Superradiance : one theory with different faces

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Abstract

Superradiance, originally theorized by Dicke in 1954, is a collective behavior of individual quantum mechanical systems interacting with their common radiation field. In this essay the main features of superradiance are briefly described in a qualitative manner. Then, along with discussion of three systems (two atoms with mirror, Bose-Einstein condensate in coherent motion, and an ensemble of quantum dots), which can be naively seen different, their connection is drawn through superradiance theory.

1 Introduction

Consider a two-level atom interacts with radiation field. When the radiation wavelength corresponds to the energy difference of the two energy levels, the atom gets excited. Then, it eventually decays to the ground state and emits light, which is known as spontaneous emission. This cycling process is usually *incoherent* such that the direction of each spontaneously emitted radiation field is random. This is the important mechanism[1] of laser cooling and trapping of atoms in atomic molecular and optical (AMO) physics [6]. However, in 1954, Dicke introduced a notion of emergent phenomenon where individual atoms spontaneously emit stronger radiation as a *coherent* (or cooperative) manner, as interacting with incident radiation field. This phenomenon is called *superradiance* [2].

In order to have a qualitative understanding of superradiance, suppose there are N number of atoms as radiation emitters. When atoms cooperatively emit radiation in phase with each other, superradiance occurs and its intensity becomes proportional to N^2 while in the case of incoherent emission the intensity is proportional only to N [9]. That is, the emission rate enhances by N in superradiance. Another interesting feature of superradiance is directionality of the emitted radiation, which is not seen in incoherent spontaneous emission [9]. Although superradiance has been so far explained with an example of atomic system, interestingly there are systems that have very different experimental settings but show close connections through one theoretical concept, superradiance.

2 Trapped atoms with mirror

A single atom is a light emitter. However, the characteristic of this emitted light and the interaction between the light and the atom are altered by the experimental conditions imposed to the atom-light system. For instance, in a cavity-QED (quantum electrodynamics) experiment where two highly reflective concave mirrors surround single atoms, strong coupling between the light and the atoms is realized, and therefore quantum information processing is thought to be feasible [5].

Eschner *et al.* experimentally investigate this environmental dependence of atom-light interaction and show (1) laser-induced self-interaction of a single atom through mirror and (2) interaction between two trapped atoms through their mirror images [3].

2.1 Single atom with mirror

The experimental procedure is described in Fig.1. When laser beam is shined at the trapped atoms, Radiation A spontaneously emitted towards and retroreflected by the mirror interferes with Radiation B (see Fig.1(a)). On the other hand, one can also make an alternative interpretation for this same process as interference by Radiation A emitted directly from

[1] In this absorption-emission process, as a moving atom continuously experiences momentum "kicks" from the counterpropagating radiation but momenta of spontaneously emitted radiation average (or cancel) out, the atom becomes slowed down.

the mirror image of atoms (see Fig.1(B)). This interference is experimentally observed and therefore shows the coherence of Radiation A and B [3].

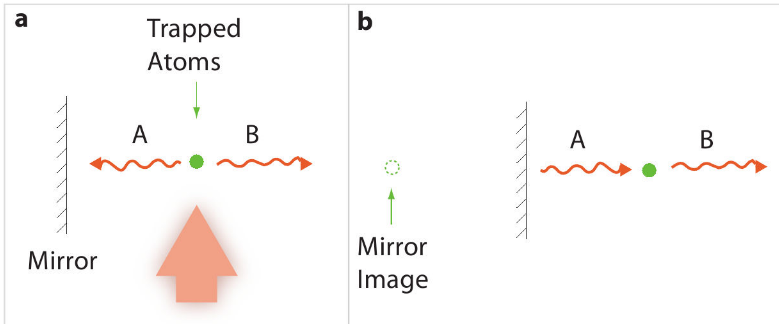

Figure 1: The single atom experiment setup. The vertical arrow represents the incident laser beam. Radiation A and B are spontaneous emission. Photodetection is done on the right in the figure. Two interpretations can be made: (**a**) Radiation A is retroreflected on the mirror. (**b**) Radiation A is emitted from the mirror image.

