INTEGRAL QUANTUM HALL EFFECT IN MBE GROWN THIN FILMS

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Abstract

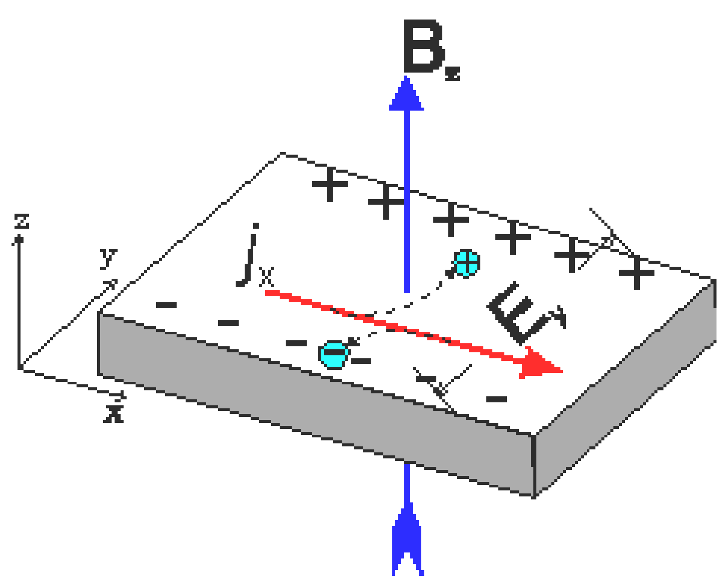
We aim to study Integral Quantum Hall Effect in MBE-grown thin films like Graphene,Bismuth,GaAs and FeSiO2.We aim to look at how the quantum size effects dominate the electrical transport properties like the resistivity, hall coefficient, magnetoresistance coefficient and low-frequency electrical noise. We are also interested to look at the measurements on the MBE-grown AlGaAs/GaAs heterojunctions which indicate a difference of nearly an order of magnitude in the transport properties.

INTRODUCTION AND BACKGROUND

The Quantum Hall Effect (QHE) is one of the most remarkable phenomena in condensed matter discovered in the second half of the 20th century. At low temperatures and in strong magnetic fields, it is found that the Hall resistance of a two dimensional electron system has plateaus as a function of the number of electrons.

