PHYS 569

Emergent States of Matter

Term Essay "Spin Glasses: Emergent Behavior of Systems with Quenched Disorder and Frustration."

Abstract

In this essay, we will consider the interplay of quenched disorder with frustration and how it gives rise to emergent behavior. The field was experimentally motivated by an observed phase transition and properties of structural and spin glass phases. The latter provides a natural setup for our discussion and we will focus on Edwards-Anderson and Sherrington-Kirkpatrick models originally suggested to describe spin glasses. In the process, we will get acquainted with replica theory, the concept of replica symmetry breaking and discuss its physical meaning.

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1 Introduction

In order to introduce the subject of spin glasses we will first define what we mean by spin glass, discuss its crucial features and pose several important questions that we will focus on.

Spin glass is a an ordered state of magnetic system with frustration due to quenched disorder.

Remark: The term spin glass is used both to mean the ordered phase of a system as well as a material.

Sometimes a picture is worth a thousand words, which is why before we dive into detailed description, we give a pictorial comparison of a square lattice ferromagnet with spin glass on Fig. 1.

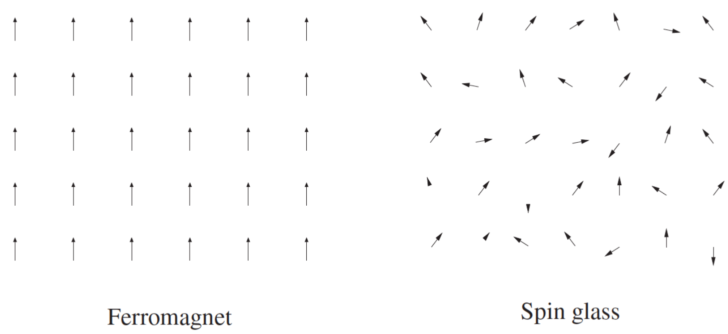


Figure 1: Spin configuration of ferromagnetic and spin glass phases.

Now lets unpack the definition. By quenched we mean disorder that is frozen and does not change on the time-scale of observation. Now imagine a random system of magnetic moments. Depending on the sign of interaction between two magnetic moments they try to align or anti-align. It is not hard to imagine that in a random system of magnetic moments many magnetic moments will be in conflict with each other. This is what we call frustration.

It is precisely the interplay between quenched disorder and frustration that causes the unusual behaviour of spin glasses, but also creates great challenges for analytical treatment. Note that frustration does not imply quenched disorder and visa versa. For instance, in a pure lattice with basis anti-ferromagnetic frustration is easily attained in cycles with odd number of members. Likewise, quenched disorder may be present without frustration in systems with ferromagnetic interaction.

Naturally, the high temperature state of spin glasses is characterized by thermal fluctuations with no magnetic order. However, as the temperature is lowered such magnetic systems undergo a transition into an ordered state, with:

$$\langle s_i \rangle_t \neq 0 \tag{1}$$

where the average is taken over times much longer than some microscopic scale. One could say that spin freezing takes place. However, due to disorder, in thermodynamic limit $N \to \infty$:

$$\frac{1}{N}\sum_i \langle s_i \rangle_t \, e^{ik\cdot r_i} = 0, \quad \forall k \tag{2}$$

Clearly some kind of ordering happens at low temperature, but its nature is hard to describe due to quenched disorder. This brings us to an important question. **Q:** *What is an order parameter characterizing the spin glass phase?* Furthermore, in the LGW paradigm we always associate a broken symmetry with a phase transition. This poses another question. **Q:** *What symmetry gets broken in the spin glass transition?*

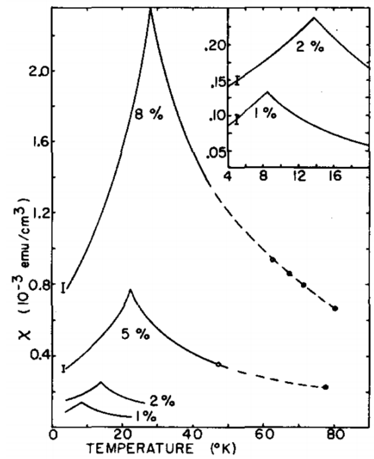
Before we start answering these questions, we will make a brief historic digression and describe how this field was motivated experimentally. This will not only provide us with physically relevant context, but also will unveil some unexpected features of spin glasses that we will try to explain in later sections.

