

Pairing in cuprate superconductors

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Abstract

Despite 25 years of intense effort and a wealth of experimental data, no clear consensus has emerged regarding the mechanism that allows pairing in the cuprate superconductors at such unprecedentedly high values of T_c . Nevertheless, it is now known, in part through the dramatic, qualitative results of dc SQUID measurements, that the d-wave pairing states are dominant in the cuprates. This result, and the constraints it places on possible microscopic theories of high- T_C superconductivity, are discussed. The $SO(5)$ model and the resonating valence bond model are described. Both models lead to an effective t - J model, the implications of which are discussed.

1 Introduction

The first of the cuprate class of high temperature superconductors was discovered in 1986 by Bednorz and Mueller [2]. Although they were awarded the Nobel Prize in Physics the next year for their discovery [6], the microscopic mechanism that allows superconductivity to persist at such high temperatures remains unknown. Initially, a wide variety of explanations were offered, but it is now generally accepted that, as in conventional superconductors, the superconducting state is caused by the formation of Cooper pairs [5]. It is also well known at this time that these pairs have zero total spin [5, 7].

The cuprate phase diagram has several universal features independent of the chosen compound. The parent compound is a Mott insulator. In hole-doped superconductors, the antiferromagnetic state associated with the Mott insulator persists over a small doping range to relatively high temperature. The superconducting state emerges as the dopant concentration increases and the antiferromagnetic state disappears. The superconducting state persists over a wide range of dopant concentrations. The material is said to be optimally doped when it achieves its maximum T_c ; below or above that dopant concentration, it is said to be under- or over-doped, respectively. The region of the phase diagram above T_c is beyond the scope of this review, but it contains the pseudogap phase (for underdoped compounds) and the strange-metal phase (for overdoped compounds). [4]

Despite the universality of their phase diagram, the cuprate superconductors have different lattice structures [7]. This suggests that superconductivity in the cuprates is governed by the common element of their lattice structure, the CuO_2 planes [7, 5]. The cuprate superconductors all show a separation between planes much greater than the spacing of atoms in-plane, so that the problem can essentially be reduced to two dimensions, as shown in Figure 1. The lattice has horizontal and vertical (as shown) reflection symmetries that the pairing symmetry must respect. That the pairs have zero total spin rules out all but the s- and d-wave pairing states. The s-wave order parameter is isotropic, so it obviously respects the symmetry of the lattice. The d-wave states which respect these symmetries are shown in Figure 2. [7] The anisotropy of the d-wave states will prove vital to a qualitative test of the pairing symmetry.

Figure 1: A CuO_2 unit cell. The a axis is in the vertical direction, the b in the horizontal. Filled circles are oxygen atoms, while open circles are copper. Note the horizontal and vertical reflection symmetries (about the oxygen atoms). (Adapted from [7].)

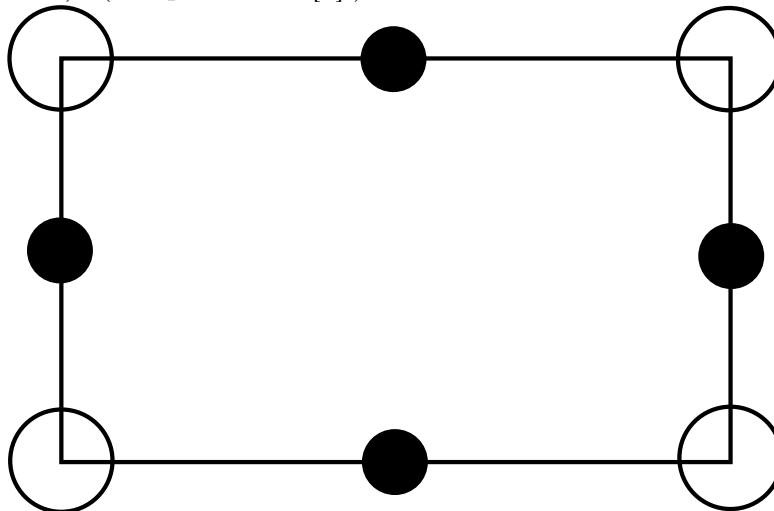


Figure 2: Diagram of order parameter anisotropy in momentum space for candidate d-wave states. On the left is the $d_{x^2-y^2}$ state and on the right is the d_{xy} state. Regions of opposite shading indicate a change in sign or a shift in complex phase by π . (Adapted from [8, 7, 9].)

