Emergence of Altruism is Public Goods Games

Barbara Stekas
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Much of evolutionary and economic theory is based on the idea that humans, animals, and even genes are inherently selfish and make decisions based on what is in their own best interest. But we know that humans and animals make self-sacrificing choices regularly, not just for the benefit of their own progeny but also to help complete strangers, through charity for example. This paper will provide an overview of public goods games (PGG) and how altruism, though harmful in the short term to the practitioners, tends to be beneficial overall . It will also explore whether altruism is evolutionarily favored and what factors can lead to its dominance or failure compared to selfishness in these models.

Introduction

Modern theories in biology and economics depend on the self-interested behavior of the subjects involved. This makes perfect sense, of course. If, for example, one individual in a population helps another by sharing food, they have less food for themselves. Their lack of nutrition would make them less capable of competing for other scarce resources, they would die, and their genetic propensity for altruism would not be passed on. Or, if altruism were a strategy rather than an inherent trait, others would note the negative effect and be less inclined to share their own food. Either way, it does not pay to be altruistic. Evolutionary theory grants some leeway for the protection of offspring and relatives in order to pass on genetic information, but there is no accounting for the random acts of selflessness present every day. From extreme examples, like risking your own life to save a stranger, to the mundane, giving money to a homeless person on the street, our propensity to help others is an important part of what makes us human. Why do we sacrifice ourselves to help others in the first place? And if only the fittest and most selfish survive, how can altruism be so common?

Public goods games offer a way to explore effects and emergence of cooperation in populations. Simulations indicate that altruistic players may be more successful in the long run than purely selfish players. Under certain conditions altruists not only survive, but also thrive and can emerge as the dominant strategy in a large population. This paper will investigate different methods that provide for the emergence of altruism in public goods games and examine their strengths and weaknesses.

Public Goods Games

In public goods games (PGGs), participants can opt to contribute a given amount to a central pot or give nothing. The pot is then multiplied by some amount and split between all participants. Those that contribute are called cooperators and that do not are called detractors. In these games, the pay out is the same for cooperators as it is for detractors, however the cooperators incur an added cost and so reap a smaller reward than detractors. This is a classic case of the prisoner's dilemma and best individual strategy is to detract. However, if all players detract then none benefit, presenting a social dilemma.

In evolutionary PGGs, players are assigned a strategy and a vertex on a graph, with each individual interacting only with those it is connected to. The benefit received is b, the cost to cooperate is c, and number of players an individual is connected to is k. The social dilemma depends on $b>c$. If a cooperator is connected to i other cooperators their pay off is $bi-ck$. Alternatively, for a detractor connected to j cooperators will receive a pay out of bj. After each round, some fraction of individuals will have the option to change their strategy. The probability of switching from strategy x to strategy y is generally given by

$$W(x \leftarrow y) = \left[1 + \exp((P_x - P_y)/\kappa\right]^{-1} \qquad (1)$$

where κ is a noise factor that allows occasional irrational choices. Alternatively, some models have the new strategy adopted simply if $P_y > P_x$. In either case, the individual is more likely to choose the strategy with the highest payoff.

Taking *k* to be constant, it can be shown that cooperation survives if *b/c>k* assuming that the total number of players *N>>k*. Consider that the payoff for a cooperator is $P_C = bq_{CC}(k-1) - ck$ and the payoff for a detractor is $P_D = bq_{CD}(k-1)$ where q_{CD} and q_{CC} is the probability that a cooperator will be next to a detractor or another cooperator respectively. By pair approximation[1] $(k-1)(q_{CC} - q_{CD}) = 1$ and we find that $P_C - P_D = b - ck$, clearly indicating that cooperation is more likely when *b/c>k*.[1] This bears out in the simulations shown in Figure 1 for different connectivities. The top line gives examples of graphs. The bottom two lines show the proportion of times cooperation won out over detraction compared to *b/c*. Runs were performed at different average values of *k* and total number of players *N*. The relation also shows that for highly connected or "well-mixed" graphs, detractors will necessarily beat out cooperators. For this reason, we look to more locally connected graphs to examine cooperation emergence.