2 Dilute Magnetic Alloys

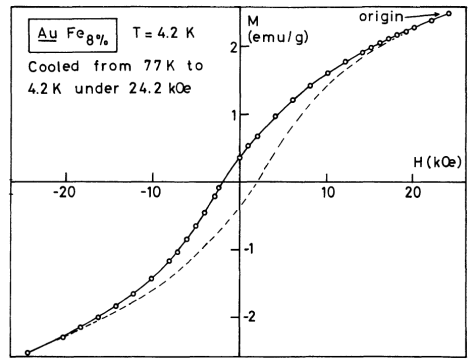
Materials that we now call spin glasses were first produced by introducing a small amount (up to several percent) of magnetic impurities into a metallic non-magnetic host. Typically the latter was a noble metal, such as gold, silver or copper. As a magnetic impurity iron or manganese were usually used. As early as the 1930's it was already know that these materials possess some unusual properties, such as the behaviour of electric conductivity at low temperatures. In the 1960's Kondo formulated a model [1] to explain a minimum in electrical conductivity as a function of temperature in very dilute magnetic alloys. This model assumes that each magnetic impurity does not feel the presence of any other. This gave birth to the Kondo problem, which played a prominent role in 20th century condensed matter physics and was only resolved with formulation of renormalization group.

Setting aside highly diluted alloys, another productive direction turned out to be the study of properties of alloys as a function of concentration of magnetic impurity. The idea was to bring the concentration high enough, so that the interaction between the impurities is not negligible. And indeed it was observed, for instance in gold-iron alloys by Cannella *et al.* [2] (Fig. 2a) at a concentration of iron of several percent, that magnetic susceptibility as a function of temperature acquires a peak. This peak was a clear evidence of magnetic ordering from a paramagnet to what we now call a spin glass.

Many other remarkable experimental observations were made on spin glasses in the 1960's to 1980's. For instance, if a spin glass material is cooled in a non-zero magnetic field, then even after removal of the field there is a *remanent magnetization* [4]. The latter relaxes away over time, but very slowly. A related feature of spin glasses is magnetic hysteresis (Fig. 2b)[3], which is a history dependent response of magnetization to external magnetic field. Both of these properties of spin glasses display dependence on history, which is characteristic of systems that are far from equilibrium. Another fascinating property of spin glasses is



(a) Magnetic susceptibility of Au-Fe alloys as a function of temperature for several Fe-concentrations: c=1%, 2%, 5%, and 8% [2].



(b) Hysteresis cycle of Au-Fe alloy with Fe-concentration c=8% at 4.2 K [3].

Figure 2

revealed in aging experiments [5], in which one quenches the system in a magnetic field below the spin glass transition temperature. The field is not switched off right away, but rather after a waiting time t_w. Remarkably, after the field is turned off, the rate of decay is dependent on the waiting time t_w.

In this essay we will not try to qualitatively describe these complicated memory effects. However, we can fairly easily get an appreciation and qualitative picture for them. Imagine a multidimensional space with each spin orientation being a parameter and an energy surface that represents a particular configuration of disorder. Due to the random nature of the latter, such a surface will be highly irregular (Fig. 3), with many troughs and crests. Then if one imagines a system performing a random walk on this *rugged landscape*, it is easy to see that the system will constantly get stuck in local minima, until a large enough thermal fluctuation will free it. This demonstrates a high level of metastability inherent to spin glasses. It also reveals the connection of spin glasses to broader complexity science and topics such as protein folding.

