Kosterlitz-Thouless Transition in RF Dressed Ultra-Cold Gases

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Abstract

Very low temperature dilute trapped atomic gases form a well known state, the Bose-Einstein condensate (BEC). Recent work has made it possible to adiabatically transfer condensates from 3 dimensional traps into RF dressed traps that hold the atoms in a small region around a specific magnetic field. For a quadrupole magnetic trap, this results in an "eggshell" potential, where atoms' movement is very limited in the radial direction. The addition of gravity causes the atoms to form a simply connected disc at the bottom of the shell. This potential can be used as a test bed for two dimensional physics. In this paper, we will review the basics of the Kosterlitz-Thouless transition. The Kosterlitz-Thouless transition occurs when a two dimensional system with continuous symmetry moves from a quasi-ordered state to a disordered one by the creation of vortices. The transition will then be considered in the context of the two dimensional dilute gas mentioned above.

1 Introduction

The fate of ultra-cold atoms, and systems with continuous symmetries in general, in two dimensions has long been studied [1][2][3]. Long wavelength phonons ruin a true Bose-Einstein condensate for any but that most elusive of temperatures. This was shown in Bugolyubov theory, and can be easily proven. However, in studies of thin films of liquid helium, superfluidity was still observed. The solution to this seeming dissonance is that, while a true condensate is not formed, a sort of quasi-order characterized by a power law correlation function reigned below a critical temperature as shown by Berezinskii [4]. Kosterlitz and Thouless further elucidated the situation by showing that the transition takes the form of an initial quasi-ordered phase ruined by the formation of isolated vortex pairs. As the pairs became more numerous they break apart and form a vortex plasma that eventually ruins any residual superfluidity and order in the system.

In a dilute atomic BEC the situation will be modified further[5]. Because of the finite size of the condensate, there is a frequency cutoff for the phonons that destroy order[6]. It was predicted that for temperatures far below the Kosterlitz-Thouless transition, there should exist a true BEC. As the temperature is increased, vortices were expected to appear. Experimental verification of vortex formation in quasi-two-dimensional systems has only very recently been achieved by the group of Jean Dalibard[7]. They show evidence of phase dislocations by interfering nearby pancakes in a one dimensional array.

In this paper I will suggest a different method to observe vortices in a quasi-two-dimensional geometry. RF dressed potentials mingle atoms and light to cause distortions in the effective potential an atom experiences[8]. One instance of these potentials is essentially two dimensional. In addition to the fact that a single pancake can be created, as opposed to very many, the parameters of the cloud can be changed very quickly, on a timescale faster than the dynamics of the system. This opens opportunities to implement different imaging techniques, which may reveal new characteristics, previously undiscovered.

2 The Kosterlitz-Thouless Transition

2.1 Two dimensional system

A two dimensional system is one in which the system's important physics are produced by degrees of freedom that are limited to two spatial dimensions. In atomic physics, one criterion for two dimensionality is that $K_B T \ll \hbar w_3$ where w_3 is the trap frequency in the third dimension. When the atom's thermal energy is less than the energy of the excited mode in the third dimension we can consider the direction frozen out. States that include an excited state in the third dimension are not substantially occupied and contribute to the system in an arbitrarily small amount as the frequency of the trap in the third dimension is increased. When loading a trap with a Bose-Einstein condensate, the temperature is significantly lower than the trap frequency energy, thus if the other two directions have much lower frequencies, the fluctuations can be thought of as two dimensional.