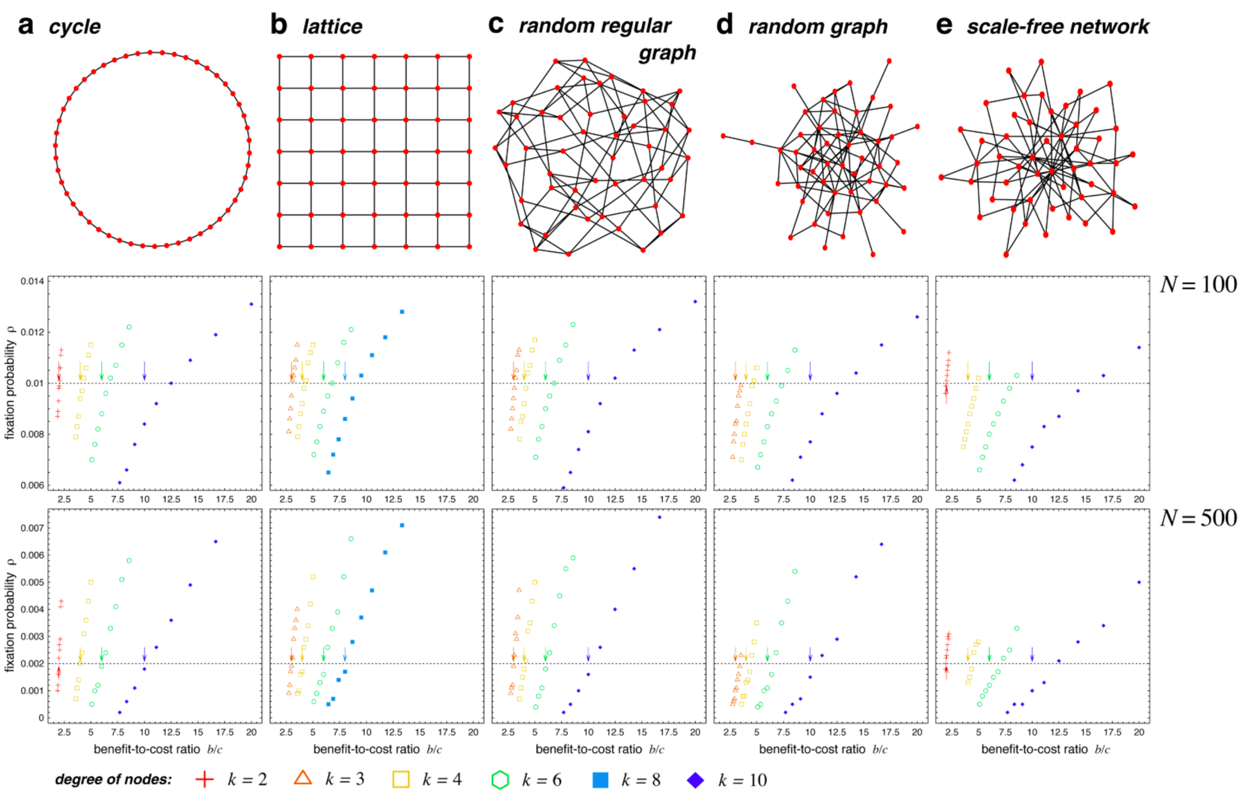

Figure 1 - Ohtsuki et al. [1] – Sample networks are shown at top. Below the corresponding graph are runs with population size N plotting fixation probability of cooperation vs. *b/c* for various values of *k* indicated by the colors above.

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1 Pair approximation is a mean field theory that tracks the frequency of strategy pairs. It will not be covered in this paper, but results do appear in reproduced graphs. Full explanation in [2]

Spatial Variation of Connections

A very simple graph to begin with is the square lattice with k = 4. We will introduce a new quantity $r = b/c$ that is the multiplication factor that amplifies the input of the cooperators. Hauert and Szabó look at this graph using Monte Carlo starting from a random initial configuration.[2][4] With each iteration, some randomly drawn players are selected to update. Player x compares it's strategy to a random neighbor y and accepts y's strategy with the probability in Eq. (1). Their results, displayed in Figure 2, show that cooperators do survive for small values of *r*, but below a threshold r_c a phase transition occurs and detractors dominate entirely. Figure 3 shows that near r_c cooperators are found in small clusters, islands in a sea of detractors. In clusters, interior cooperators only interact with other cooperators and so gain more from the PPG than cooperators on the boundary playing against detractors and more than those detractors. They gain sufficiently more to stave off invasion until r_c at which point the gains on the interior cannot offset the cost on the boundary and detractors creep in.

