Emergent Gravity

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Abstract

The idea of the emergent gravity came from the study of black hole thermodynamics. Basically by inversion the logic in the derivation of the black hole entropy, we may take the entropy as the fundamental object and the gravity as something emerges from the microscopic interactions of fundamental particles. We discuss the entropic force description of gravity, its relation to the AdS/CFT correspondence, some possible consequences in cosmology, and a criticism based on the gravitational quantum well experiment of neutrons.

1 Introduction

Black hole physics gives us a interesting relationship between gravity and thermal physics [1]. It is shown that Einstein's general theory of relativity can be derived as an equation os state from the point of view of thermodynamics [2]. In 2010, Verlinde purposed the idea of entropic gravity, by reversing the logic of black hole entropy, treating the entropy come from the information of the microscopic physics as the fundamental object and the phenomena of gravity as emergent [3]. The argument is strongly relying on the framework of AdS/CFT correspondence [4]. The paper has drawn some attentions and leads to various following works, including the application to cosmology [6] [7] [9]. It has also drawn disagreement about the idea [10].

In section 2 we introduce the concept of the entropic force and Verlinde's theory of entropic gravity, with a short note on the relationship with AdS/CFT. In section 3 we look at some cosmological predictions, particularly on the non-Gaussianity of the primordial perturbation. In section 4 we discuss a critical paper using the gravitational quantum well experiment of neutrons as a disproof of the entropic gravity.

2 Gravity from the Entropic Force

2.1 Entropic Force

The entropic force is an effective force manifests the tendency of a system toward its minimal entropy. An classic example is the random walk model for a rubber band or a polymer molecule. By calculating the entropy and the free energy of the system, the effective force can be shown to satisfy Hooke's law:

$$Force = -(constant) \cdot k_B T \Delta x \tag{1}$$

, where k_B is the Boltzmann constant, T is the temperature, and Δx is the distance of the endpoint of the object (rubber band or polymer) from its equilibrium position.

There are some important things we can learn from this example. We know that statistical mechanics is a tool to derive the macroscopic behavior of a system from a specific microscopic model. Usually we have experimental data macroscopically, but we know nothing about the microscopic side. This is the case we have with gravity. However, gravity is something so universal that effects all kinds of energy-momentum, and up to our knowledge the law governs gravity is independent of the microscopic description of beings. In other words, if we want an entropic force description, then

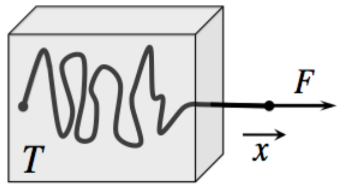

Figure 1: The entropic force of a polymer or a rubber band. This graph is taken from [3].

there should be an very special definition for the micro states, which leads to an entropy related to "anything" in a similar way.

Another point to notice is that, the entropic force should disappear if we look at some small enough scale. An emergent model for gravity should also satisfy this property.

2.2 Newton's Law of Gravity

The discussion here follows [3]. We will go through this in more detail, since similar philosophy can also be applied in a relativistic version to get Einstein equations.

Consider the following system. In 3-dimensional Euclidean space, a test particle with mass m is near a 2-dimensional spherical surface of radius R. The microscopic physics lives on this so called "holographic screen." The radial direction is an emergent spatial dimension, which corresponds to the coarse-graining of the microscopic physics, as in the AdS/CFT correspondence [4].

On the screen there are information storage bits, which give rise to the thermodynamics variables, the energy, the entropy, and the temperature. The details of how the bits work is not important here. Indeed, the result here should be independent of the microscopic description of these bits.

To derive Newton's law of gravity we need the following four assumptions:

1. The area law: the number of bits on the screen is proportional the the area of the screen

$$N = \frac{c^3}{G\hbar} A \tag{2}$$

, where c is the speed of light, G is the gravitational constant, $\hbar$ is the Planck constant,and $A = 4\pi R^2$ in this case.