2.2 Order

A long range ordered state is one that exhibits correlations across the entire system. As an example of the different types of order that will be considered in this paper, imagine a football stadium filled with fans (it is unimportant for the purposes of this paper what kind of football). At the beginning of the game the stadium is in ordered state with everyone sitting down. When a goal is scored by the home side everyone stands up and cheers. Both the sitting and standing state exhibit long range order because the state of a person sitting in *section A* will predict the state of a person in *section LL*. If the game gets a little boring, the crowd may start a "wave". This is a state in which a group of supporters stands up and then sits down followed directly after by the same action in the people on their left. As the perturbation moves around the stadium we can see that the state is still ordered because, on average, the state of someone predicts the state of a different person very far away. As the match grows more intense, a person who has a strong interest in the game (perhaps a gambling wager) may become so nervous that he or she stands up to get a better view. The person in the next row back, who may previously, before the game became so intense, have been content to continue sitting with an obstructed view, now decides to stand as well. The process plays out in a similar way with the surrounding people. The intense nature of the game and a gambling addict (as an analogy to quantum or thermal fluctuation) has now caused a chain reaction that created a whole section of fans standing. We now can observe that the long range order of the system has been broken. We can no longer predict the state of a person far away by the state of a person close by. However if a person is still sitting, there is a good chance the people very close to him or her are also still sitting. This state is analogous to what we will call a quasi-ordered state. The correlation obeys a power law. This means that domains of all sizes will exist throughout the stadium (in the thermodynamic limit). At the end of the game everyone will grab their belongings and start to get up and head for the exits. Now whether a person is sitting or standing is completely uncorrelated with the people around them. This is analogous to what we call a disordered state.

2.3 Long range order in a 2D system with continuous symmetry

Order must be stable against fluctuations, otherwise the fluctuations will become so numerous that order will be lost. For a system with continuous symmetry, we can use an XY model. In this model, a lattice of spins is coupled such that the spins of neighbors have a minimum in energy the spins point the same direction in the plane. If this system were in 3 dimensions the energy associated with a fluctuation in the system grows as $E \cong \frac{L^3}{2\pi L^2}$. In the thermodynamic limit these fluctuations become very energetic and do not occur at low temperatures. In one dimension, the energy is $E \cong \frac{L}{2\pi L^2}$. For a very large system, these fluctuations cost no energy, and long range order cannot be maintained. The case of two dimensions is borderline $E \cong \frac{L^2}{2\pi L^2}$. Thus the energy remains a constant as the system grows. What results is a state that exhibits a quasi-ordered state with a power law dependence ($G \sim r^{-\eta}$). The fact that this state is confused about what it wants to do makes it interesting.

3 Vortex formation in a 2D BEC

In a system defined by an order parameter, certain types of defects cannot be changed by local deformations. A specific type of topological defect is a vortex in two dimensions. Bose-Einstein condensates have an order parameter defined as a complex number that can be written as $A(r)exp^{i\gamma(r)}$ where $\gamma(r)$ is the phase of the order parameter, and A will be called the amplitude. One configuration of the order parameter has a constant amplitude over most of the plane and a phase that makes an integer (m) trips from 0 to 2π as it winds around a closed loop once. The order parameter must satisfy the condition that $\gamma(r(\theta) = mod_{2\pi}(\gamma(r(\theta + 2\pi))$. As the distance from the origin increases the phase still must wrap the same number (m) of times around a loop that contains the first loop. When the loop gets smaller it must either cross a point where the amplitude vanishes, or it reduces in size till the whole $m \times 2\pi$ wraps around in an infinitesimal point. This point must have zero amplitude because of the stipulation of a continuous order parameter. The point where the amplitude falls to zero is the vortex. Forming a vortex costs energy. One can either think of this energy coming from a coupling between adjacent spins in a lattice (a liquid crystal for example). In the context of a dilute atomic gas this energy is most easily thought of as coming from momentum $p = \nabla\gamma$. The topological defect can be thought of as a spinning fluid with a whirlpool in the center. There is an energy associated with this momentum proportional to p^2. Also, we can think of our winding number as an angular momentum quantum number, which must be conserved.