Before we proceed to the issue of how we can build a statistical model that describes spin glasses, lets take a closer look at the interaction that produces spin glass behaviour. Although all spin glasses are magnetic by definition, not all them are magnetic alloys. Many spin glasses are insulators with magnetic impurities. So what is the unifying feature of the interaction in spin glasses? It turns out that direct dipole-dipole interaction between magnetic moments is too weak to produce the observed behavior. Rather then interacting directly, it is mediated by electron scattering, which leads to the RKKY exchange interaction.

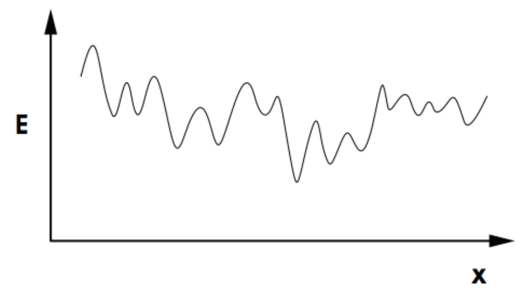


Figure 3: Sketch of a rugged landscape.

The latter oscillates with distance:

$$J(r) = J_0 \frac{\text{Cos}[2k_F r + \phi_0]}{(k_F r)^3} \tag{3}$$

where J_0, ϕ_0 are constants and k_F is the Fermi momentum of the host material. It is precisely this oscillation that results in ferromagnetic and anti-ferromagnetic interaction being equally likely between a pair of magnetic impurities. And since the impurities do not migrate on the timescale of experiment, this results in frozen-in disorder.

3 Averages and Replicas

Given the set of behaviours that we described in the previous section, one is faced with a question: **Q:** *What is the simplest model that incorporates universal features of spin glasses?* Such a model was suggested by Edwards and Anderson in 1975 [6]:

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} \vec{s}_i \cdot \vec{s}_j - \vec{h} \cdot \sum_i s_i \tag{4}$$

where $\langle i, j \rangle$ designates summation over the nearest neighbours and $\vec{s}_i$ is an n-vector. The EA Hamiltonian resides on a d-dimensional cubic lattice with one spin per site. Note that the disorder is incorporated in the interaction between spins rather then their position. One typical example of the probability distribution of interaction strength is Gaussian:

$$P(J_{ij}) = \frac{1}{\sqrt{2\pi(\Delta J_{ij})^2}} \exp\Big[-\frac{(J_{ij} - \bar{J}_{ij})^2}{2(\Delta J_{ij})^2} \Big]. \tag{5}$$

Although we spoiled the role of disorder and frustration from the get go, it was the insight of Edwards and Anderson to suggest the simplest model that displays these features. An even simpler model that exhibits glasslike behavior is an Ising EA model:

$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} \sigma_i \sigma_j - \vec{h} \sum_i \sigma_i \tag{6}$$

where $\sigma = \pm 1$ and J_{ij} takes values from a random distribution analogously to EA model. Note the difference in our description of thermal and quenched disorder. "Thermal" disorder is conventionally taken into account by assignment of different probability weights derived from a Hamiltonian, but quenched disorder parameterizes the Hamiltonian itself. In fact there is infinitely many such Hamiltonians. At first sight this presents a formidable challenge.

Q: *How can one derive universal features of systems with frozen-in disorder that are not produced by a specific instance of Hamiltonian?*

The answer to this question naturally leads to a discussion of statistical averages and the method of replicas. Let's assume that quenched disorder can be described by a highly-multidimensional random variable X, which instantiates the Hamiltonian $H[X]$. One common quantity of interest is the *annealed average*. For instance, the free energy of a system given by:

$$F = -k_B T \ \mathrm{Log}\big\{Z[X]\big\}_X \tag{7}$$

$$Z[X] = \mathrm{Tr}\ \mathrm{Exp}\Big[-\beta H[X]\Big] \tag{8}$$

where $\{\cdots\}_X$ gives the average over all configurations of disorder. Here the average is taken over all partition functions, rather than the free energy. However, it turns out the annealed average is not appropriate for describing spin glasses. One can use the annealed average if an observation time is much larger than the fluctuation time of random variables. Since spin glasses do not exhibit disorder fluctuations on the timescale of observation, we do not have the luxury of straightforward averaging over all configurations of x. We are thus faced with the problem of calculating properties of a system in a particular configuration of disorder.