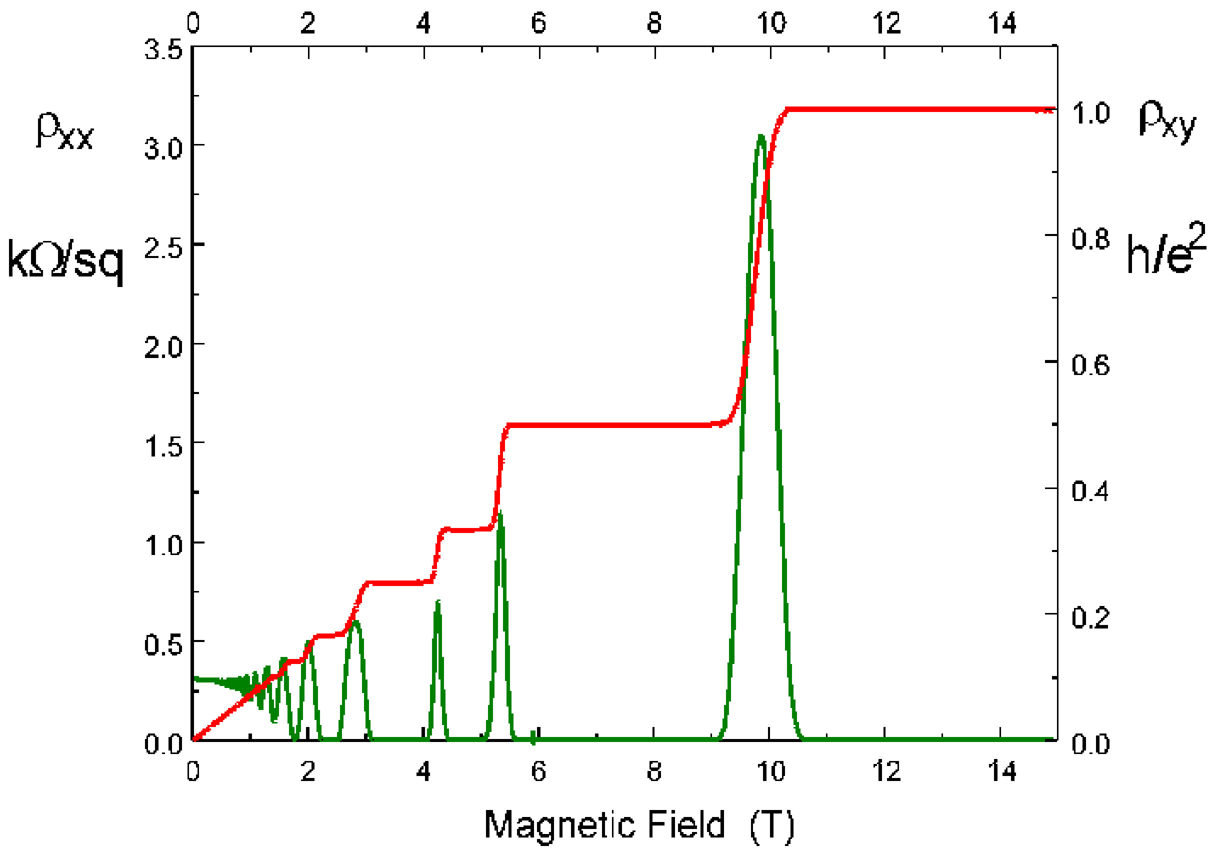
In 1985 Klaus von Klitzing won the Nobel Prize for discovery of the quantized Hall effect. The previous Nobel Prize awarded in the area of semiconductor physics was to Bardeen, Shockley and Brattain for invention of the transistor. Everyone knows how important transistors are in all walks of life, but why is a quantized Hall effect significant?

It was over 100 years ago E.H. Hall discovered that when a magnetic field is applied perpendicular to the direction of a current flowing through a metal a voltage is developed in the third perpendicular direction. This is well understood and is due to the charge carriers within the current being deflected towards the edge of the sample by the magentic field. Equilibrium is achieved when the magnetic force is balanced by the electrostatic force from the build up of charge at the edge. This happens when $E_y = v_x B_z$ .The Hall coefficient is defined as $R_H = E_y / B_z j_x$ and since the current density is $j_x = v_x Nq$, $R_H = 1/Nq$ in the case of a single species of charge carrier. R_H can thus be measured to find N the density of carriers in the material. Often this transverse voltage is measured at fixed current and the Hall resistance recorded. It can easily be seen that this Hall resistance increases linearly with magnetic field.



The FIGURE 1 above shows the normal Hall Effect experimental set-up.

In a two-dimensional metal or semiconductor the Hall effect is also observed, but at low temperatures a series of steps appear in the Hall resistance as a function of magnetic field instead of the monotonic increase. What is more, these steps occur at incredibly precise values of resistance which are the same no matter what sample is investigated. The resistance is **quantized** in units of h/e^2 divided by an integer. This is the QUANTUM HALL EFFECT.



The FIGURE 2 shows the integer quantum Hall effect in a GaAs-GaAlAs heterojunction, recorded at 30mK.

The basic experimental fact characterizing QHE is that the diagonal electrical conductivity of a two-dimensional electron system in a strong magnetic field is vanishingly small $\sigma_{xx} \to 0$, while the non-diagonal conductivity is quantized in multiples of e^2/h: $\sigma_{xy} = -\nu e^2/h$, where ν is an interger [The Integer Quantum Hall Effect (IQHE)] or a fractional number [The fractional Quantum Hall Effect].

