Superfluid state for photons?

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Photons, which are bosons, can form a 2D superfluid due to Bose-Einstein condensation inside a nonlinear Fabry-Perot cavity filled with atoms in their ground states. The effective mass and chemical potential for a photon inside this fluid are non-vanishing. The dispersion relation for this photon gas is identical in form to the Bogoliubov relation describing elementary excitations of weakly-interacting bosons. I will summarize the basics behind and the ongoing numerical and experimental tests on the possibility of a superfluid state for photons.

Introduction

Experimental discoveries in the last two decades have shown that Bose-Einstein condensation can in fact be realized in laboratories using dilute atomic gasses. Superfluid state of matter has been observed for many decades in many systems. At the heart of all the experiments lies the quantum statistics of particles. It is the Bose statistics in this case, and it does not matter if the bosons are whether alkali atoms, ^{4}He atoms, or Cooper pairs. One naturally asks the question: Since, photons – particles of light – are also Bosons, is it possible to get similar phenomena with photons as well? If this is the case than the nature of these phenomena can also be tested by optical experiments. Can Bose-Einstein condensation of photons be observed? Can light behave like a superfluid? The obvious rejection is that photons are quite different than other bosons. In the usual 3D Planck blackbody configuration, which consists of an empty cavity, the photons are massless and there is no number conservation, and the chemical potential is zero. Also photons do not interact with each other, therefore how can one talk about superfluidity?

In expense of reducing the dimensionality of photons to 2D by placing them inside of a Fabry-Perot cavity, the photons can acquire an effective mass. In addition, the cavity can be filled with a nonlinear material exhibiting Kerr type nonlinearity (either self-focusing: attractive interactions or self-defocusing: repulsive interactions). Light propagating in this a Kerr medium behaves like bosonic atoms evolving with a delta-function type nonlinearity. In each case it's governed by a nonlinear Schrodinger equation (NLSE) (or Gross-Pitaevskii equation). This would also imply the existence of a Bogoliubov dispersion relation for the low-lying elementary excitations of a photon fluid.

One may also expect to observe a phase transition with photons in this situation. In 2D, it is not the Bose-Einstein phase transition, but rather the Kosterlitz-Thouless transition, which creates a quasi-long range order. The phase transition only happens for repulsive nonlinearities, as a self-focusing nonlinearity leads to the collapse of the field to a point, or breaking into filaments. One could also experimentally test for superfluidity.

One of the advantages of the optical experiments would be that one could directly sample and interfere the fields, and thus directly measure correlation functions of the system.
Although more than a decade has passed since these ideas were born, no concrete experiment has shown any of the predictions due to technical difficulties. The difficulty mainly lies in generating the initial conditions and sufficient nonlinearity for anything interesting to be observed. However, experiments are still on their way. Here I will explain the basics, expected results and the experimental progress.

Basics

Dispersion relation for free photons in a Fabry-Perot cavity [1, 2, 4]:

We are about to see that the dispersion relation for photons in a short Fabry-Perot cavity is the same as the dispersion relation of non-relativistic massive particles (Fig 1).

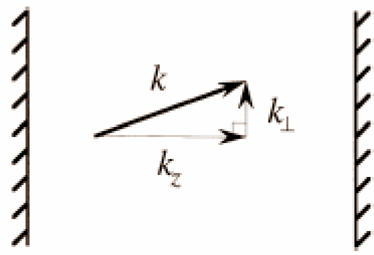

Figure 1 A planar Fabry-Perot cavity. Momentum in z direction is quantized. For a plane wave traveling at a small angle with respect to z axis, one gets the dispersion relation for massive bosonic particles in 2D.