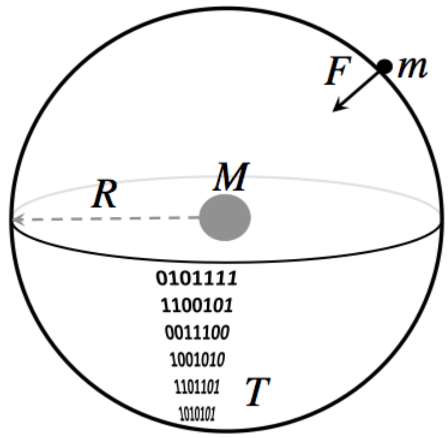

Figure 2: A test particle on a holographic screen feels an entropic force which is identical to Newton's law of gravity. This graph is taken from [3].

2. The total energy of the system is distributed evenly to the bits on the screen, and we may use the equipartition theory to relate it to the temperature

$$E = \frac{1}{2} N k_B T \tag{3}$$

3. The total energy is equivalent to a rest mass M at the center of the sphere:

$$E = Mc^2 \tag{4}$$

The three statements above are all about the physics on the screen. The fourth assumption we need is about the coarse-graining direction and the test particle .

4. The entropy formula: if we move the particle by a distant Δx in the radial direction, then the change of entropy is given by

$$\Delta S = 2\pi k_B \frac{\Delta x}{\lambda_c} \tag{5}$$

where $\lambda_c = \frac{\hbar}{mc}$ is the Compton wavelength of the test particle. This formula is inspired by the change of black hole entropy when a test particle passes through the event horizon.

In the process the energy E is not changed, so from the thermodynamics $0 =$

$\Delta E = T\Delta S - F\Delta x$ then we get the entropic force

$$F = T\frac{\Delta S}{\Delta x} \tag{6}$$

$$= \frac{GMm}{R^2} \tag{7}$$

This is Newton's law of gravity.

Even though we have a motivation for the expression of the change of entropy, the fourth assumption is a-priori. It serves as the basis of the entropic gravity idea, and as mentioned before it is independent of the microscopic physics.

The derivation of the Poisson equation for the general mass distribution can be found in [3]. The derivation of Einstein equations as a thermodynamical equation of state is from [2] and [3] has a shorter discussion.

2.3 AdS/CFT Correspondence

The derivation above is essentially by the spirit of AdS/CFT. The radial foliations of the spacetime describe the boundary field theory operators in different scales, and moving along the radial direction corresponding to moving along the renormalization group flow. The fields in (d+1)-dimensional bulk spacetime just obey classical gravity theory and their boundary values can give the partition function of quantum field theory in d-dimensional spacetime, or so-called boundary field theory, by

$$< e^{\int_{\partial M} \phi_0 \hat{O}} >= e^{-S_E[\phi]} \tag{8}$$

where ∂M is the boundary of the spacetime manifold, ϕ_0 is the boundary value of some field in the bulk, $\hat{O}$ is the corresponding boundary field theory operator, and S_E is the Euclidean action for the classical gravity theory of the bulk spacetime. Here we write down the statement in Euclidean signature. In the Lorentzian signature, because the complexity of the boundary of the manifold, extra care is needed[5].

3 Consequences in Cosmology

Since we can re-derive Newton's and Einstein's theory of gravity, it is not difficult to recover the standard cosmology model, i.e., the Friedmann-Robertson-Walker metric and the Friedmann equations, from the entropic description [6]. The interesting question is that if there is any new observable prediction in cosmology, or if we can solve some mysteries in cosmology through this point of view.

In particular, it is shown in [7] that a double-screen model of entropic cosmology gives a non-Gaussianity of the primordial perturbation spectrum

$$f \simeq \frac{5\epsilon}{36\sqrt{2}\pi c_v} \tag{9}$$

where ϵ is the slow-roll parameter and c_v is a constant related to the heat capacity of the outer holographic screen. The number c_v should be determined by the microscopic description of the quantum gravity. So the non-Gaussianity may or may not be observable, depending on the relative value of ϵ and c_v.