THEORETICAL EXPLANATION OF QUANTUM HALL EFFECT

Derivation of the Quantum Hall Coefficient

In the Drude theory of the electrical conductivity of a metal, an electron is accelerated by the electric field for an average time τ before being scattered by impurities, lattice imperfections and phonons to a state which has average velocity zero. The average drift velocity of the electron is

$$v_d = -eE\tau / m$$

where E is the electric field and m is the electron mass. The current density is thus

$$j = -nev_d = \sigma_0 E \text{ where } \sigma_0 = ne^2\tau / m$$

where n is the electron density.

In the presence of a steady magnetic field, the conductivity(σ) and resistivity(ρ) become tensors: $j = \sigma.E$ and $E = \rho.j$ always hold.

From the Lorentz Force Law, we find

$$v_d = -e(E + \frac{v_d \times B}{c})\frac{\tau}{m}$$

We assume that the magnetic field is in the z direction, then in the x-y plane

$$\sigma_0 E_x = \omega_c \tau j_y + j_x \quad \text{and} \quad \sigma_0 E_y = -\omega_c \tau j_x + j_y$$

where σ_0 is defined above and $\omega_c = \frac{eB}{mc}$.

Finally, we get the relationship between conductivity and resistivity as follows

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \quad \text{and} \quad \sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$$

Here we see that if $\rho_{xy} \neq 0$, then conductivity σ_{xx} vanishes when ρ_{xx} vanishes.

On the other hand, we have $\sigma_{xy} = -\frac{nec}{B} + \frac{\sigma_{xx}}{\omega_c \tau}$.

So, when $\sigma_{xx} = 0$, $j_x = \sigma_{xy} E_y$ where σ_{xy} which is the Hall Conductivity is given by

$$\sigma_H = \sigma_{xy} = -\frac{nec}{B}.$$

Now, in Quantum Mechanics, the Hamiltonian is

$$H = \frac{1}{2m}(p + \frac{eA}{c})^2 + eEx \quad (E \text{ is along the x direction})$$

For this problem it is convenient to choose the Landau gauge, in which the vector potential is independent of y coordinate: $A = (0, B_x, 0)$.

Solving the Schrodinger Equation which can be transformed to the familiar harmonic oscillator equation, the eigen values are as follows:

$$E_i(\mathrm{E}) = (i + 1/2)\hbar\omega_c + e\mathrm{E}(I_c^2 k_y - e\mathrm{E} / 2m\omega_c^2)$$

where $I_c = (\frac{\hbar c}{eB})^{\frac{1}{2}}$ (classical cyclotron orbit radius) and i=0,1,2,…

The different oscillator levels are also called Landau Levels. The electric field simply shifts the eigen values by a value without changing the structure of the energy spectrum.

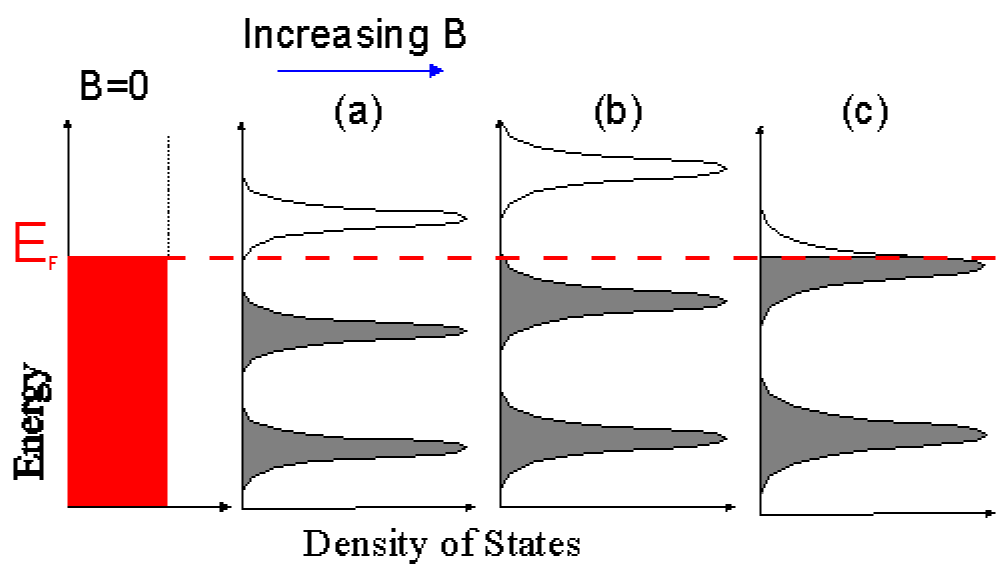
From the wave functions, we can calculate the mean value of the velocities as