Here the thermodynamic limit comes to our rescue. The random variable has macroscopic dimensionality and therefore in certain cases we can average over its probability distribution $P[X]$. The subtlety lies in which quantities can be averaged in this way. Consider a system that can be split into a macroscopic number N_1 of macroscopic subsystems of size N_2. Since the coupling between their interfaces is suppressed by a factor of $1/N_2$ in comparison to the bulk, extensive quantities can be averaged over all subsystems. This property is called *self-averaging*:

$$\lim_{N_1 \to \infty} \Big\{A[X] - \{A[X]\}_X\Big\} = 0 \tag{9}$$

Therefore in our aforementioned example of free energy calculation, rather than averaging over partition functions one has to average over free energy, which is an extensive quantity. It can be shown [7] that the annealed average, over partition functions, provides an upper bound for quenched averages of extensive quantities. It the fact that we have to average the free energy, which contains the random variable inside the logarithm, that makes analytic treatment of spin glasses so tricky. To resolve this issue several methods were introduced, one of which is the *replica method*.

To motivate the replica method consider the following relation:

$$\{\mathrm{Log}[Z[x]]\}_X = \lim_{n \to 0} \frac{1}{n}\Big(\big\{Z^n[X]\big\}_X - 1\Big) = \lim_{n \to 0} \frac{\partial}{\partial n}\big\{Z^n[X]\big\}_X \tag{10}$$

where we used $a^n \simeq 1 + n\text{Log}[a]$ for small n. Using this simple relation we moved from averaging logarithms to averaging over n-replicas of the same system, and the consequent limit of $n \to 0$. The new partition function that we average over is:

$$Z^n[X] = \prod_{\mu=1}^{n} \text{Exp}[-\beta H[X, S^\mu]] = \text{Exp}[-\beta \sum_{\mu=1}^{n} H[X, S^\mu]] \tag{11}$$

where μ is a replica index and S^μ a collective variable for all the spins in μ's replica. This partition function may be calculated for an integer n and needs to be analytically continued to real n in order to take the limit of $n \to 0$. According to our simple derivation, we need to take the limit $n \to 0$ before $N \to \infty$. However, we would like to perform the average in thermodynamic limit before analytic continuation to real n. It is currently believed that such reversal of order of limits does not cause any problems [8]. However, as we will see soon, another very important issue related to symmetry upon interchange of replicas may still arise when we analytically continue replica number.

To demonstrate the issue, consider an integer number of replicas n. We first take a thermodynamic limit, which allows us to perform the average:

$$Z(n) = \left\{Z^n[X]\right\}_X = \text{Tr Exp}[-\beta H_{eff}(n)] \tag{12}$$

where we introduced an effective Hamiltonian. Unlike the original Hamiltonian, it has no disorder and is translationally invariant. But there is a price to pay. The original independent replicas are now coupled to each other. However, it is clear that the effective Hamiltonain is invariant under permutation of replicas, it has *replica symmetry*. This may cease to be the case when we analytically continue and send n to 0, causing *replica symmetry breaking*.

4 Sherrington-Kirkpatrick Model and Replica Symmetry Breaking

In the previous section we have offered a minimal model, EA model, that contains all the essential ingredients for description of spin glasses. However, we still have not given an order parameter that becomes non-zero at the onset of the spin glass transition. Such an order parameter was proposed by Edwards and Anderson [6] along with their model:

$$q_{EA} = \lim_{N\to\infty} \frac{1}{N} \sum_i \langle \sigma_i \rangle_t^2 . \tag{13}$$

This definition pertains to Ising-type models and assumes taking the thermodynamic limit. As before, the average is taken over a time much longer than microscopic scale. In the high temperature phase, $\langle \sigma_i \rangle_t = 0$ and thus $q_{EA} = 0$. In the spin glass phase ,$\frac{1}{N} \sum_i \langle \sigma_i \rangle_t = 0$ due to random orientation of frozen-in spins, but this difficulty is circumvented by squaring the magnetization at each site. In principle, now we have all the tools to describe the thermodynmics of EA spin glass. However, analytic treatment of such models is very challenging.