$$L = r \times p \tag{1}$$

$$\Rightarrow p = L/r \tag{2}$$

$$\Rightarrow p^2 = L^2/r^2 \tag{3}$$

$$\tag{4}$$

integrating the energy in a closed loop in the plane

$$\int_0^R E(r)rdr \simeq \int_0^R dr/r \simeq ln(R) \tag{5}$$

The energy associated with the creation of a vortex increases logarithmically as the size of the system increases, and in the thermodynamic limit, the energy is unbounded. To calculate whether a vortex will form we must calculate the free energy difference when we form a vortex. The free energy is of course $F = E - TS$. The entropy also turns out to be proportional to $ln(R)$. Thus the free energy of the vortex is

$$F_v = J\pi ln(\frac{R}{a}) - K_B T ln((\frac{R}{a})^2) \tag{6}$$

Thus one would expect a vortex to form when $K_B T = \pi J/2$. This simple calculation fails when we consider the possibility of the simultaneous creation of two vortices. In the case of a single vortex, we showed that even at very large distance the order parameter still retained angular momentum and energy associated with the vortex. If two vortices were to form that line integral of the phase around the vortices could be zero if the two vortices had opposite charge. Since the perturbation to the system dies out in a finite distance, the energy associated with forming these two vortices in much less than before. From the calculation of free energy, the system forms vortices at lower temperature. The energy of many vortices can be calculated

$$H_v = -\pi k \sum_{|r-r'|<a} s(r)s(r')ln(\frac{|r-r'|}{a}) + E_c s^2(r) \tag{7}$$

This equation is equivalent to a two dimensional two component Coulomb gas with a chemical potential E_c.

As more and more of these vortices are created they eventually come unbound from each other. To continue the analogy to the screened Coulomb gas, it will become a vortex plasma. This is the nature of the Kosterlitz-Thouless transition. On one side there is a quasi-ordered fluid and on the other we have something akin to a disordered vortex plasma.

4 Trapping Bose-Einstein Condensates in 2D

4.1 BECs as a starting point

To trap a dilute gas, one must create a potential that is deep enough to keep the atoms from escaping by thermal excitation (i.e. $K_B T < V(r_{max})$). When atoms start out in a Bose-Einstein condensate , the ultra-low temperature makes it possible to adiabatically fill the ground states of other potentials that may or may not support off diagonal long range order. An example of this is a the Mott insulator - superfluid transition. When a BEC is in a harmonic trap, it is a superfluid. However, if a optical standing wave is imposed on top of the harmonic potential, the atoms will undergo a phase transition to an insulating state, where there is no long range order in the phase, but rather, number squeezing at each site in the optical lattice. BECs are a good starting point to explore other quantum systems that require a very low temperature.

4.2 Recent Experimental Work

The group of Jean Dalibard has recently shown that vortices are created in a two dimensional gas of atoms. They accomplish this by first creating a stack of pancake shaped traps stacked next to each other. The traps are made two dimensional by carefully dealing with the tunnelling between traps. At the temperatures studied, a few vortices will form in the small clouds. The group was unable to directly image the vortices because the clouds expand so quickly in the "frozen" direction and so slowly in the plane which constituted the 2D world. The vortices remain very thin but get long, making them hard to see. Instead of direct imaging, the group interfered the clouds and relied on the fact that the topological defects cause phase winding over a much larger area than the depleted core of the topological defect. The detection method is somewhat indirect and does not give great information about the number, orientation, or dynamics of the vortices.

4.3 Dressed atoms

We will consider now putting atoms into a 2 dimensional trap by transferring the atoms into a "dressed state". A dressed state is created by applying an external oscillating field. The oscillating field changes the eigenstates of the atom. For example, if an atom were originally in state A, it might become one half state A and no photon mixed with one half state B plus a photon. We can also think of this change not occurring in the atoms but rather, the state of the atoms stays the same, and the shape of the potential the atom sees changes.

