisted of Gibbs-preserving operations, or operations that leave the thermal equilibrium state unchanged. More formally, a map $\mathcal{E}$ is a Gibbs-preserving operation if, for a Gibbs state $\gamma$, $\mathcal{E}(\gamma) = \gamma$. Similarly, a matrix operator $\Lambda$ is Gibbs-preserving if $\Lambda\gamma = \gamma$.

The thermal ordering of a set of states is defined by its thermal cone, named in analogy to a light cone in special relativity. A state $\sigma$ is said to be in the future thermal cone $\mathcal{T}_+$ of a state $\rho$ if $\rho$ may be evolved into $\sigma$ using only Gibbs-preserving maps. Similarly, if a state $\sigma$ may be evolved into $\rho$ using only Gibbs-preserving maps, $\sigma$ is in the past thermal cone $\mathcal{T}_-$ of $\rho$. A thermal cone diagram is displayed in Figure 3. In analogy to the light cone in special relativity, events outside the thermal cone cannot be reached using only Gibbs-preserving operations.

Consider two events, $\rho$ and $\sigma$. Let the state $\mathcal{T}_-(\rho, \sigma)$ consist of the set of all light cones that contain both $\rho$ and $\sigma$ in the past. Similarly, let $\mathcal{T}_+(\rho, \sigma)$ consist of all light cones that contain both $\rho$ and $\sigma$ in the future. Physical, $\mathcal{T}_-(\rho, \sigma)$ is the set of all possible past states consistent with the current forms of $\rho$ and $\sigma$, while $\mathcal{T}_+(\rho, \sigma)$ is the set of all possible futures consistent with the current forms of $\rho$ and $\sigma$. For any $\mathcal{T}_-(\rho, \sigma)$, there is a unique $\tau_-$ such that for every event $\tau$ in the past light cone $\mathcal{T}_-(\rho, \sigma)$, $\tau$ precedes $\tau_-$. Thus, $\tau_-$, the "join" of $\rho$ and $\sigma$, represents the final state in the overlap between the past thermal light cones of $\rho$ and $\sigma$ that may evolve into either $\rho$ or $\sigma$; that is, it is their last shared past state. Similarly, there is a unique $\tau_+$ in $\mathcal{T}_+(\rho, \sigma)$ such that for every $\tau$ in $\mathcal{T}_+(\rho, \sigma)$, $\tau_+$ precedes $\tau$. $\tau_+$, the "meet" of $\rho$ and $\sigma$, represents the earliest future state that may be evolved into by both $\rho$

and $\tau$. A visualization of the join and meet of a set may be seen in Figure 4.

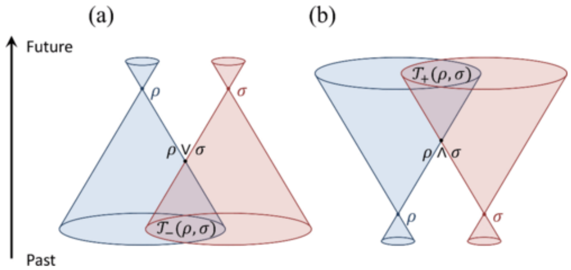


**Figure 4:** A visualization of a join (a) and a meet (b). The join is the last place of intersection in the past thermal cones of two states, while the meet is the first place of intersection between the future light cones of two states [1].

Using these definitions, a thermodynamic lattice may be defined. For a set of thermodynamically-ordered states $S_d$, if for each pair of states $(\rho, \sigma)$ there exists a join and a meet, $S_d$ is a thermodynamic lattice, where $\rho, \sigma \in S_d$. This may be interpreted as follows: suppose an initial state $\alpha$ evolves into a final state $\beta$, both of which are on the thermodynamic lattice $S_d$. For any other state $\gamma$ in the lattice, there is a thermodynamic "path" between $\alpha$ and $\beta$ (a sequence of thermodynamically-ordered Gibbs-preserving transformations that begins at $\alpha$ and ends at $\beta$) that "passes through" $\gamma$. Note, however, that for a partially ordered state, it may not be possible for each state to evolve into any other. For example, it may be possible to construct thermodynamic paths $\alpha \to \gamma \to \beta$ and $\alpha \to \delta \to \beta$, but it is not possible to construct both of the paths $\alpha \to \gamma \to \delta \to \beta$ or $\alpha \to \delta \to \gamma \to \beta$.