The non-Gaussianity can be analyzed from the cosmic microwave background observational data of WMAP [8]. However, it is not enough to determine the validity of the entropic gravity model here. We can only use it as an input to get the unknown constant c_v in this case.

Entropic cosmology is also used to explain the dark matter and the dark energy [9], but we do not get into that here.

4 Gravitational Quantum Well Experiment

At the end of our discussion, it is worthy to point out that there is an criticism [10] based on the ultra-cold neutrons experiment in the gravitational field of the Earth [11].

The system can be described by the time-independent Schrödinger equation:

$$[\frac{p_z^2}{2m} + V(z)]\psi(z) = E\psi(z) \tag{10}$$

, where

$$V(x) = \begin{cases} mgz & \text{if } z \geq 0; \\ \infty & \text{if } z < 0. \end{cases}$$

By thinking about the density matrix $\rho_S(z) = \Sigma_{i(z)}|i(z)><i(z)|$ of the holographic screen and the von Neumann entropy $S_S(z) = -k_B Tr[\rho_S(z)\ln(\rho_S(z))]$, and using equation (5), one can calculate the change of entropy of the neutron state $S_N(z+\Delta z) - S_N(z)$. Since this quantify is non-zero, the translation operator $e^{ip_z\Delta z}$ evolves a pure state into a mixed state, and it is not unitary. Hence we can deduce that the generator p_z is not a hermitian operator. Indeed,

$$p_z = -i\hbar\frac{\partial}{\partial z} - i2\pi mc \tag{11}$$

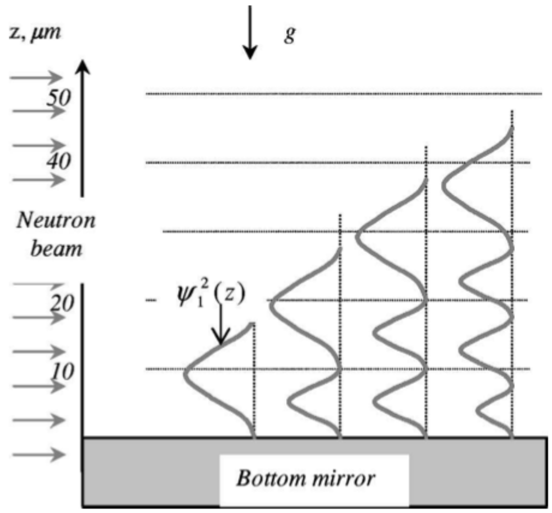

Figure 3: An uniform horizontal neutron beam moves in the gravitational field of the Earth. This graph is taken from [11].

Put this into the Hamiltonian and solve the Schrödinger equation, the non-hermitian part will lead to an exponential decay of the wave function in the z direction. This suppression is huge since the characteristic length $\sim \mu m$ is much larger then the Compton wavelength of neutron $\sim 10^{-9}\mu m$, yet the suppression is not observed in the real experiment.

Further discussion could be found in [12], which says that there is a wrong identification of the characteristic length in [10]. The detail is beyond the scope of this paper.

5 Conclusion

In this essay we go through the idea of entropic gravity, its relation with the AdS/CFT, and also some possible observational and experimental issues. Even though the idea seems to be interesting and ground breaking, in the scope of our investigation no new physically observable prediction is found yet. Some cosmological predictions are made, but the non-Gaussianity, dark matter and dark energy can also be explained by other models. There is a disagreement based on experimental fact, yet the argument itself seems to have flaw. So far, the entropic gravity is more like a change in the philosophical point of view rather than a new physics.

Is there a good way to verify the trueness of it? An simple answer is yes, by doing experiment in the quantum-gravity scale, by a direct and good observation of the black holes, we may find the natural of the gravity, and that it is emergent or fundamental. We need more experimental and observational works before we can make a definite statement on this topic.

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