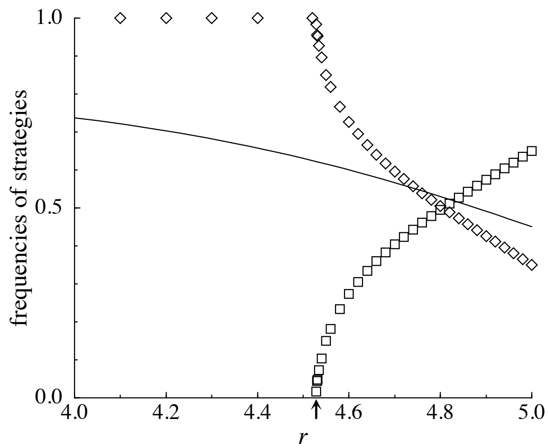

Figure 2 -Hauert and Szabó [4] –Frequency of cooperators (squares) and detractors (diaomns) vs. r
The line represents the frequency of defectors in pair approximation

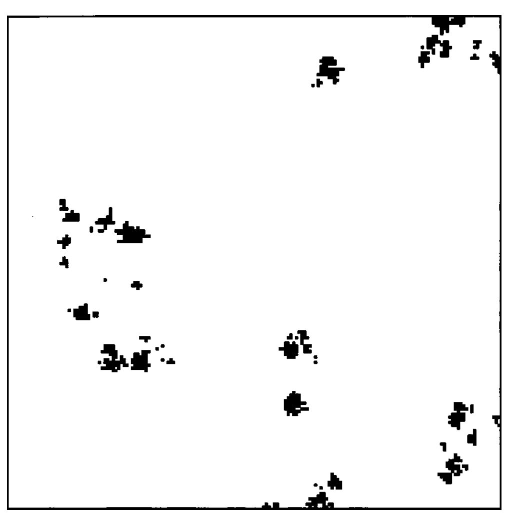

Figure 3 - Hauert and Sazabó [2] – Spatial configuration of cooperators (black) and detractors (white)

A bit more complex is a regular small world network (RSW), which has the same number of connections (k=4), but a certain fraction reach beyond the nearest neighbor of the simple lattice. More complex still is the random regular graph (RGG), which again has the same number of connections, but here all the connections are randomly selected from the entire population, not just the nearest neighbors. The two graphs have increasing the spatial variety of the connections. Examples are shown in Figure 4 below.

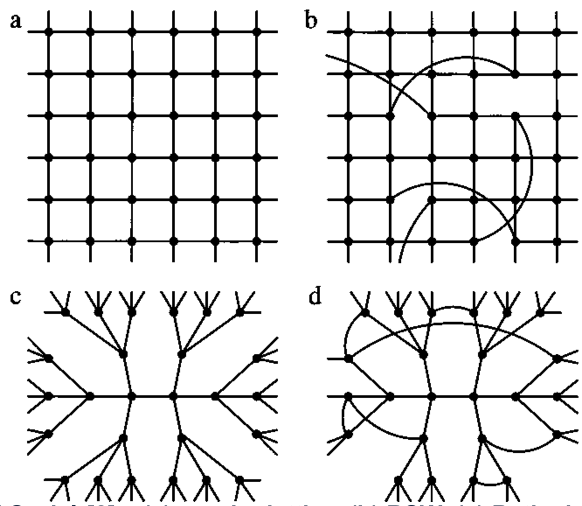

Figure 4 - Hauert and Szabó [2] – (a) regular lattice, (b) RSW, (c) Bethe lattice or tree, (d) RRG

One might expect that for the heterogeneous graphs the proportion of cooperators would decrease, as their interactions are no longer confined to clusters of neighboring cooperators. However, this is not the case. Cooperators actually have better chances on the random regular graphs.[2] The cooperators hold on for smaller benefit-to-cost ratios and the nature of their eventual extinction changes. Figure 5 shows the sharp fall off of the lattice compared to the more linear decrease of RSW and RRG graphs. It is also evident that our results are highly sensitive to topological features of the graph.