In this paper, the authors created conditions for the existence of partially-ordered thermodynamic lattices. In addition, they elucidated the relationship between the arrow of time and partially-ordered thermodynamic lattices. For any system, a partially-ordered thermodynamic lattice may be created at infinite temperature. For two-state systems, a thermodynamic lattice may also be created at finite temperatures; however, for systems with more than two states, the symmetry allowing the creation of the lattice is broken.

A thermal lattice provides substantial amounts of information on the arrow of time: the existence of a thermal lattice describing a system implies that for any pair of states in that system, there exists a unique state in the past consistent with those states and any other state in the past. Similarly, it asserts the existence of a unique state in the future that is consistent with both of those states and any other future states. Thus, a thermal lattice defines a thermodynamic clock that maps exactly to the arrow of time by allowing one to rigorously determine the thermodynamic order of each state in the system. Unless there are partially ordered processes, the thermodynmaic order will correspond exactly with the temporal order. The fact that a thermodynamic lattice cannot be created in three dimensions at finite temperature implies that it is not possible to create an exact arrow of time from a thermodynamic lattice, as there will always be some ambiguity in the thermodynamic order of states. Note that this only precludes the emergence of an exact arrow of time from thermodynamic lattices; other thermal constructions may be able to accomplish this goal.

Like the models discussed previously, this construction fails to account for quantum and thermal fluctuations. These local disturbances can destroy the thermodynamic lattice by altering the thermodynamic order through performing spontaneous non-Gibbs-preserving transformations. In addition, the existence of partially-ordered processes implies that even if a lattice may be constructed, it may not be possible to temporally order the states.

However, even though an arrow of time that lacks any ambiguity cannot be created from a thermodynamic lattice, this does not preclude its emergence at large scales. Macroscopic systems contain an enormous number of states, so inaccuracies on the temporal ordering of individual processes will not affect the large-scale arrow of time substantially. In this way, an effective arrow of time emerges for macroscopic systems. In addition, as the temperature of the system increases, the thermodynamic order approaches that of a thermal lattice, so for the high temperatures and large phase spaces observed in everyday life, this construction does appear to explain the emergence of an arrow of time. In addition, if the number of partially-ordered processes is small, then overall, the thermodynamic order will correspond with temporal ordering, again allowing the emergence of a statistical arrow of time at macroscopic scales.

## 6 Reversing the Arrow of Time with Quantum Correlation

Except where otherwise noted, all information in this section was found in [11].

Thus far, all models detailing the emergence of an arrow of time from thermodynamics fail to assign temporal succession due to the possibility of quantum and thermal fluctuations. As all of these methods map entropy to temporal succession, a quantum or thermal fluctuation may cause local, time-limited disruptions in entropy. This may lead to a local decrease in entropy over time. While this does not violate the second law of thermodynamics (as the second law only applies to large-scale observations), fluctuations would create problems when assigning an arrow of time to quantum processes, as they would create the appearance of temporary time reversal. However, other effects may lead to the appearance of time reversal at the quantum level, raising questions about the fundamental nature of time in microscopic systems.

In this paper, the authors demonstrate that the arrow of time may be reversed at the quantum level using initially correlated systems, resulting in heat flowing from cold to hot. The authors began with two quantum correlated qubits, each of spin 1/2, created using $^1$H and $^{13}$C nuclei drawn from a $CHCl_3$ liquid that has been diluted in Acetone-d6. The qubits initially have different temperatures, having been created in two different environments using NMR techniques. The system is placed in a constant magnetic field and manipulated using radio-frequency fields in order to study processes that take substantially less time than the decoherence time.

Using the modulated magnetic field, an initial state is created of the form

$$\rho_{AB}^0 = \rho_A^0 \otimes \rho_B^0 + \chi_{AB},$$

where state $A$ is the hydrogen nucleus, state B is the carbon nucleus, $\chi_{AB} = \alpha \left|01\right\rangle \left\langle 10\right| + \alpha^* \left|10\right\rangle \left\langle 01\right|$ is a correlation term, and $\rho_i = \frac{e^{-\beta_i T_i}}{\mathcal{Z}}$. $\left|0\right\rangle$ represents the ground state of the Hamiltonian $\hat{H}$, while $\left|1\right\rangle$ represents the excited state. When $\alpha = 0$, the systems are initially uncorrelated; however, when $\alpha > 0$, the systems exhibit an initial correlation.