$$< v_y > = -\text{E}c / B \text{ and } < v_x > = 0$$

Thus, $j_y = -ne\text{E}c / B$ which is the same as classical results derived above.The current along the direction of electric field (x) is zero at Landau levels.

Explanation of the plateaus and zeroes (FIG 2)

The zeros and plateaus in the two components of the resistivity tensor are intimately connected and both can be understood in terms of the Landau levels (LLs) formed in a magnetic field.



In the absence of magnetic field the density of states in 2D is constant as a function of energy, but in field the available states clump into Landau levels separated by the cyclotron energy, with regions of energy between the LLs where there are no allowed states. As the magnetic field is swept the LLs move relative to the Fermi energy.

When the Fermi energy lies in a gap between LLs electrons can not move to new states and so there is no scattering. Thus the transport is dissipationless and the resistance falls to zero.

The classical Hall resistance was just given by *B/Ne.* However, the number of current carrying states in each LL is *eB/h*, so when there are *i* LLs at energies below the Fermi energy completely filled with *ieB/h* electrons, the Hall resistance is h/ie^2. At integer filling factor this is exactly the same as the classical case.

The difference in the QHE is that the Hall resistance can not change from the quantised value for the whole time the Fermi energy is in a gap, i.e between the fields (a) and (b) in the diagram, and so a plateau results. Only when case (c) is reached, with the Fermi energy in the Landau level, can the Hall voltage change and a finite value of resistance appear.

EXPERIMENTAL SETUPS FOR QHE

Quantum Hall Effect has been observed in two types of 2-D electron systems achieved in experiment. They are as follows:-