Q: *How can we simplify the EA model while preserving spin glass behaviour?*

Quenched disorder in spin glasses is a manifestly non-perturbative effect. We are thus compelled to resort to another common method in statistical physics, mean field theory. Such an analysis was first given by Sherrington and Kirkpatrick [9] in the form of an infinite-range EA Ising model:

$$H[J,S] = -\sum_{\langle i,j\rangle} J_{ij}\sigma_i\sigma_j - \sum_i h_i\sigma_i \tag{14}$$

where $\langle i,j\rangle$ designates summation over each pair in the sample and J_{ij} is drawn from a Gaussian distribution:

$$P(J_{ij}) = \sqrt{\frac{N}{2\pi\Delta J^2}}\mathrm{Exp}\Big[-\frac{NJ_{ij}^2}{2\Delta J^2}\Big] \tag{15}$$

which is the same for arbitrary i and j. Note the first moments of this distribution:

$$[J] = 0 \quad [J^2] = \Delta J^2/N \tag{16}$$

which were chosen to ensure the independence of energy density from the number of spins N. Using this probability distribution:

$$Z(n) = \prod_{i,j}\int_{-\infty}^{\infty} dJ_{ij}P(J_{ij})\sum_{\{\sigma\}}\mathrm{Exp}\Big[\beta\sum_a\sum_{i,j}J_{ij}\sigma_i^a\sigma_j^a + \beta\sum_a\sum_i h_i\sigma_i^a\Big] \tag{17}$$

To clarify this loaded expression, the integral over J_{ij} is taken to average over disorder. The sum over $\{\sigma\}$ is taken to account for all spin configurations. The sum over a represents summation over all replicas. And lastly, there are summations in the exponent over all the sites of the spin glass. Now since disorder is assumed to be Gaussian distributed, we can easily perform the integral over J_{ij}. Since we will eventually take the thermodynamic limit $N \to \infty$, we only keep the terms that are leading order in N. Furthermore, we can also perform the Hubbard-Stratonovich transform in order to turn quartic terms into quadratic using the identity:

$$\mathrm{Exp}[ax^2/2] = \frac{1}{\sqrt{2\pi a}}\int_{-\infty}^{\infty} dy\ \mathrm{Exp}[-y^2/2a + xy] \tag{18}$$

If we then assume the magnetic field to be constant, after all these steps we arrive at:

$$\begin{aligned} Z(n) = \mathrm{Exp}[-Nn\beta^2\Delta J^2/4]\int_{-\infty}^{\infty}\prod_{a<b}\Big\{\beta\sqrt{\frac{N}{2\pi}}dq_{ab}\Big\}\mathrm{Exp}\Bigg[\frac{N}{2}\beta^2\Delta J^2\sum_{a<b}q_{ab}^2 + \\ + N\ \mathrm{Log}\Big[\sum_{\{S\}}\mathrm{Exp}\big[(\beta\Delta J)^2\sum_{a<b}q_{ab}S^aS^b + \beta h\sum_a S^a\big]\Big]\Bigg] \end{aligned} \tag{19}$$

where we have introduced n variables $S^a = \pm 1$, one for each replica. Furthermore, there are $\frac{n(n-1)}{2}$ variables q_{ab}, that may be thought to constitute a symmetric matrix with 0's on the diagonal. The latter fact is caused by $(S^a)^2 = 1$.