Allowing heat to be pass from the hot qubit to the cold qubit will not require any work; thus, the change in heat may be found by measuring the change in internal energy of the two particles $\Delta E_i$. This interaction will follow

$$\frac{Q_A}{k_B T_A} + \frac{Q_B}{k_B T_B} \geq I(A:B),$$

where $I(A:B)$ is the change in mutual information between the particles. $I(A:B)$ is defined by

$$I(A:B) = S_A + S_B - S_{AB} \geq 0,$$

where $S_i$ is the von Neumann entropy of the state $i$ and $S_{AB}$ is the entropy due to their correlation. For an initially uncorrelated system with $T_A > T_B$, the initial shared information is 0; thus, the shared information can only increase. This implies that $Q_A + Q_B = 0$, so

$$\frac{Q_B}{k_B}\left(\frac{1}{T_B} - \frac{1}{T_A}\right) \geq 0,$$

indicating that heat flows from the hot qubit ($A$) to the cold qubit ($B$), as would be expected from classical thermodynamics. However, if the qubits are initially correlated, it may be that

$$\frac{Q_B}{k_B}\left(\frac{1}{T_B} - \frac{1}{T_A}\right) \leq 0$$

because the mutual information may decrease during contact. This would imply that heat could flow from the cold qubit ($B$) to the hot qubit ($A$), decreasing the entropy of the system and indicating that the flow of time has spontaneously reversed.

To test these scenarios, the setup in Figure 5 was created, with $k_BT$ on the scale of peV to ensure that any interactions require substantially less time than the decoherence time. To measure the initial "quantumness" of the system (that is, to measure $\alpha$), the authors measured the normalized geometric discord $D_g = \min_{\psi\in\mathcal{C}} 2||\rho - \psi||^2$, where $\mathcal{C}$ is the set of all classically correlated states $\psi$. The normalized geometric discord is the minimum Bures distance between the particles, where the Bures distance measures the difference in amount of statistical information carried by the two particles. The geometric quantum discord may be measured directly using QST techniques.

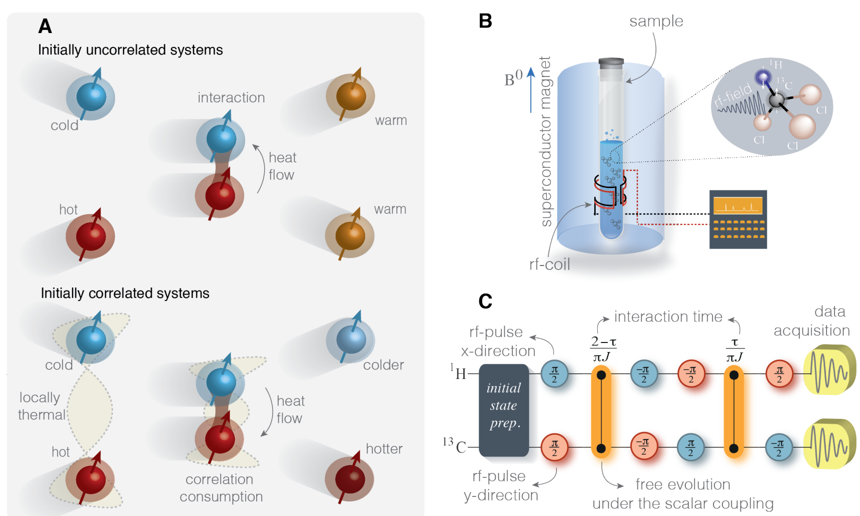


**Figure 5:** The experimental setup for [11]. In A, the upper diagram represents $\alpha = 0$, while the lower shows $\alpha > 0$. B shows a diagram of the NMR setup, and C shows the electronic components used to perform the measurements.