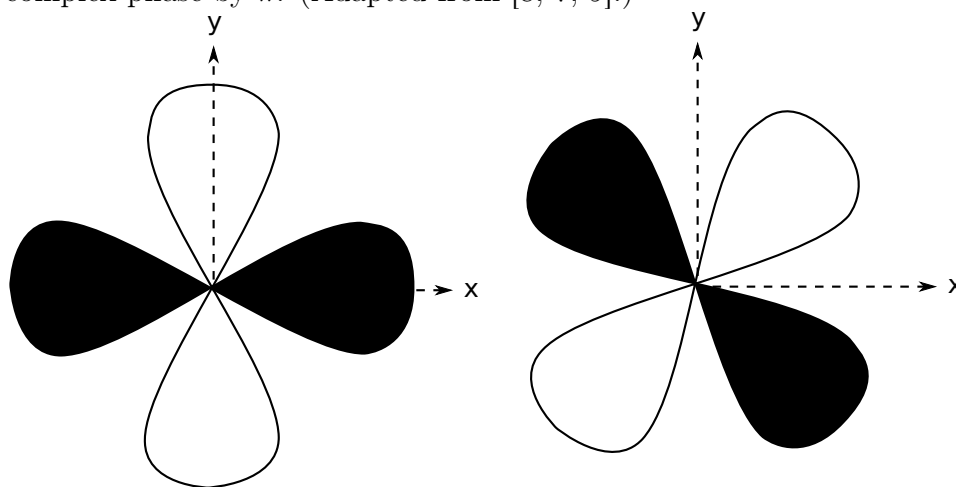
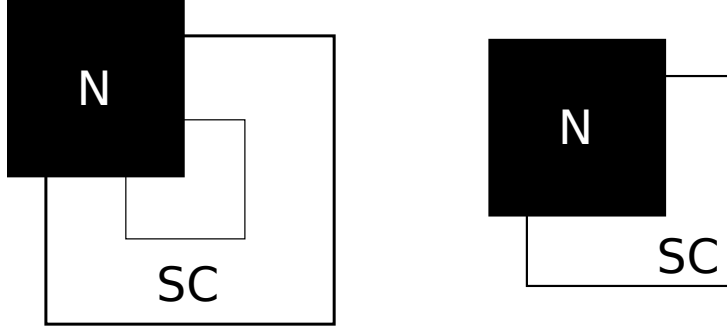


Figure 3: Corner configurations. On the left, corner ring. On the right, single corner junction. **N** denotes normal metal and **SC** denotes superconductor. (Adapted from [7, 9, 8].)

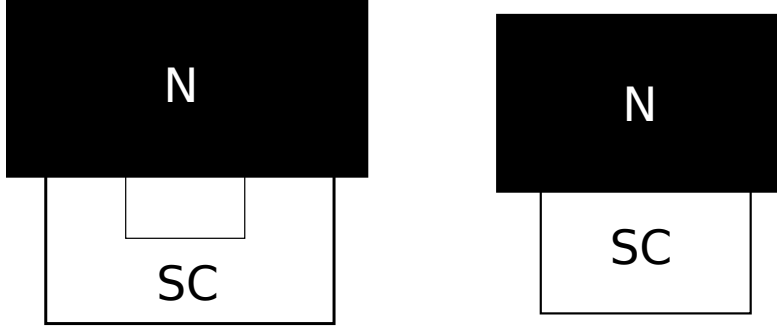


2 Evidence of d-wave pairing

2.1 Josephson effect

A Josephson junction allows Cooper pairs to travel through a barrier from one superconductor to another [7]. In this case, the barrier is a region of normal metal, as shown in Figures 3 and 4 [7, 9, 8]. First, consider the ring configurations in each diagram. The rings can be classified according to the phase difference between the order parameter values at the normal metal junctions; a 0-ring has no phase shift between junctions and a π -ring has a π phase shift (or sign change) between junctions. Because the s-wave order parameter is isotropic, its phase does not depend on the orientation of the junction; therefore, both the edge and corner configurations are 0-rings if the pairing state is s-wave. However, the d-wave order parameter is anisotropic in momentum space. Although this produces no phase shift between junctions in the edge configuration (since the junctions have the same orientation with respect to the lattice), it does produce a π phase shift for the corner ring (where the junctions are orthogonal). This suggests a qualitative test of the pairing state of the superconductor: s-wave pairing superconductors will produce 0-rings in both configurations, but d-wave pairing superconductors will produce a 0-edge-ring and a π -corner-ring. (0-rings and π -rings exhibit different flux quantization behaviors, as will be discussed in the next section.) [7, 8, 9]

Figure 4: Edge configurations (left, ring; right, single junction). **N** denotes normal metal and **SC** denotes superconductor. (Adapted from [7, 8, 9].)