For high-reflectivity mirrors, the vanishing of the electric field at the reflecting surfaces of the mirrors imposes a quantization condition on the allowed values of the *z*-component of the photon wave vector, $k_z = n\ \pi\ /\ L$, where n is an integer, and L is the distance between the mirrors. Therefore the usual frequency-wavevector relation

$$\omega(k) = c\left[k_x^2 + k_y^2 + k_z^2\right]^{1/2}$$

after multiplication with $\hbar$ becomes the energy-momentum relation for the photons

$$\begin{aligned} E(p) &= c\left[p_x^2 + p_y^2 + p_z^2\right]^{1/2} \\ &= c\left[p_x^2 + p_y^2 + \hbar^2 n^2 \pi^2 / L^2\right]^{1/2} \\ &= c\left[p_x^2 + p_y^2 + m^2 c^2\right]^{1/2} \end{aligned}$$

Where $m = \hbar n\pi/Lc$ is the effective mass of the photon. For waves traveling at a small angle with respect to z axis, one can Taylor expand this relativistic relation, and obtain a non-relativistic energy-momentum relation in 2D.

$$p_\perp = \left[p_x^2 + p_y^2\right]^{1/2} << p_z = \hbar k_z = \hbar n\pi / L$$

$$E(p_\perp) \cong mc^2 + p_\perp^2 / 2m$$

Where $m = \hbar n\pi/Lc = \hbar\omega / c^2$ is the effective mass. One can redefine the zero of the energy for convenience, leaving only the effective kinetic energy; $\varepsilon(p_\perp) \cong p_\perp^2 / 2m$.

As long as one is experimentally exciting the fundamental mode (n=1) of the cavity the situation can be represented with a fixed effective mass in 2D. However, when one accounts for the interaction in the nonlinear medium, due to the coupling in longitudinal and transverse directions, one would need a renormalization in the effective mass [5]. We are not going to be concerned with this here.

Interacting photons in a nonlinear Fabry-Perot cavity [1, 2]:

Nonlinearities for photon-photon interactions are in general very small. This makes photons an ideal candidate for treating them as weakly interacting bosons. We are going to be concerned with the zero-temperature case. This problem is widely known as the Bogoliubov problem. The Bogoliubov Hamiltonian reads as follows:

$$H = H_{free} + H_{\text{int}}$$
$$H_{free} = \sum_{p} \varepsilon(p) a_p^{\dagger} a_p$$
$$H_{\text{int}} = \tfrac{1}{2} \sum_{\kappa\, p\, q} V(\kappa) a_{p+\kappa}^{\dagger} a_{q-\kappa}^{\dagger} a_p a_q$$

Here a_p and $a_p^{\dagger}$ are the annihilation and creation operators respectively for the state with momentum p, and satisfy the Bose commutation relations. Note that the momentum p represents the transverse momentum and for convenience we dropped the 'perpendicular' index. H_{free} is the energy of the non-interacting system with $\varepsilon(p)$ given as expressed in the previous section. H_{int} is the interaction between the photons arising from the potential energy $V(\kappa)$ which is the Fourier transform of the potential energy $V(r_1-r_2)$ in coordinate space. This interaction represents the scattering of two particles with initial momenta p and q to final momenta $p+\kappa$ *and* $q-\kappa$. The interaction is created by the atomic vapor inside of the cavity excited to the red side of the atomic resonance with the incident light field. Since the excitation is only near resonant, the population of the atoms mainly stays in the ground state, yet one gets enough nonlinearity. The atomic vapor produces a self de-focusing Kerr non-linearity, corresponding to repulsive photon-photon interactions.

Here we will sketch the main approximations going into the particular Bogoliubov problem and the main results coming out.

It is important to note that the system is an open one, in contact with a reservoir. The mirrors of the Fabry-Perot cavity have large but non-unit reflection coefficients. Therefore a photon that enters the cavity will, bounce off the mirrors many times and stay inside of the nonlinear cavity long enough so that thermalization will occur with other photons (See Fig. 2). The incident light field E_{inc} fills the cavity, and the field, after many bounces leaks out from the right end which is shown as E_{trans}.