Now a thermodynamic limit may be taken allowing us to calculate the integral in Eq. 19 using the method of steepest descent. Then the free energy density is:

$$f = -k_B T \lim_{n\to 0} \left\{ \frac{\beta^2 \Delta J^2}{4} - \frac{\beta^2 \Delta J^2}{4n} \sum_{a,b} q_{ab}^2 + \frac{1}{n} \mathrm{Log}\Big[\sum_{\{S\}} \mathrm{Exp}[L] \Big] \right\} \tag{20}$$

where we defined:

$$L = (\beta \Delta J)^2 \sum_{a<b} q_{ab} S^a S^b + \beta h \sum_a S^a \tag{21}$$

and q_{ab} satisfy the saddle-point conditions:

$$\frac{\partial f}{\partial q_{ab}} = 0. \tag{22}$$

At this point Sherrigton and Kirkpatrick guess the q_{ab} matrix to have the following form:

$$q_{ab} = q \quad \text{for} \quad a \neq b \quad \text{and} \quad q_{ab} = 0 \quad \text{for} \quad a = b \tag{23}$$

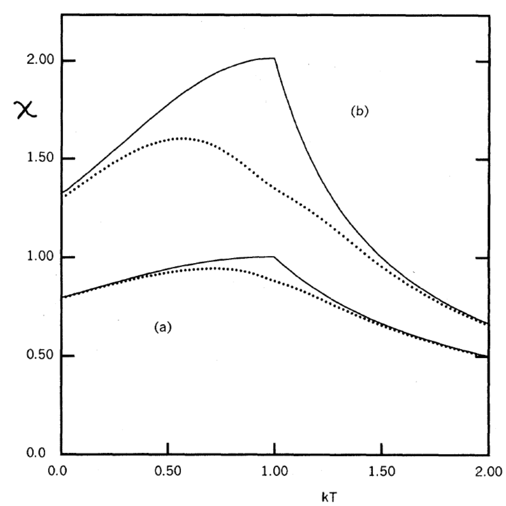


Figure 4: Magnetic susceptibility without external field (solid) and with magnetic field $H = 0.1\Delta J$ (dotted) for a) $\frac{[J]}{\Delta J} = 0$ and b) $\frac{[J]}{\Delta J} = 0.5$ [9].

Note that this form of q_{ab} is consistent with "replica symmetry". Using this form of matrix q_{ab} we can easily evaluate the expression for free energy in Eq. 20

$$f = -\frac{\Delta J^2}{4 k_B T}(1-q)^2 - \frac{k_B T}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dz \ \mathrm{Log}\Big[2\mathrm{Cosh}\big[\frac{\Delta J \sqrt{q} z + h}{k_B T} \big] \Big] \mathrm{Exp}[-z^2/2] \tag{24}$$

where q is given by the self-consistency condition:

$$q = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dz \ \mathrm{Tanh}^2 \big[\frac{\Delta J \sqrt{q} z + h}{k_B T} \big] \Big] \mathrm{Exp}[-z^2/2] \tag{25}$$

and is equal to the Edwards-Anderson order parameter (Eq. 13).

We thus used the replica method to derive the free energy of the SK model. Our solution was an inspired guess by Sherrigton and Kirkpatrick motivated by replica symmetry. From our final expression for free energy in Eq. 25, one can see that without an external magnetic field, there is only one solution above the critical temperature $T_c = \Delta J$, which is $q = 0$. Above T_c another solution for q appears, which corresponds to the spin glass transition.

Furthermore, from the free energy all thermodynamic parameters of the model may be derived. Remarkably, as was shown by Sherrigton and Kirkpatrick in [9] (Fig. 4), magnetic susceptibility has a cusp at the transition temperature T_c. These is great news! Not only does the model predict a spin glass transition, it also reproduces some of its experimentally observed features. But there is bad news too. At small temperatures the Sherrington-Kirkpatrick (SK) solution predicts negative entropy. Clearly this is an unphysical result that was realized already by the authors of the model. It was quickly realized that this issue stems from taking the limit $n \to 0$, and resolving this problem necessitated an introduction of a new concept, replica symmetry breaking.