- **MOSFET-Metal Oxide Semiconductor Field Effect Transistor**

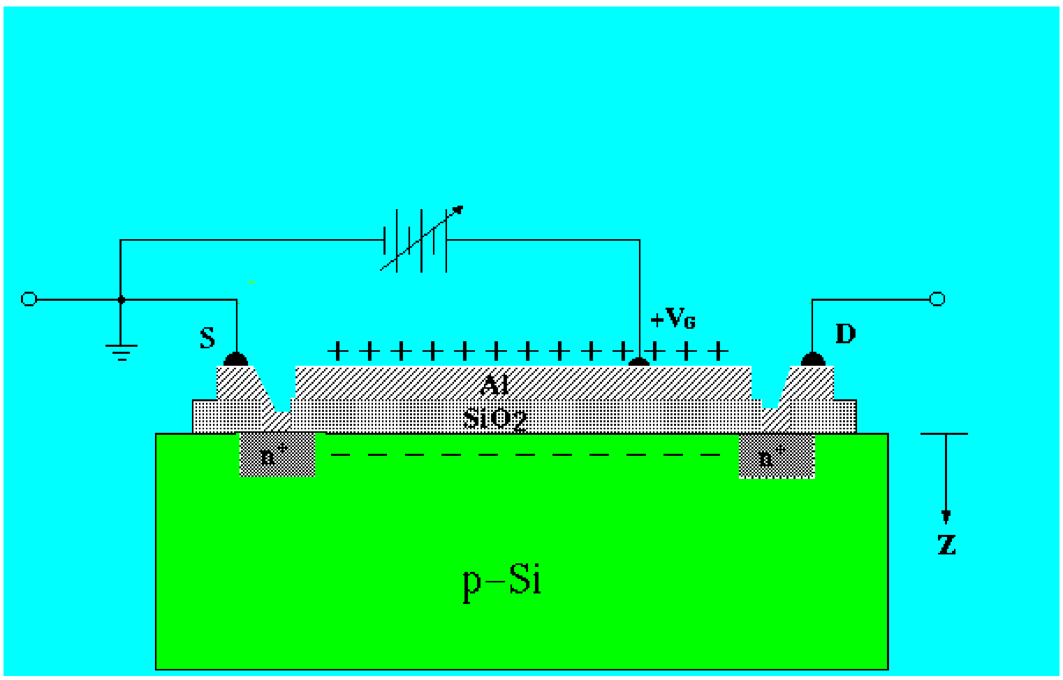


FIG 4: Schematic side view of a silicon MOSFET

In MOSFET, inversion layers are formed at the interface between a semiconductor and an insulator or between two semiconductors, with one of them acting as an insulator. The system in which the Quantum Hall Effect (QHE) was discovered has Si for the semiconductor, SiO_2 for the insulator. Figure 2 is a schematic side view of a silicon MOSFET showing the aluminum gate, the SiO_2 insulator and the p-type Si crystal substrate. The principle of the inversion layer is quite simple. It is arranged that an electric field perpendicular to the interface attracts electrons from the semiconductor to it. These electrons sit in a quantum well created by this field and the interface. The motion perpendicular to the interface is quantized and thus has a fundamental rigidity which freezes out motional degrees of freedom in this direction. The result is a two-dimensional system of electrons.

- **Superlattice**

Another type of two-dimensional electron system is formed in the heterostructures of two semiconductors. Using molecular beam epitaxy (MBE) technique, people can grow two semiconductors alternately to form a one dimensional sandwich like structure. Each layer has a width of about several nanometers. They are called superlattice. They can also be grown by metal-organic chemical vapor deposition (MOCVD). For example, in $GaAs - Ga_{1-x}Al_xAs$ superlattice, a certain controlled number of layers of GaAs is followed by an almost perfectly matched sequence of layers of GaAlAs. The GaAlAs is deliberately doped n-type, which puts mobile electrons into its conduction band. These electrons will migrate to fill the few holes on the top of the GaAs valence band but most of them will end up in states near the bottom of the GaAs conduction band. However, there is a positive charge left on the donor impurities which attracts these electrons to the interface and bends the bands in the process. This is the source of the electric field in this system. The transfer of electrons from GaAlAs to GaAs will continue until the dipole layer formed from the positive donors and the negative inversion layer is sufficiently strong. This dipole layer gives rise to a potential discontinuity which finally makes the Fermi level of the GaAs equal to that of the GaAlAs. Figure below shows the band structure.