2.2 Half-integer flux quantization

Consider the free energy of a ring with a Josephson junction

$$U(\Phi, \Phi_a) = \frac{\Phi_0^2}{2L} \left[\left(\frac{\Phi - \Phi_a}{\Phi_0} \right)^2 - \frac{L|I_c|}{\pi\Phi_0} \cos \left(2\pi \frac{\Phi}{\Phi_0} + \phi \right) \right]$$

where Φ_0 is the flux quantum, Φ_a is the applied flux, Φ is the flux through the ring, L is the self inductance of the ring, I_c is the Josephson junction critical current, and ϕ is 0 for 0-rings and π for π -rings [7]. Minimizing the free energy in the $L|I_c| \gg \Phi_0$ limit gives the result

$$\Phi = \begin{cases} n\Phi_0 & \phi = 0 \\ (n + \frac{1}{2})\Phi_0 & \phi = \pi \end{cases}$$

This result indicates that 0-rings exhibit normal flux quantization, but π -rings exhibit half-integer flux quantization [7]. Considering the phase coherence of the superconducting state shows particular modulation of the critical current with applied magnetic field according to

$$I_c = 2I_0 \left| \cos \left(\pi \frac{\Phi_a}{\Phi_0} + \phi \right) \right|$$

where I_0 is the applied bias current. The obvious phase difference in the dependence of the critical current on the applied flux provides a qualitative distinction between 0- and π -rings, and therefore between s- and d-wave pairing states. [8, 9]

2.3 Results

The aforementioned modulation of the critical current has been repeatedly measured for the cuprate superconductor $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$ (YBCO). Experiments show that the edge configuration is a 0-ring, but the corner configuration is a π -ring, indicating that the superconducting state has d-wave pairing symmetry. However, a number of difficulties have been reported performing these experiments reliably. The observed phase shift in the modulation of I_c is highly dependent on asymmetries in the ring, background magnetic fields, and magnetic vortices that can be trapped in the ring. [8, 9] None of these effects appear to be large enough to invalidate the conclusion of these experiments, but they do obscure the phase shift to the point that only a qualitative comparison can be made [8].

In response to these difficulties, another, similar method has been developed using single junctions rather than actual rings (see again Figures 3 and 4). This eliminates some of the difficulties with the ring methods without destroying the sharp, qualitative contrast between pairing states. Comparing Figures 5 and 6 once again reveals an unmistakable difference between the pairing states. The edge junction exhibits the Fraunhofer diffraction-like behavior in Figure 5; regardless of the pairing state, there is no phase shift in the order parameter along the junction because there is no change in orientation. The corner junction exhibits the same behavior for s-wave pairing states, but d-wave pairing produces the pattern in Figure 6. Rather than the single, central peak of the Fraunhofer diffraction pattern, the d-wave pattern has two offset peaks and half the oscillation frequency. As with the ring configurations, experiments using the single-junction configurations consistently show the presence of d-wave pairing. [8, 7] The single-junction methods are still not reliable enough to allow direct, quantitative comparison [8]. Methods that produce better quantitative agreement exist, but the results are not as dramatic [7].

3 Microscopic models

The BCS Hamiltonian can be generalized to

$$H = \sum_{\mathbf{k}, \sigma} \epsilon(\mathbf{k}) c_{\mathbf{k}\sigma}^\dagger c_{\mathbf{k}\sigma} + H_{pairing}$$

Figure 5: Predicted critical current I_c (y-axis, arbitrary units) as a function of $\frac{\Phi}{\Phi_0}$ for a single junction with no phase shift. (Adapted from [8].)