5 Breaking the Replica Symmetry

Q: Where did our replica symmetric calculation for the SK model go wrong?

If we go through the calculation and "check our premises", we can spot the resolution. It lies in our cavalier assumption that the replica symmetry is preserved in the limit $n \to 0$. Although the SK solution satisfies:

$$\frac{\partial f}{\partial q_{ab}} = 0 \tag{26}$$

that does not imply that the free energy is minimized. We can clearly see the origin of the problem by looking at the stability of the SK solution. Near the stationary point the free energy is equal to:

$$f[q_{ab}] = f[q_{ab}^{SP}] + k_B T \lim_{n \to 0} \frac{1}{n} \sum_{a<b,c<d} R^{ab,cd} \delta q_{ab} \delta q_{cd} \tag{27}$$

where $f(q_{ab}^{SP})$ is a saddle point free energy. Depending on $R^{ab,cd}$, the saddle point free energy may or may not be stable with respect to fluctuations around q_{ab}^{SP}. Stability of the saddle point is synonymous with having a local minimum, which is realized when the Hessian matrix $R^{ab,cd}$ has only positive eigenvalues.

Careful analysis of the eigenvalues of R has been done by Almeida and Thouless [10]. It shows that for positive integer number of replicas all the eigenvalues are positive. However, when $0 \leq n < 1$, there always exist a small enough temperature such that the SK solution becomes unstable. The result of their calculation is shown on the Fig. 5. This plot depicts two phases in the parameter space of temperature and magnetic field. Above the AT line,

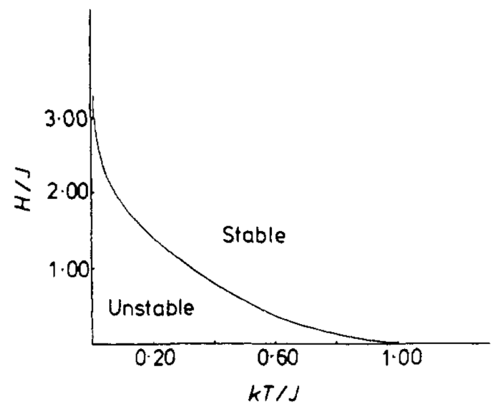


Figure 5: Almeida-Thouless (AT) line for SK model that separates the paramagnetic and spin glass phases [10].

the SK solution is unique and exact. This region corresponds to the paramagnetic phase. Below the AT line, the SK solution becomes unstable and gives way to the replica symmetry broken solution. This region of the parameter space corresponds to the spin glass phase. One can say that the replica symmetry gets broken to make the eigenvalues positive, stabilizing solutions. The AT line itself signals the spin glass transition.

Unfortunately, there are many ways how replica symmetry can be broken, and there is no established way for determining q_{ab}. In practice one makes a guess of q_{ab} with some variational parameters and checks it *a posteriori*. Such an ansatz for SK model was formulated by Parisi [11]. To arrive at Parisi solution, we start from the SK form of the $n \times n$ matrix q_{ab}. Then we split this matrix into blocks of size $m \times m$ along the diagonal (Fig. 6) and change the values of the off-diagonal elements in these blocks to a new constant. Now we repeat this step for the new blocks $(N-1)$ more times. We arrive at a sequence of block sizes and corresponding q-values:

$$n \equiv m_0 \geq m_1 \geq m_2 \geq ... \geq m_N \geq 1 \qquad q \equiv q_0, q_1, q_2, ..., q_N \tag{28}$$

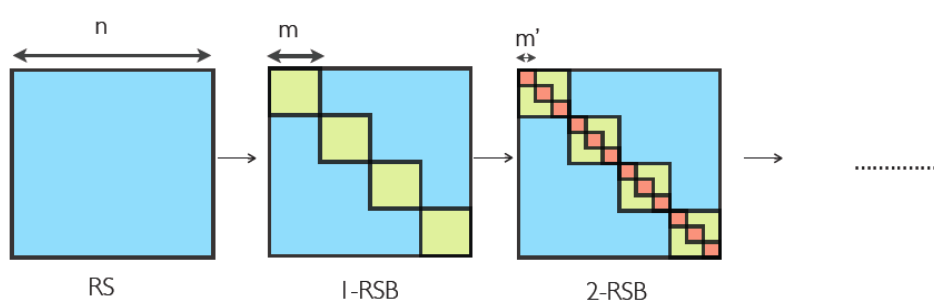


Figure 6: Schematic representation of replica symmetry breaking [12].