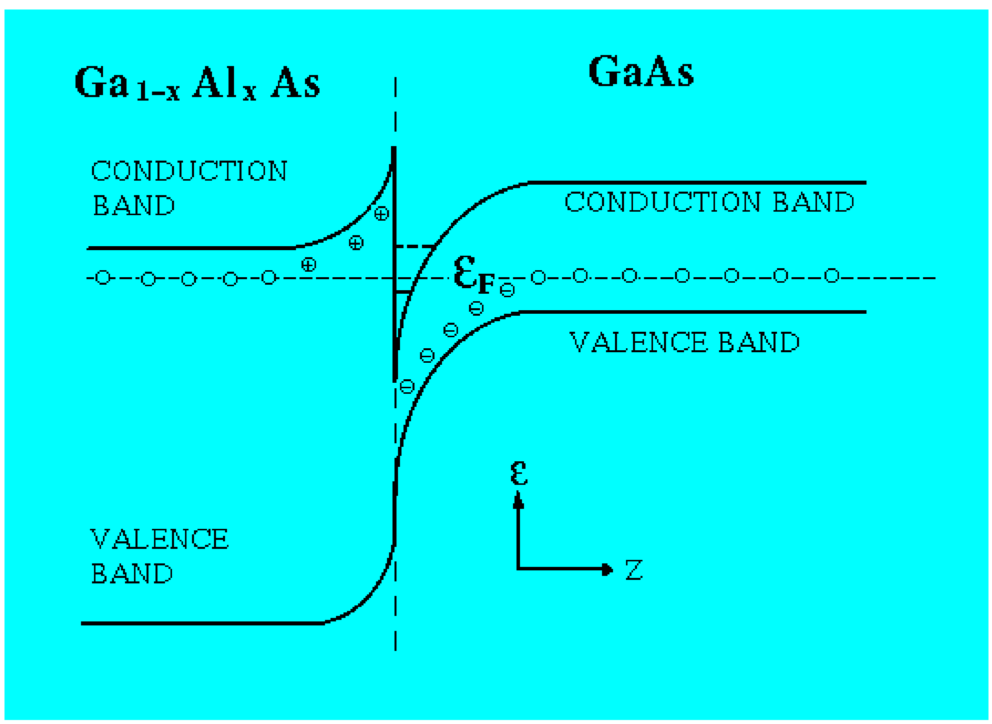


FIG.5 Electron energy level diagram of a GaAs-AlGaAs heterostructure device

This report will concentrate on the 2nd type of QHE system ie the MBE grown superlattices.

ROLE OF MBE IN QHE EXPERIMENTS

Molecular beam epitaxy is used in making ultra thin epitaxial films of of lead, Graphite, GaAs, Bismuth etc. The sample preparation and the measurements were performed in an ultra-high vacuum system with a base pressure < 7×10^{-11} mbar. It was equipped with a He cryostat and a reflection high-energy electron diffraction (RHEED) system. The crystal structure of the growing film was monitored with RHEED. By very efficient computer control we can effectively control the deposition of the number of mono-layers of different films on the substrate, thus making a good super-lattice.

Ultra-thin films provide a promising starting point for obtaining a general understanding of the influence of the Quantum Size Effects (QSE) on the Hall Effect. QSE oscillations were observed in all the transport properties of the thin films at low temperatures.

If the sample thickness becomes of the order of the carrier wavelength, the resulting discreteness of the energy spectrum appreciably modifies the density of states. This influences the relaxation time and hence the mobility. All the transport properties which depend upon the mobility will oscillate as a function of sample thickness with a period of approximately one-half of the wavelength of the carriers at the Fermi surface.

RESULTS OF SOME OF THE EXPERIMENTS PERFORMED ON QHE IN THIN FILMS

The figure below shows the temperature dependence of the Hall Coefficient showing data for several samples of different thicknesses of Bismuth films.

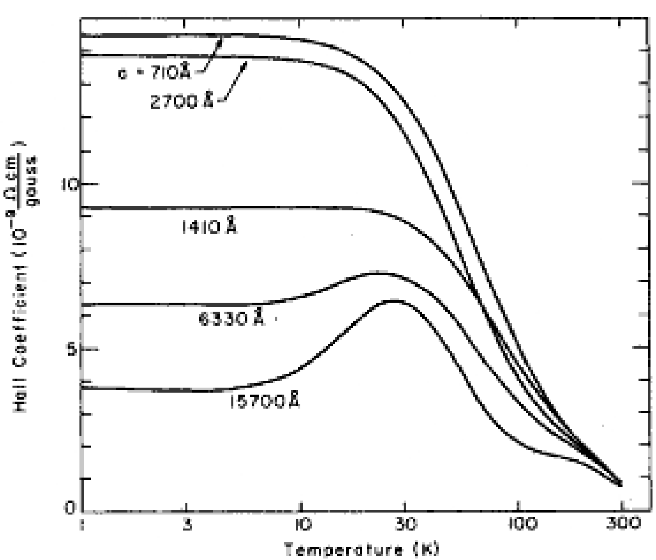


FIG. 4. Temperature dependence of the Hall coefficient for films of various thicknesses.