First, the situation in which $\alpha = 0$ was tested. In this state, the normalized geometric discord was statistically indistinguishable from 0, indicating that there was no statistically significant initial correlation. The authors measured the internal energy, mutual information, and geometric quantum discord for each qubit (plots are displayed in Figure 6), finding that heat flowed from the hot qubit ($A$) to the cool qubit ($B$). In this case, the second law of thermodynamics was obeyed, as entropy increased over time. This provided a control for when $\alpha > 0$.

Next, a non-zero $\alpha$ was tested. Measurements of the geometric discord of the system yielded a non-negative, non-zero value for the initial state, corresponding with $\alpha > 0$. Performing the same measurements as before, it was observed that heat flowed from the cold qubit ($B$) to the hot qubit ($A$); these results may be seen in Figure 6. As the arrow of time corresponds with increasing entropy, this implies that the arrow of time was reversed at a quantum level. This demonstrates that the arrow of time cannot be produced entropically at the microscopic level, drawing into question the effectiveness of mapping the increase in entropy to the march of time for quantum correlated systems. This does not directly contradict the results of the previous paper: these results solely apply to intially correlated systems, while the previous paper only examined driven, uncorrelated quantum systems. However, as there does appear to be at least one quantum phenomenon in which an increase in entropy cannot be mapped to the flow of time, this indicates that a thermodynamic arrow of time cannot be generally applied to quantum systems [11].

However, note that this does appear to produce an arrow of time at large scales. As may be seen in Figure 6, the reversal in the direction of time was only temporary; after enough time, the net flow of was from hot to cold in both correlated and uncorrelated quantum systems. As the scales of import when considering the second law of thermodynamics are much longer than the duration of time reversal ($\sim 1$ ms), the effects of time reversal due to quantum correlation are expected "wash out" in classical settings.

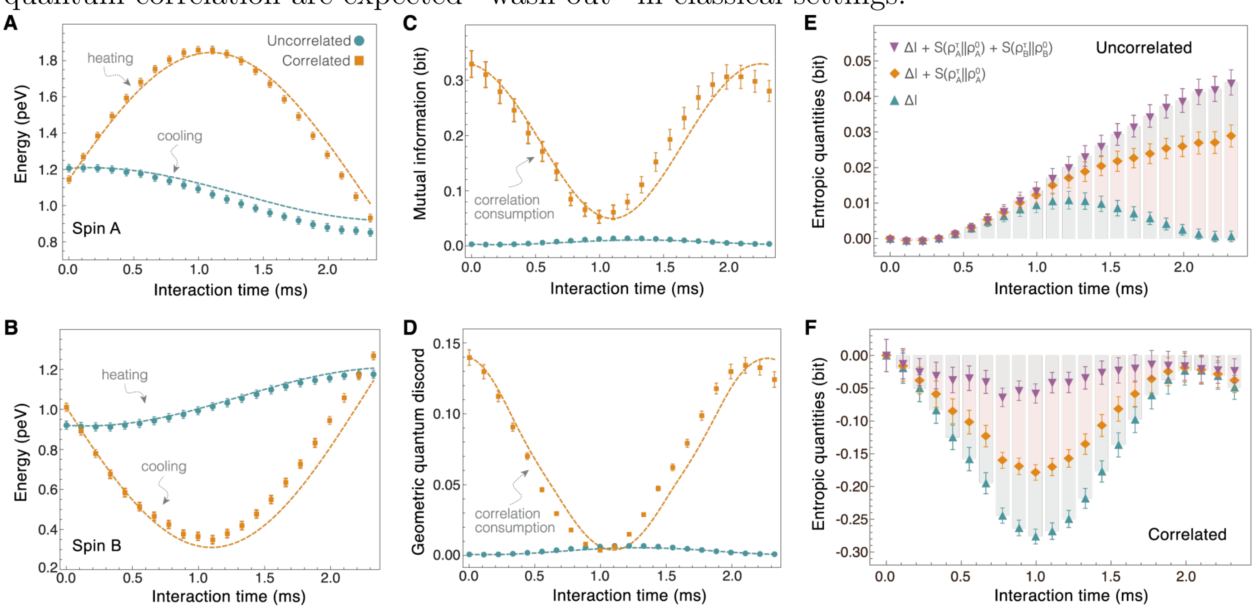


**Figure 6:** The data for [11]. A and B shows the time evolution of the energies of qubit $A$ and $B$, respectively. For uncorrelated system, heat travelled from the hot qubit to the cold qubit; however, for the correlated system, energy initially flowed from cold to hot as the correlation between the states broke down. This description corresponds well with C and D, which show the mutual information and relative quantum discord of the two qubits: heat flowed from cold to hot while the mutual information and quantum discord were low, then reversed when these quantities increased. E and F show various contributions to the entropy in the system.