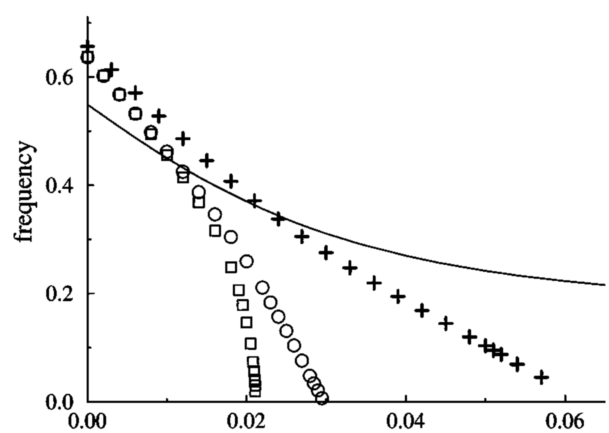

Figure 5 - Hauert and Szabó - Frequency of cooperators vs. 1/*r* for the lattice (squares), RSW (circles), RRG (+), and predictions of pair approximation (line)

Social Diversity

To this point, the graphs introduced have had a constant connectivity, *k*, between all vertices. Santos et. al. have introduce heterogeneous graphs that include highly connected individuals alongside sparsely connected ones; closer to true human interactions in their diversity.[3][5] Their graphs are scale-free, meaning the proportion of vertices with k connections $P(k) \sim k^{-3}$. In this case, the cost-benefit measure must be renormalized to reflect the varied degree of connectivity. Santos et. al. use a renormalized enhancement factor $\eta = [r(z+1)]^{-1}$ where z in the average connectivity of the graph. Their results are shown in Figure 6. Also shown is a drawing of the various PGGs at play for a particular individual and their cost to a cooperator in two different cases.

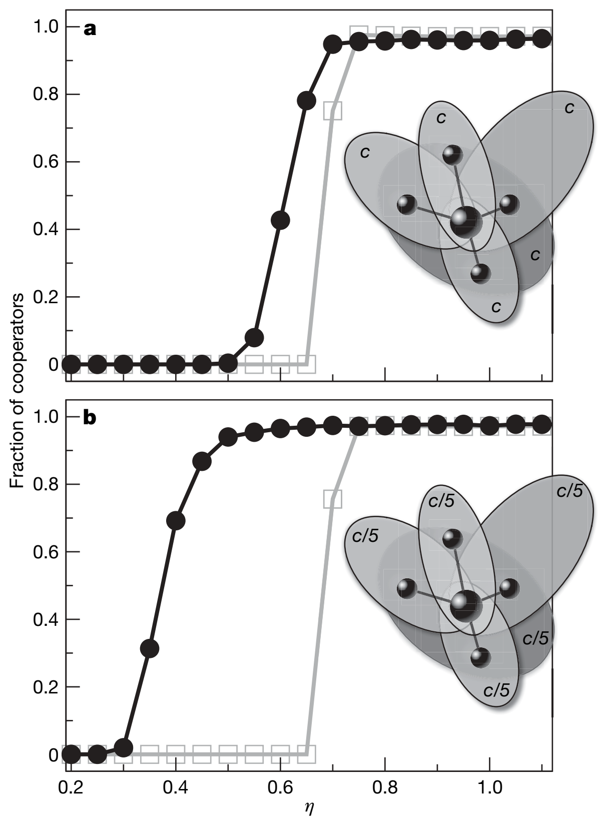

Figure 6 - Santos et. al. [3] - Fraction of cooperators vs. η for scale-free (black line) and regular (grey line) graphs. In all cases z=4. (a) Fixed cost model. (b) Shared cost model.