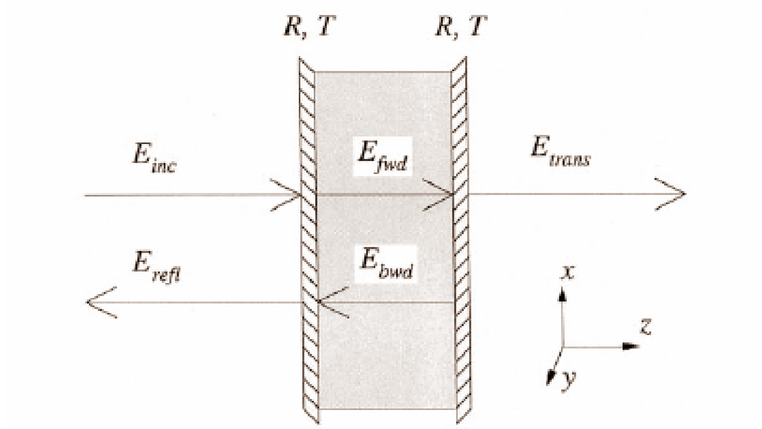

Figure 2 Fields entering and leaving the nonlinear Fabry-Perot cavity.

The number of total particles, i.e., photons, inside of the cavity is not constant. It can fluctuate. In statistical mechanics this is described in Grand canonical ensemble, by subtracting a chemical potential term μN_{op} from the Hamiltonian:

$$H \to H' = H - \mu N_{op}$$

Here $N_{op} = \sum_p a_p^\dagger a_p$ is the total number operator and μ (which we shall determine) represents the chemical potential, i.e., the average energy for adding a particle to the open system described by H.

In the non-interacting case, condensation is expected to happen to the zero momentum ($p=0$) state. The effect of weak interactions is to slightly populate other states too; however there will still be a macroscopic occupation in the $p=0$ state. When this state is macroscopically occupied the operators a_0 and $a_0^\dagger$ essentially commute and both can be replaced by the c-number $\sqrt{N_0}$, where N_0 is the occupation number of the $p=0$ state. In calculations it is useful to separate the zero momentum state out. Then, one sees that the main contribution to the ground state energy comes from the interaction energy of the photons in the $p=0$ state, that is $E_0 = \langle \Psi_0 | H | \Psi_0 \rangle \approx \frac{1}{2} N_0^2 V(0) \approx \frac{1}{2} N^2 V(0)$. The chemical potential is given by $\mu = \partial_N E_0$. Therefore, $\mu \approx N_0 V(0)$ implies that the effective chemical potential of a photon is given by the number of photons in the condensate times the repulsive pairwise interaction energy between photons with zero relative momentum difference.

Here, it is important to note that Bose-Einstein condensation with truly long-range order occurs strictly at T = 0. However it is known that in 2D a quasi condensate can exist with an algebraic decay of long range coherence, and it is believed that a phase transition of the Kosterlitz-Thouless type should be observed with a 2D photon fluid.

Applying the canonical Bogoliubov transformations to diagonalize the quadratic part of the Hamiltonian *H'*, leads us to the non-interacting quasi-particles in the condensate with the well known Bogoliubov dispersion relation for the energy-momentum relation

$$\tilde{\varpi}(\kappa) = \left[\left(\frac{\kappa^2}{2m} \right)^2 + \frac{\kappa^2 N_0 V(\kappa)}{m} \right]^{1/2}$$

The elementary excitations, which are density fluctuations in the fluid described by the dispersion relation above, are phonons. In the classical large phonon number limit these are the sound waves with the speed

$$\upsilon_s = \lim_{\kappa \to 0} \frac{\tilde{\varpi}(\kappa)}{\kappa} = \left(\frac{N_0 V(0)}{m} \right)^{1/2} = \left(\frac{\mu}{m} \right)^{1/2}$$

Putting in some real world numbers, one sees that the speed of the sound is on the order of thousands of times slower than the speed of light.

Now, we will look at some simulations and experimental situations.