[Courtesy: PRB Vol 3, Num 6, 15 Mar 1971]

As an example of Quantum Size Effect, the figure below shows the thickness dependence of the resistivity ratios $E_i(\mathrm{E}) = (i+1/2)\hbar\omega_c + e\mathrm{E}(I_c^2 k_y - e\mathrm{E}/2m\omega_c^2)$ and ρ_{77}/ρ_{300K} for Bismuth films .The ratios were plotted in order to eliminate the error in the absolute resistivity resulting from the thickness measurement. The Hall and magnetoresistance coefficients were also found to exhibit the same type of oscillatory behavior as the resistivity ratio at low temperatures.

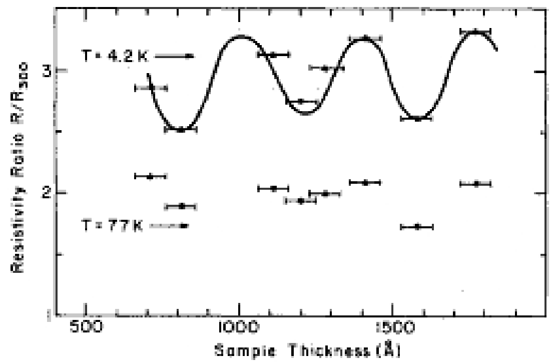
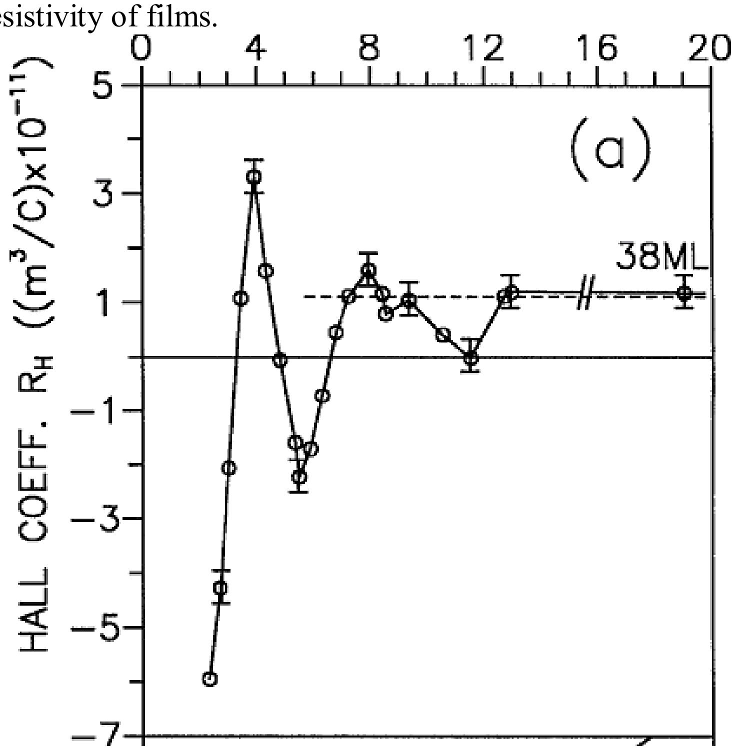


FIG. 5. Thickness dependence of the resistivity ratio at 4.2 and 77 K.

[Courtesy: PRB Vol 3, Num 6, 15 Mar 1971]

The data below shows the Hall Coefficient calculated from the data obtained from a set of ultra-thin Pb layers prepared at 80K and measured at 20K.Each circle is from a separate film. Below 2.5 ML the noise rapidly increased with decreasing thickness due to the large resistivity of films.