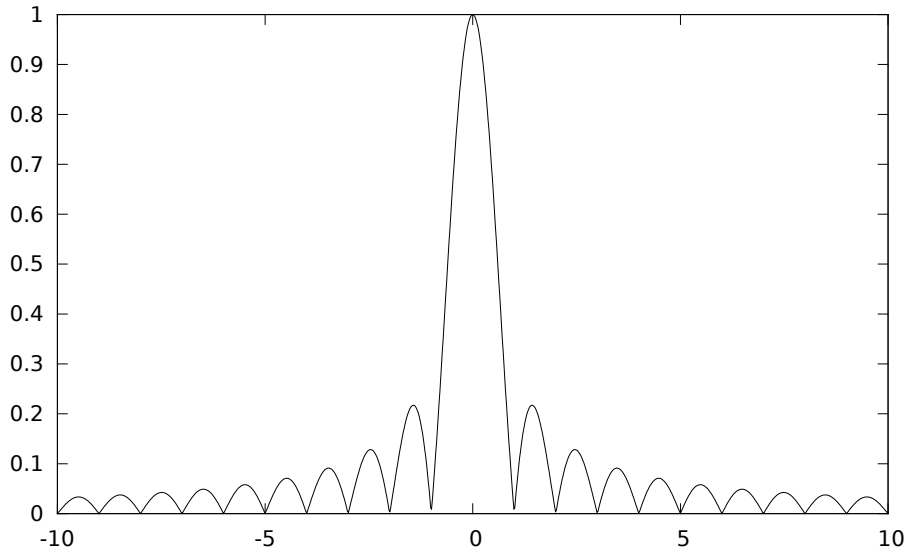
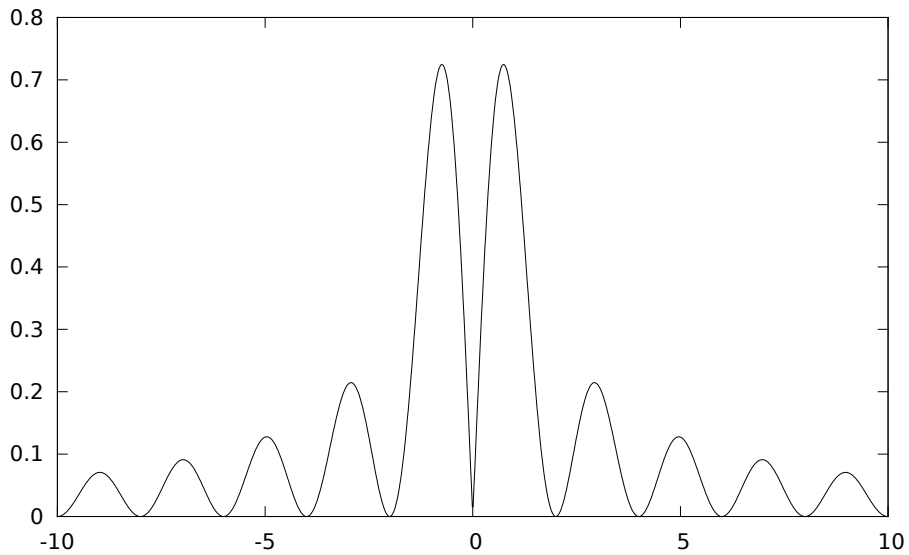


Figure 6: Predicted critical current I_c (y-axis, arbitrary units) as a function of $\frac{\Phi}{\Phi_0}$ for a single junction with a π phase shift. (Adapted from [8].)



where

$$H_{pairing} = \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} V(\mathbf{q}) c_{\mathbf{k}+\mathbf{q}, \uparrow}^\dagger c_{-\mathbf{k}-\mathbf{q}, \downarrow}^\dagger c_{-\mathbf{k}, \downarrow} c_{\mathbf{k}, \uparrow}$$

One of the consequences of the d-wave pairing state established in the previous section is that the pairing potential $V(\mathbf{q})$ should change from attractive to repulsive as \mathbf{q} , the momentum transferred, increases. [7]

One proposal to explain high T_c superconductivity in the cuprates is the resonating valence bond (RVB) model. The RVB model is based on the construction of spinons (spin- $\frac{1}{2}$ fermions carrying no charge) and holons, the combination of a spinon and a hole (dopant) carrying charge but no spin. A strong, on-site Coulomb repulsion drives this theory. [1] The repulsion can be included by the introduction of a slave boson that serves to prevent double-occupancy of any site [4]. Alternately, the Coulomb repulsion can be included by beginning with the Hubbard model Hamiltonian in which the only interaction is on-site repulsion [1]. The Hubbard Hamiltonian can be transformed to prevent double occupancy [1]. In either case, the Hamiltonian is effectively transformed into a t - J Hamiltonian [4]:

$$H = - \sum_{\langle ij \rangle, \sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + J \sum_{\langle ij \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right)$$

Another proposed model is the $SO(5)$ theory which unifies antiferromagnetism and superconductivity in the cuprates. This is achieved by combining the three-component order parameter for antiferromagnetism with the two-component order parameter for superconductivity into a single five-component order parameter, based on the hypothesis that both types of order are caused by the same interaction. It can be shown that under this model, the condensation energy comes from a decrease in the antiferromagnetic interaction energy. [3] This is, in fact, not entirely unlike the RVB model, in which condensation is caused by a spin interaction.

4 Conclusion

Despite the fact that there is no generally accepted microscopic model of high temperature superconductivity in the cuprates, even after more than two decades of intense study, definite progress has been made. There exists today a very large body of empirical data about many materials in the

cuprate class. Even though no particular pairing mechanism has been firmly established, the experiments described here provide strong evidence of the pairing state, placing strong constraints on the possible pairing interactions. It is clear that a spin interaction, possibly combined with on-site Coulomb repulsion, is compatible with these constraints. The case for this interaction is strengthened because similar effects can be derived from theories which are very different on the surface. However, the compatibility of this type of interaction with the experimental evidence does not ensure its suitability to explain pairing at high temperatures, nor does it rule out other possible interactions. Much more work will be required in this area before a consensus can be reached.

References

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