Experimental and numerical tests

Dissipative optical flow in a nonlinear Fabry-Perot cavity [3]:

It is extremely important to note that the Bogoliubov dispersion relation also follows from classical non-linear optics. This should be expected since the classical limit of bosonic particles corresponds to classical waves. Similarly, solving the non-linear Schrodinger equation, i.e., the Gross-Pitaevskii equation, (ignoring the many-body nature and using a mean field approximation) for weakly interacting Bose-Einstein condensates usually captures the physics of these dilute gasses. However note that these classical calculations do not give us any hint about the quantum states of the underlying fields, i.e., the statistics of particles; whether it is a coherent state, squeezed state, a Fock state, or a thermal state, etc.

Without going into details I would like to summarize the proposed experiment and the performed numerical tests for the superfluidity and its breakdown in a nonlinear optical Fabry-Perot cavity. The essential modification to the previous cavity that we described is that now, there is a cylindrical obstacle in the cavity as shown in Fig. 3. This obstacle is basically a cylindrical region with a different index of refraction (particularly a defocusing region) as compared to the rest of the cavity. The entire cavity is still filled with a nonlinear material. The waves are incident at an oblique angle θ. This is equivalent to the waves coming at normal incidence and the obstacle moving instead.

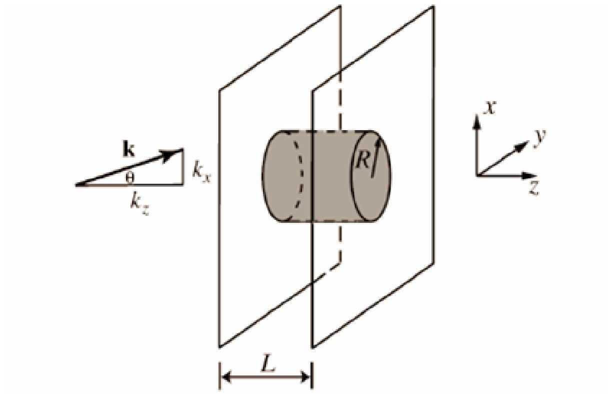

Figure 3 The proposed configuration for observing dissipative optical flow [3].

If the state is superfluid, the obstacle should not experience any drag force as the fluid and the obstacle are moving relative to each other. This condition is satisfied only for time-independent fields where the transverse momentum is constant throughout time. A force is exerted only in the case of a time-dependent field.

Below a certain critical velocity, one expects no scattering off the obstruction. Above that velocity, one would see waves radiating out from the obstruction and/or formation of vortices. Because the velocity depends on density, one could as well change the density rather than the velocity. To test superfluidity, one can send a laser field at an oblique angle, and the field can build up in the cavity. After the transient oscillations due to the build up, one would reach a steady state in the cavity. Given the photon density, if the corresponding critical velocity is above the transverse velocity of the fluid provided by the oblique incidence angle of the field, then the flow around the obstacle should be dissipationless, i.e., superfluidity must be observed. The flow inside the cavity can be monitored by the light leaking out from the right hand side of Fig. 3. The numerical evaluation of the superfluid case is shown in Fig. 4.