Figure 6a shows the added benefit of social diversity to cooperation compared to the previously discussed regular graphs. In this fixed cost method, cooperators begin to dominate the population at smaller enhancement values.

However, there is no reason that an individual must contribute the same amount to every game. Figure 6b shows the results of cooperative contributions that depend on group size; the cost is $c/(k+1)$. Here there is a large improvement of cooperation compared to regular graphs and the fixed cost, scale-free graph. In this case, the payoff of an individual is not only determined by their designation as a cooperator or detractor, but also by their connectivity. Highly connected individuals are natural advantage over the sparsely connected and it happens that large hubs tend to be vulnerable to cooperator take over. Consider a defector hub. Its payoff or fitness will increase more dramatically than the less connected vertices around it. Hence, its neighbors will begin to convert to detractors as well. This produces negative feedback as it leaves the hub in a sea of detractors that contribute nothing to the game, decreasing the hub's fitness and leaving it vulnerable. Figure 7 illustrates this phenomenon for a simple, 2-hub graph. In this way, cooperation can dominate quite quickly.

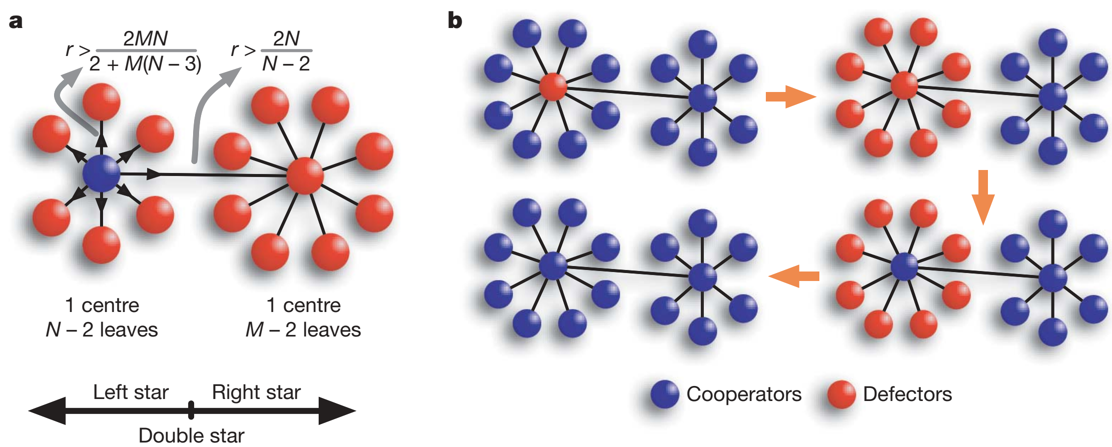

Figure 7 - Santos et. al. [3] - Detractor hub take over. (a) The initial scenario, r is the value above which the hub's fitness exceeds all of its neighbors. (b) Evolution of the hub.

Voluntary Participation

Another variation on the standard model allows players, known as loners, to opt out of participation. Cooperators and detractors still engage in their games while loners sit out and receive a flat benefit σ, where σ must be smaller than the payoff of two cooperators, but larger than zero (the payoff of two detractors). The loners produce a Red Queen effect, in which all three types of players coexist in a dynamic equilibrium for certain values of r.[2][4][6][7]

The loner alternative provides a balance natural balance to the system. When there are many detractors in the population, it becomes more favorable to opt out and become a loner. When detractors become scarce, cooperation is more favorable, because there are no free riders to split the benefits with. And when cooperators begin to dominate, more detractors will pop up to take advantage and the cycle will repeat.

Figure 8 shows sample trajectories of the evolution of frequencies of cooperators, defectors, and loners in the PGG that bear this out. Well-mixed populations, shown in Figure 8a, relax to an all loner state rather than the all detractor state seen earlier. Figure 8b shows RRG trajectories that could potentially end on any of the three absorbing states, but tend to end in favor of the loners. Figure 8c represents RSW networks and leads to an asymptotically stable limit cycle, oscillating between the three strategies. The regular lattice case is in Figure 8d and evolves toward a stationary state with all three strategies coexisting. Figure 9 shows the change in the frequency with time between the three strategies in the oscillatory limit of RSW networks. Notice the succession of dominant strategies is as described; large numbers of cooperators cause an increase in the number of detractors, more detractors paying cause more loners to opt out, and the lack of PGG players bring out the cooperators once again.