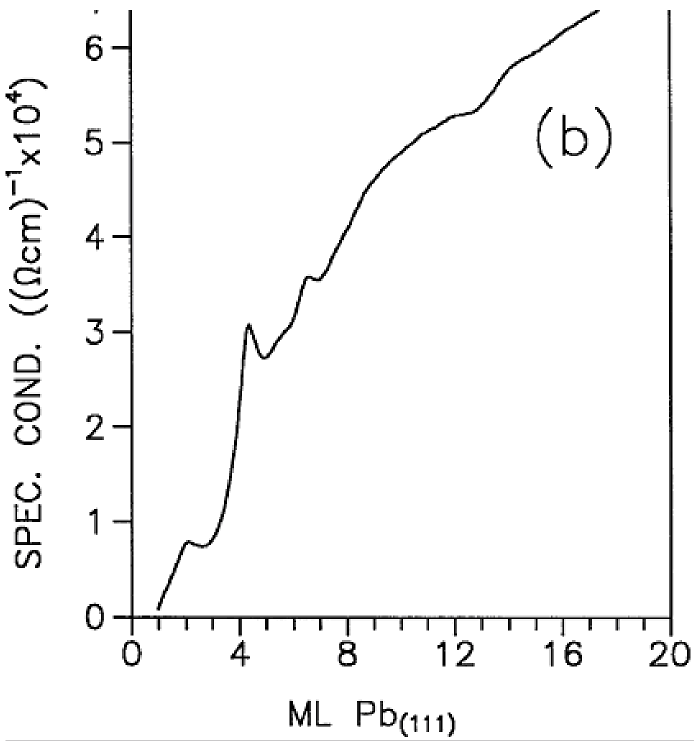


FIG: Hall coefficient of ultrathin Pb(111) films deposited at 80K on Si(111)-$(\sqrt{3}\times\sqrt{3})R30^0$ Ag, measured at 20K(a).The dashed line is data for bulk Pb measured at 20K.(b) shows the thickness dependence of the specific conductivity, measured during deposition in the same condition as in (a).

[Courtesy:Vol 76, Num 22, PRL, 27 May 1996]

Other experimental progress

Since 1992, several experimental groups have succeeded in making low resistance contacts between superconducting leads and a two-dimensional electron gas (2DEG) in a semiconductor heterostructure. Evidence has been found for Andreev reflection, in which Cooper pairs are converted into unpaired electrons and transported into the 2DEG.These experiments raise the exciting possibility of making and studying a tunnel junction between a superconductor and a dissipationless quantum Hall fluid. One can see the immediate obstacle that arises when considering a tunnel junction between a superconductor and a Hall fluid. The large magnetic field needed to put the 2DEG into the quantum Hall fluid will tend to suppress the superconductivity. This is the reason why this is an extremely challenging area of research.

CONCLUSION

After the Nobel Prize winning work in Quantum Hall Effect, this research has crossed several frontiers. QHE has been studied in many two dimensional electron systems like MOSFET and superlattice. Liquid Helium surface is one more 2-D electron system where Quantum hall effect is not yet seen which explores another promising area of research. Here we have looked at the influence of the Quantum Size Effects of the thin films on the electronic transport properties. Even after so much experimental progress, it is still a challenge for any experimentalist to prepare a good tunneling heterojunction to make a 2-D electron gas system.

REFERENCES

[1] J.P Harrang et al, Physical Review B, Vol 32, Num 12, 15 Dec 1985
[2] R A Hoffman and D R Frankl, Physical Review B, Vol 3, Num 6, 15 Mar 1971
[3] M Jalochowski et al, Vol 76, Num 22, PRL, 27 May 1996
[4] B Raquet et al, Physical Review B, Vol 62, Num 24, 15 Dec 2000-2
[5] V P Gusynin and S G Sharapov, PRL 95, 146801(2005), 30 Sept 2005
[6] Matthew P A Fisher, Physical Review B, Vol 49, Num 20, 15 May 1994-2
[7] Website of Lucent Technologies, the Fractional Quantum Hall Effect