## 7 Conclusion

Though it has been hypothesized that the arrow of time arises from classical thermodynamics (namely, entropy) [2] [4], classical mechanics [6], and resource theory [1], various barriers, including those discussed in [11], prevent its generalization to the microscopic level. Quantum and thermal fluctuations cause time to appear to flow backward in quantum systems for short periods of time [2], and though these effects may be accounted for [4], additional effects, such as quantum correlation[11], may fundamentally preclude its applicability for microscopic systems. As these constructions demonstrate that time only emerges at large scales as a result of spontaneous symmetry breaking at the microscopic level, this draws into question the fundamentality of time. Also, if our basic understanding of time cannot be applied to quantum systems, there must be an alternative construction that allows this extension because the Schrödinger equation is time-dependent.

Additional research should be performed to construct a mechanism that produces time at both the quantum level and classical level. Seeing that a thermodynamic approach may not account for certain quantum effects, the emergence of an arrow of time from other theories is an active area of research; for example, some authors have demonstrated the existence of a time-reversal asymmetry arising from cosmological principles [12]. In addition, experiments can be used to test the limits of a thermodynamic arrow of time. As discussed previously, quantum correlated systems allow the reversal of time at microscopic levels, so it would be of benefit to determine whether other quantum effects lead to similar situations. Answering these questions may provide the information needed to explain the existence of time at the macroscopic and microscopic scales, potentially yielding great insight into a theory of quantum gravity and the fundamental properties of the universe itself.


[1] K. Korzekwa. Structure of the thermodynamic arrow of time in classical and quantum theories. , 95(5):052318, May 2017.

[2] C. Y. Chen. Boltzmann's H-theorem and time irreversibility. *ArXiv Physics e-prints*, November 2003.

[3] J. Dressel, A. Chantasri, A. N. Jordan, and A. N. Korotkov. Arrow of Time for Continuous Quantum Measurement. *Physical Review Letters*, 119(22):220507, December 2017.

[4] T. B. Batalhão, A. M. Souza, R. S. Sarthour, I. S. Oliveira, M. Paternostro, E. Lutz, and R. M. Serra. Irreversibility and the Arrow of Time in a Quenched Quantum System. *Physical Review Letters*, 115(19):190601, November 2015.

[5] J.S. Waugh, W.-K. Rhim, and A. Pines. Spin echoes and loschmidt's paradox. *Journal Pure Applied Chemistry*, 32:317–324, 1972.

[6] K. Rèbilas. Origin of the thermodynamic time arrow demonstrated in a realistic statistical system. *American Journal of Physics*, 80:700–707, August 2012.

[7] S. Vaikuntanathan and C. Jarzynski. Dissipation and lag in irreversible processes. *EPL (Europhysics Letters)*, 87:60005, September 2009.

[8] S. Kullback and R. A. Leibler. On information and sufficiency. *Ann. Math. Statist.*, 22(1):79–86, 03 1951.

[9] H. Tasaki. Jarzynski Relations for Quantum Systems and Some Applications. *eprint arXiv:cond-mat/0009244*, September 2000.

[10] Alfréd Rényi. On measures of entropy and information. In *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Contributions to the Theory of Statistics*, pages 547–561, Berkeley, Calif., 1961. University of California Press.

[11] K. Micadei, J. P. S. Peterson, A. M. Souza, R. S. Sarthour, I. S. Oliveira, G. T. Landi, T. B. Batalhão, R. M. Serra, and E. Lutz. Reversing the thermodynamic arrow of time using quantum correlations. *ArXiv e-prints*, November 2017.

[12] M. Castagnino, L. Lara, and O. Lombardi. The cosmological origin of time asymmetry. *Classical and Quantum Gravity*, 20:369–391, January 2003.