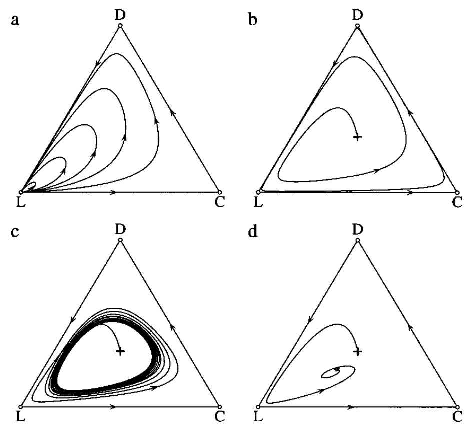

Figure 8 - Hauert & Szabó - Evolution of frequencies of loners (L), defectors (D), and cooperators (C). (a)Well-mixed case. (b)RRG case. (c)RSW case. (d)Lattice case.

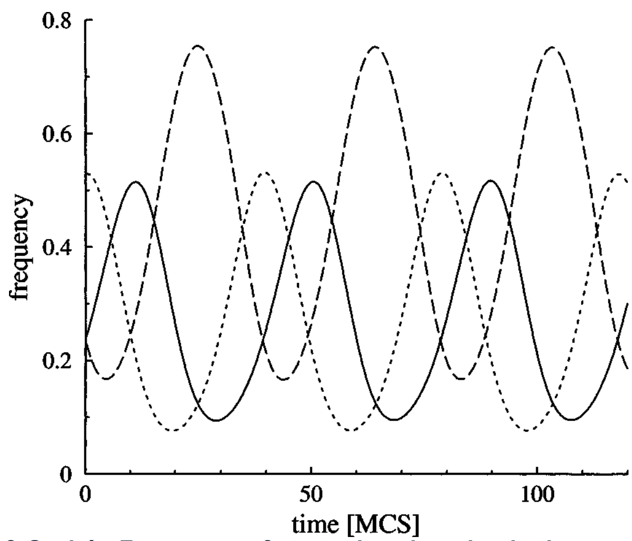

Figure 9 - Hauert & Szabó - Frequency of strategies changing in time on an RSW network. Cooperators (dotted line), detractors (solid line), and loners (dashed line)

The value of r also has a significant effect on the relative dominance of strategies. In the simple lattice case, there are three distinct regimes. For a significantly small value of r, the cost will be too high to allow cooperators. Therefore below a transition point, r_D, loners will dominate as cooperators disappear since opting has a larger payoff than detracting and receiving nothing. At a significantly high benefit level, r_L, cooperation will reign as in the compulsory case since defection will not pay off and σ is smaller than joint the cooperative case. For $r_C<r<r_L$, the three strategies coexist. Figure 10 clearly shows these

three regimes and compares the compulsory case (A) to the voluntary case (B). It also shows at in all cases the cooperative strategy provided the highest payoff. This is somewhat surprising because to start we determined the best paying option in the prisoner's dilemma was to defect. Figure 11 shows the spatial configuration in the $r_C<r<r_L$ regime. Notice the distinct regions strategy regions that appear to invade each other for dominance.