Although the simple description above helps one visualize what is happening in the experiment, it does not really explain why the spontaneous emission rate (or experimentally measured photon count in Fig.2) changes as the mirror shifts. Instead, we should think of this physics as if the mirror creates the modulation of spontaneous emission *modes* (represented as blue in Fig.3) as a function of space coordinate, the position of atoms with respect to the mirror. Since the spontaneous emission rate is proportional to the mode intensity, the atoms experience the maximum spontaneous emission rate at antinode and the minimum at node [3].

This variation of spontaneous emission rate with the mirror position indicates change in population of the excited states of atoms. Therefore, by adjusting the distance between atoms and the mirror (25 cm in the experiment) one alters the internal dynamics of atoms [3].

2.2 Two atoms with mirror

Now two atomic systems are laser-trapped with the same experimental setup, and the mirror distance is varied. With an interpretation different from self-interference we have discussed until now, spontaneously emitted light from one atom and retroreflected from the mirror is absorbed by the other atom. Again the interference signal same as Fig.2 but with lower contrast is observed. This indicates “communication” between these two atoms. Under this condition these two atoms no longer behave as individual atoms but as a composite system, which is a signature of superradiance. Despite of limits from acoustic vibration, the thermal motion of trapped atoms, and imperfect phasefront matching of the two interfering radiations, this atom-mirror system has a potential application in determining the precise position of trapped atoms [3].

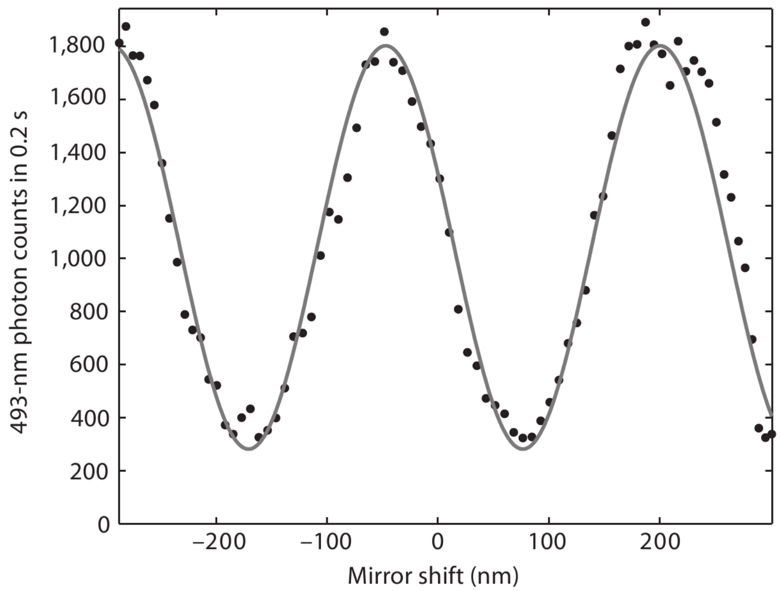

Figure 2: The spontaneously emitted photon count collected for 0.2 s as a function of mirror shift [3]. The wavelength of spontaneous emitted light is 493 nm, and the "period" of interference is half the wavelength of radiation.

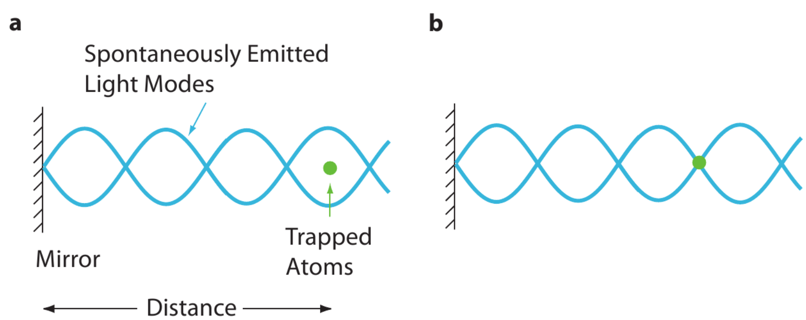

Figure 3: Coupling between spontaneously emitted light mode and single atoms. The spontaneous emission rate is proportional to the mode intensity (blue). Note that the spontaneous emission rate is now a function of the position of atoms. **(a)** The atom is at the antinode. **(b)** The atom is at the node.