To understand why the potential changes, lets think about what happens when an atom is "flipped" and expelled from a trap. As it travels through the trap comes to the point in the magnetic field where the difference in energy from being in the "up" state (for example) vs. the "down" state is equal to the energy from the RF photon. At this point the atom can undergo a Landau-Zener transition. The harmonic trap that normally looked like a parabola, now has its edges flipped over and pointed downward (see figure). On the other hand the energy manifold that was once the anti-trapped state, now becomes the mirror image of the other energy state. Near the point where the Landau-Zener transition occurred in the in the initially untrapped states, there is now a tight trap. The tightness of this trap can be controlled by raising and lowering the amount of power in the RF field. The position of the new trap can be adjusted by changing the frequency of the RF field.
The new trap occurs everywhere the magnetic trap has a specific magnitude. For a 3D harmonic trap, the locus of a certain magnetic field magnitude will fall along the surface of an ellipsoid, or on an "eggshell". The addition of gravity to the equation adds a twist, the atoms will all fall to the bottom of the eggshell. In practice the atoms appear in a very thin very flat bowl shape below the center of the trap. By adjusting the strength of the RF field, one can change the thickness of this pancake. However, the heating goes up and

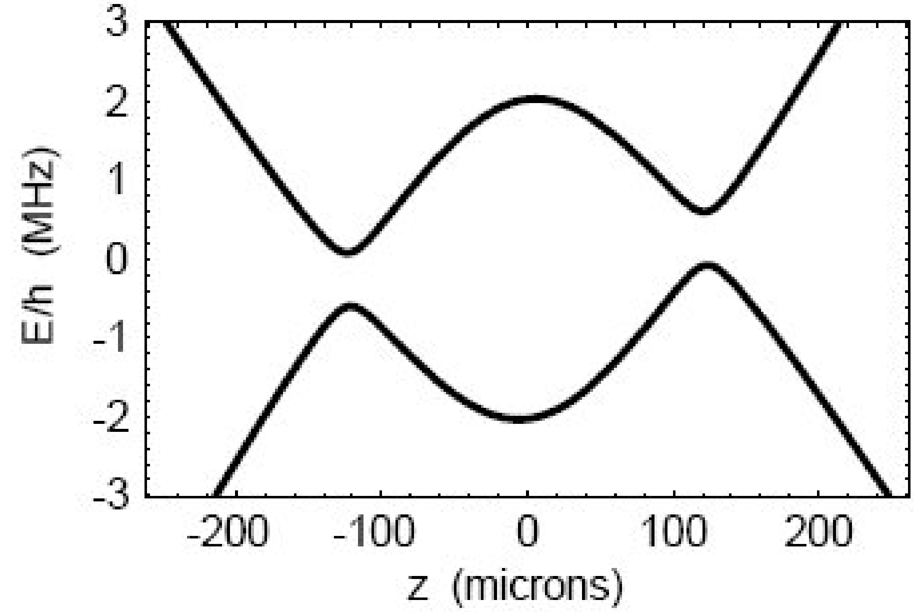

Figure 1: an RF dressed potential.

the lifetime of the cloud decreases rapidly when the trap is made thin.

4.4 Suggested new method using RF technique

Based on research recently performed in our group at UIUC, I believe that the type of machine in use in our lab may be capable of exploring more directly the Kosterlitz-Thouless transition. The group of Dalibard was not able to image vortices directly because they expand so quickly in the direction normal to the pancake. If one were able to slow down the expansion of the (lets call it z) z direction, the XY direction could expand significantly and allow the vortices to be directly imaged. With the RF technique we have control over the width of the pancake, by quickly ramping the RF power up before imaging, we can adiabatically expand the cloud in the z direction and thus slow its time of flight expansion. If the vortex dynamics don't change much over the time scale of adiabatic expansion, one would still be essentially imaging the smaller scale vortex. In addition, by increasing the frequency at the same time, one would expand the radius of the ellipsoid that the cloud sits on, thus increasing the XY extent of the cloud. The core size of the