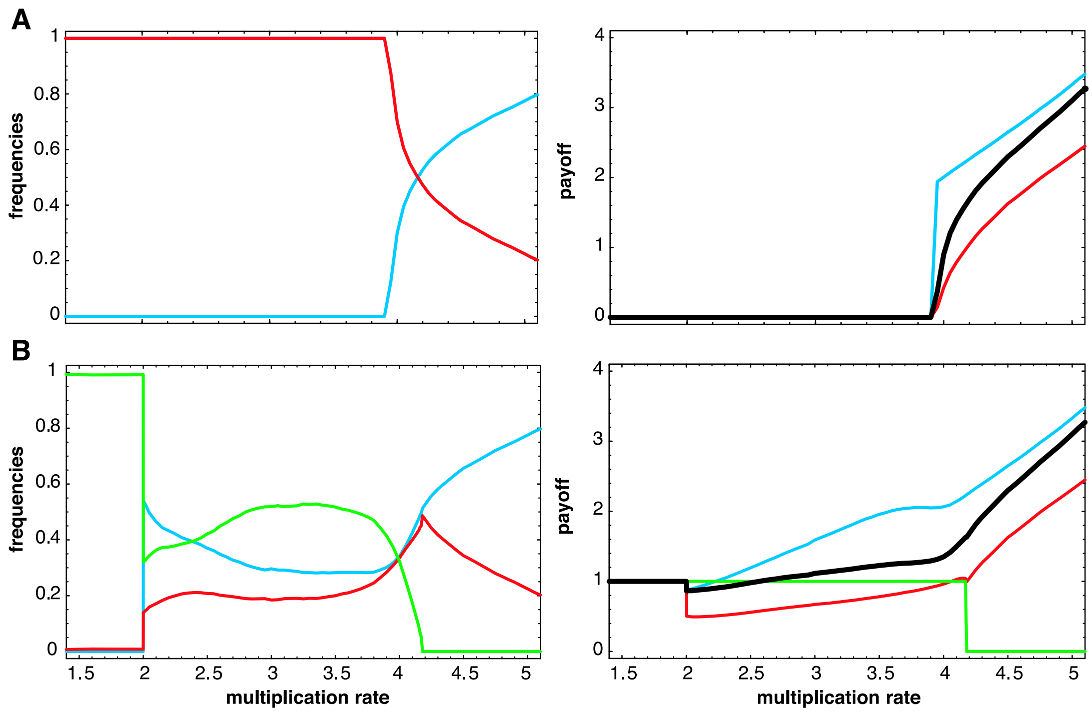

Figure 10 - Hauert et. al. [6] – Average frequencies and payoff of each strategy plotted vs. r in the (A) compulsory case and the (B) voluntary case. Cooperators (blue), Defectors (red), Loners (green), Average payoff (black)

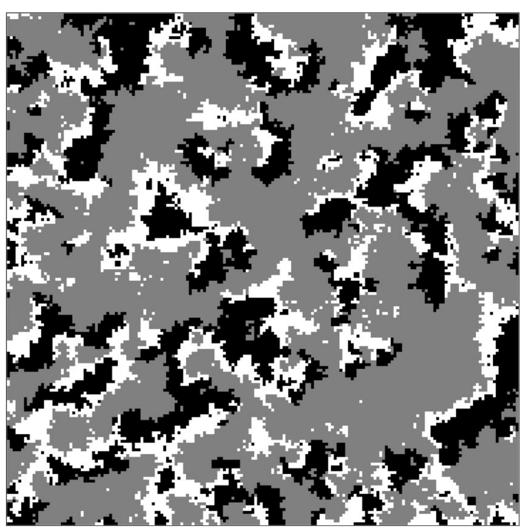

Figure 11 - Hauert & Szabó [2] - Snapshot of the spatial configuration of cooperators (black), defectors (white), and loners (grey) in a state of dynamic equilibrium

Conclusions

Public goods games present a social dilemma in that if all participants did what is individually most beneficial, no one benefit at all. To examine the problem, we assigned individuals randomly to be selfish detractors or altruistic cooperators and ran simulations to see which emerged as the dominant or best choice. It turned out that for large enough payoffs, the cooperators would outnumber the detractors despite the initial conclusion that rational selfishness was the safest bet. We then layered that with non-local spatial connections and variation in vertex connectivity and found that heterogeneous graphs produced a stronger and stronger tendency toward cooperation. And when participants were allowed voluntary participation, this allowed for dynamic equilibriums that cycled through the possible strategies and allowed cooperation to survive under conditions of lower r.

These models are necessarily simplistic and cannot account for emotions or similar human traits, but it is interesting that without changing the fundamental rules of the game, we were able to see surprisingly decisive results that show a dominance of cooperation. Just rearranging connections between players and the choice or whether to play or sit out, significantly changed the results of the game.

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