This sequence is constructed for some positive integers n and N. We now analytically continue this sequence to $n \to 0$ and $N \to \infty$. This flips the signs in the inequality for block sizes:

$$0 \leq m_0 \leq m_1 \leq m_2 \leq ... \leq m_N \leq 1 \tag{29}$$

and makes m a continuous variable:

$$m_i \to m[x],\ m[x] \in [0, 1]. \tag{30}$$

The information about the replica symmetry breaking is then encoded in the function $q[x]$ on a unit interval. This function is called the Parisi order parameter and the free energy may be expressed in terms of this function [13]:

$$f = -\frac{\Delta J^2}{4k_B T}\Big(1 - 2q[1] + \int_0^1 dx\ q^2[x]\Big) - \frac{k_B T}{\sqrt{2\pi}} \int_{\infty}^{\infty} dz\ G\big[0, H + \sqrt{q[0]}z\big] e^{-z^2/2} \tag{31}$$

where $G[x, y]$ is given by the differential equation:

$$\frac{\partial G}{\partial x} = -\frac{\Delta J^2}{2} \frac{\partial q}{\partial x} \left(\frac{\partial^2 G}{\partial y^2} + x \left(\frac{\partial G}{\partial y} \right)^2 \right) \tag{32}$$

with boundary condition $G[1, y] = \mathrm{Log}[2\mathrm{Cosh}[\beta y]]$. The Parisi ansatz for replica symmetry breaking results in an expression for the free energy, which as advertised, needs to be evaluated variationally by varying $q[x]$. Furthermore, as was shown by C. De Dominicis *et al.* [14], the Hessian matrix for Parisi solution has only non-negative eigenvalues and thus it is stable. This resolves the central issue that prompted the introduction of the replica symmetry breaking in the first pace.

6 Physical Meaning of Replica Symmetry Breaking and Concluding Remarks

To summarize our discussion of the mean field theory of the spin glass transition, we introduced the SK model and suggested a solution using the replica method. We consequently discussed the applicability of the SK solution and stressed the necessity of replica symmetry breaking. The latter corrected the failings of the SK solution, but it has the downside of being seemingly very abstract. On the face of it, it is not at all clear how one should interpret the breaking of the permutation symmetry between replicas. Somewhat surprisingly, this mathematical trick has a definite physical interpretation. It has to do with the rugged energy landscape that we introduced before. Above the AT line, the only minimum of the energy landscape of the SK model is the replica symmetric saddle point. However, as we cross the AT line, many saddle points appear, some of which are minima. In thermodynamic limit the energy barriers between these minima diverge forming valleys or "pure states". The system gets trapped in these valleys, breaking ergodicity. This is precisely the effect that necessitates breaking the replica symmetry.

In this essay, we focused on how quenched disorder and frustration, which define spin glasses, give rise to a multitude of complex behaviors. To describe these behaviors, we used mean field theory. In the context of spin glasses, mean field theory displays rich behavior, which required introduction of replicas and the breaking thereof. However, this essay barely scratches the surface of mean field theory of glasses and did not consider the theory of spin glasses beyond mean field.

Furthermore, although the field was established in the attempt to describe the magnetic behavior of diluted magnetic alloys, it has found many applications beyond its original scope. Many connections to complexity theory were found, which include protein folding, neuronal networks, and many other. It is therefore a fascinating example of why deep scientific mysteries are worth pursuing irrespective of their immediate applicability or glamour.

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