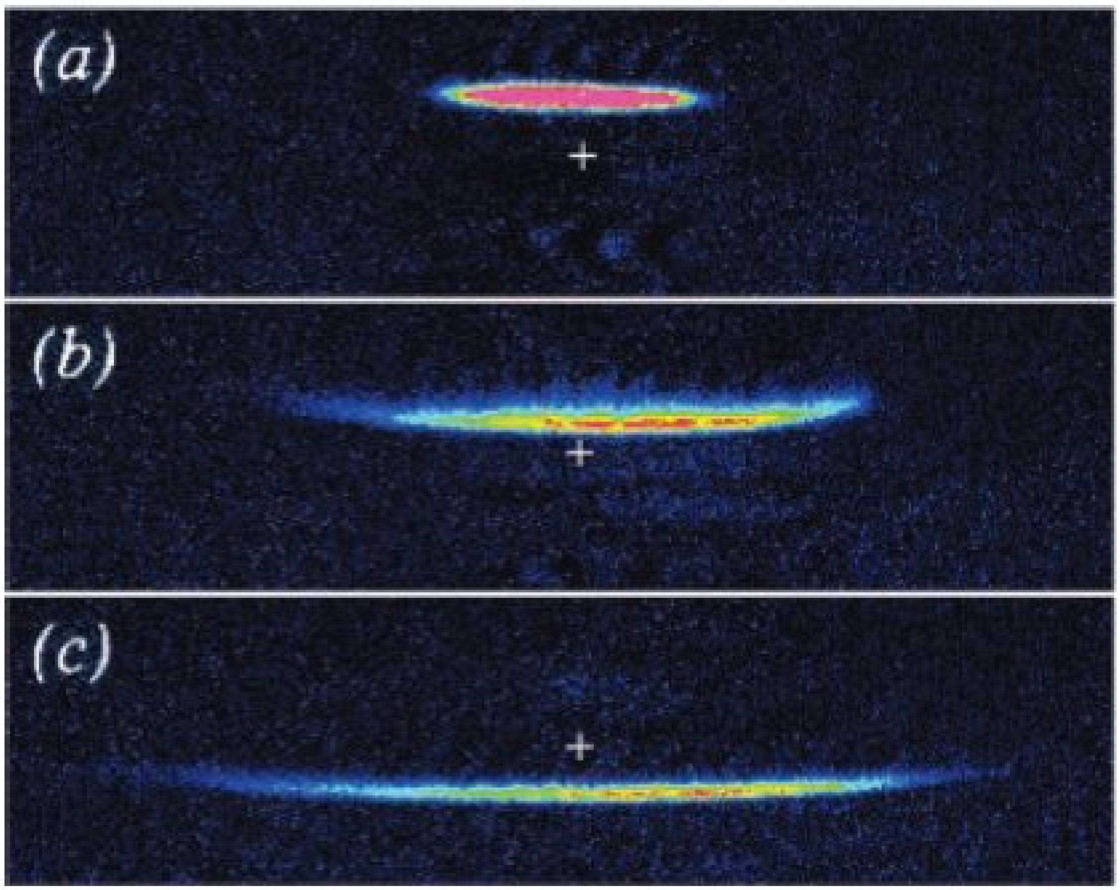

Figure 2: pictures of BEC in an RF dressed trap at different RF frequencies

vortex would increase, again allowing for more direct imaging.

5 Conclusion

The Kosterlitz-Thouless transition is a widely studied and interesting physical phenomenon. It has been observed in a wide variety of systems with continuous symmetry. The techniques being developed with Bose-Einstein condensates have recently been able to study the KT transition directly. A new RF technique suggested in this paper may be helpful in studying more phenomenological behavior that was previously unaccessible. The information gained from the unique abilities of ultra-cold atoms will augment the already well developed body of knowledge on this subject and perhaps provide critical insight and lead to new discoveries.

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