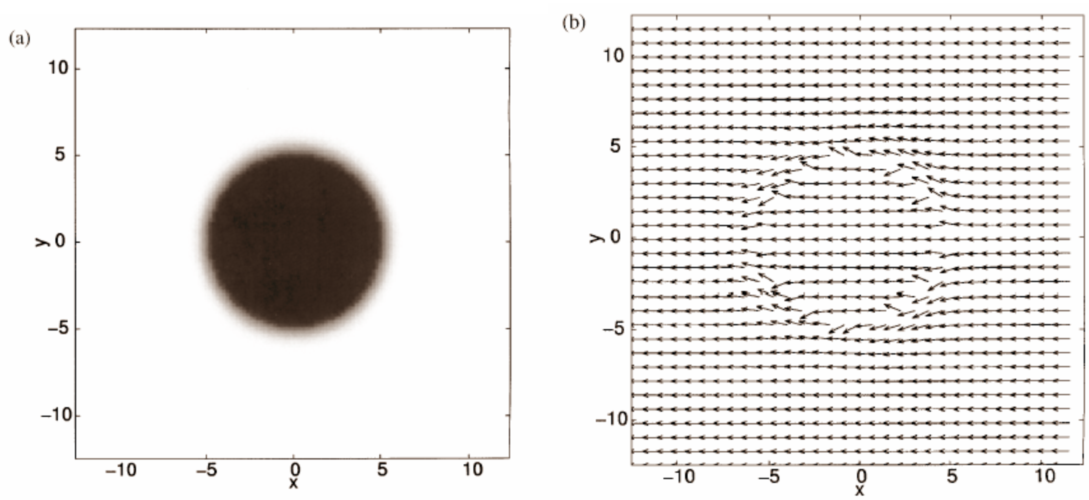

Figure 4 Transmitted field at the right end of the cavity. Superfluid state. Below the critical velocity. **a)** Magnitude; lighter regions correspond to higher intensity. **b)** Direction of transverse gradient of phase.

If the incident laser is now turned of the field inside the cavity is going to decay, which is to say that the fluid density is going to decrease, and the critical velocity is going to decrease as well. At some moment when the critical velocity is lower than the fluid velocity, the numerical evaluation of the cavity output field is shown in Fig. 5.

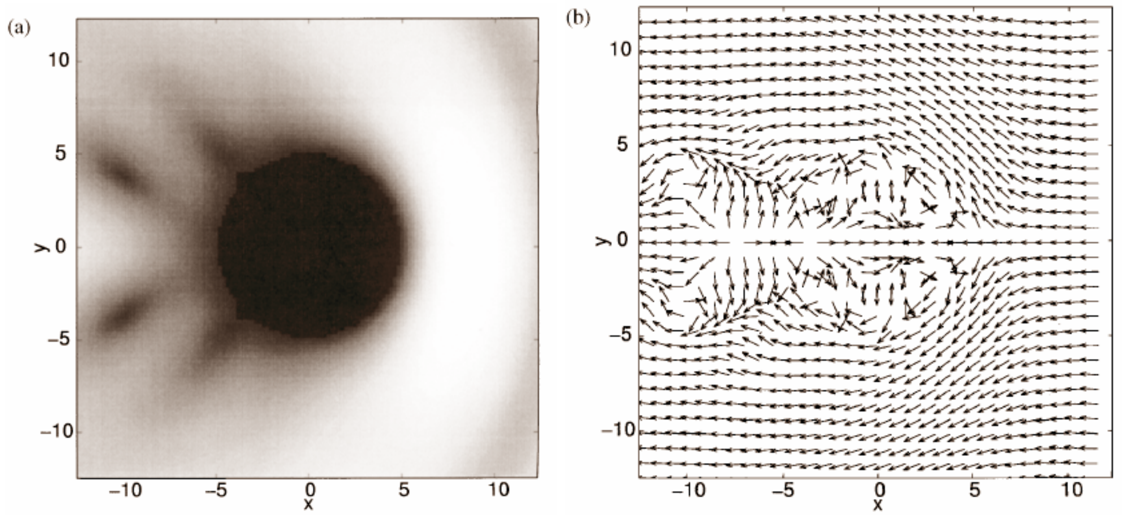

Figure 5 Transmitted field at the right end of the cavity. Dissipative state. Above the critical velocity. **a)** Magnitude; lighter regions correspond to higher intensity. **b)** Direction of transverse gradient of phase.

As evident from Fig. 5, there is scattering and vortex pairs form (small dark spots on the left side of Fig. 5a). At even lower densities, i.e., lower critical velocities, the system approaches to a linear, geometrical optics regime.

Measuring the speed of sound [1, 2]:

One can also measure the speed of sound, therefore the Bogoliubov dispersion relation. One possible experiment is shown in Fig. 6.

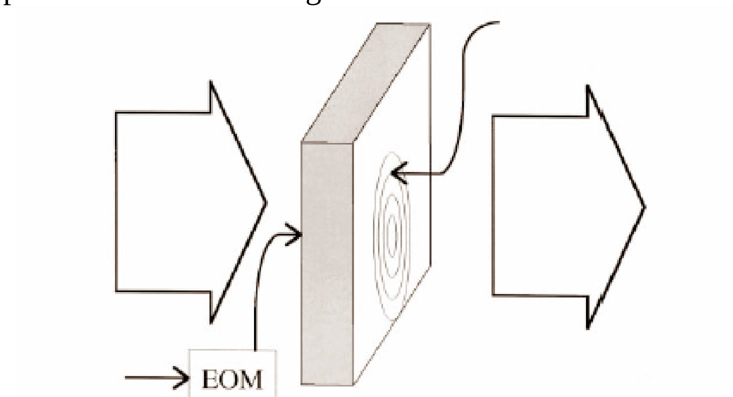

Figure 6 Schematic of an experiment to observe the sound waves in a photon fluid in a non-linear Fabry-Perot cavity. The gray volume is the nonlinear medium.

In this scenario, there are who incident optical waves. The first one is a wide beam to form the nonlinear background fluid. The second wave is supposed to be injected from a single point (from a fiber tip), and should be amplitude-modulated via an electro-optic modulator (EOM) at the sound frequency (corresponding to radio frequencies) to resonantly create sound waves in the nonlinear background fluid, propagating out from a fixed point. The sound waves can be phase sensitively detected by another fiber tip at the exit face. The velocity and the dispersion relation of the sound waves can be obtained from these kinds of measurements, and therefore the critical velocity can be inferred.

One of the problems encountered in this type of an experiment was that only very lately that it was realized that the dispersion relation would be changed by any (coherent) field entering the cavity. Thus in order to do a clear-cut measurement, it would be necessary to fill the cavity with light, and then turn off any driving fields and observe the dynamics. This was beyond the capabilities at the time, so it was not done. However, time has passed, and better control may be achieved.

Achieving sufficient nonlinearities [4, 5]:

There has been work going on in the literature which can achieve sufficient nonlinearities for the photon-photon scattering. One necessity is that the interaction is strong enough and yet still fast enough. Two systems which fit the description are, first, Rydberg atoms in a microwave Fabry-Perot cavity, and second, alkali atoms in an optical Fabry-Perot cavity. Although experimentally, the system of Rydberg atoms have the advantages of relatively large cavity dimensions, and a strong dipole coupling, the obvious disadvantage is that Rydberg atoms are much harder to create and manipulate then alkali atoms in their ground state. With the current experimental values of the nonlinearities in alkali vapors and cavity mirror reflectivities, thermalization of the system should be feasible and interesting effects should be seen.

Conclusion

Clearly light in a Kerr medium is expected to show many of the features of classical superfluids, being described by the same differential equations. Theoretical calculations show that, the photon fluid formed by the many photon-photon collisions in a nonlinear Fabry-Perot cavity should be a superfluid. Furthermore, a Kosterlitz-Thouless type phase transition can be observed in the system described. Also, the Bogoliubov dispersion relation for the low-lying quasi-particle excitations should experimentally observable. The nonlinear cavities for the photon-photon collisions have already been demonstrated. It seems feasible to demonstrate the expected effects.

In this article, we concentrated on repulsive interactions. However, it has also been noted [5] that in the attractive interaction regime, bound states of photons may be realizable, which has no classical counterpart.

Despite all the similarities with the massive bosons, perhaps the most "super" of the superfluid features, the true zero viscosity, may not be present for a photon fluid in a nonlinear Fabry-Perot cavity. Any loss of photons will effect the superfluidity, and there will always be ‘some’ loss associated with an optical nonlinearity.

Acknowledgements

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