3 Bose-Einstein condensate in coherent motion

In the previous section two atoms with a mirror show a hint of superradiance when one atom is positioned at the antinode of spontaneous emission modes generated by the other atom. Considering superradiance as a phenomenon from a coherent system, it may be natural for one to wonder whether there is superradiance indication from a Bose-Einstein condensate, which has a long coherence time and consists of atoms all in one ground state. Inouye *et al.* looks into this problem and observes that superradiant Rayleigh scattering is coupled with the coherent motion of condensate [4].

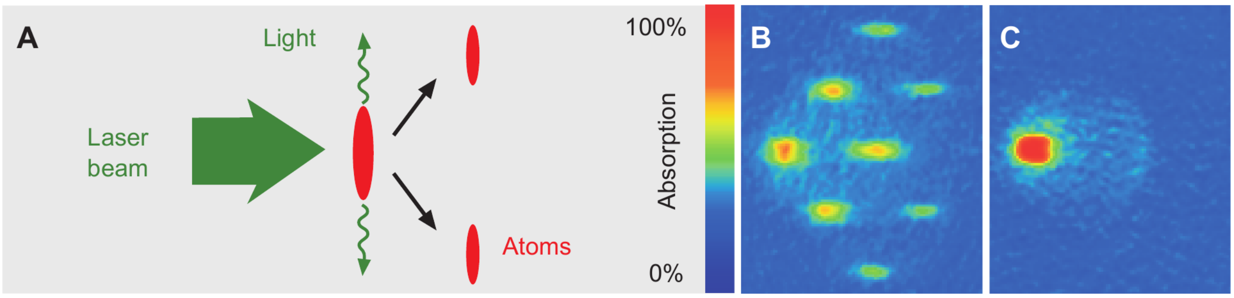

Figure 4: (**A**) Description of the experiment for superradiant Rayleigh scattering (see text). (**B** and **C**) Time-of-flight absorption images of atoms at 20 ms after exposed to a pulse of laser beam with 100 μs duration. Time-of-flight imaging is a technique where trapped atoms are released from the trap and images are taken at a specific time. (B) Polarization of incident laser beam is perpendicular to the long axis of the condensate (superradiant Rayleigh scattering). (C) Polarization is parallel to the long axis (normal Rayleigh scattering) [4].

Fig.4(A) describes the experiment. When laser beam with the wave vector $\mathbf{k}_0$ is shined at the condensate, photons[2] with the wave vector $\mathbf{k}_i$ scatter off the condensate and atoms recoil with the wave vector $\mathbf{K}_j$. As this scattering occurs, the recoiling atoms interfere with the condensate at rest, forming a moving matter wave grating with $\mathbf{K}_j$ [4]. As long as the phase-matching condition (basically momentum conservation equation),

$$\mathbf{k}_0 = \mathbf{k}_i + \mathbf{K}_j, \tag{1}$$

is satisfied and laser beam diffracts from the wave matter grating, the number of recoiling atoms increases as self-amplification. (i.e. The contrast of matter wave grating becomes more and more visible.) Because of the directionality of dipole pattern[3] and the increase in gain of this amplification[4], superradiant Rayleigh scattering is observed along the long direction of the anisotropic condensate (see Fig.4(A)). By thinking of this process as elastic

[2]The words *photon* and *laser beam* are used interchangeably.

[3]Rayleigh scattering power is maximized when the polarization of incident light is perpendicular to the direction of scattered radiation [4].

[4]The gain is proportional to the length of condensate along the direction of scattering [4].

scattering ($|\mathbf{k}_0| = |\mathbf{k}_i|$) it is understood to see a highly directional 45° motion of the recoiling atoms (Fig.4(B)). Otherwise, when the polarization of laser beam shined at the condensate is rotated to parallel to the long axis, superradiance is suppressed (Fig.4 (C)).

The connection of this experiment to Dicke's superradiation theory follows: Imagine that the condensate before interacting with incident laser beam was "the excited state" of two-level atoms and the collection of recoil atoms was "the ground state" [4]. Then, this moving matter wave grating is in fact a coherent behavior and the diffracted light is analogue to the coherent spontaneous emission in Dicke theory. Enhancement in emission rate expected from superradiance theory [9] is also observed in this experiment (see Fig.5).

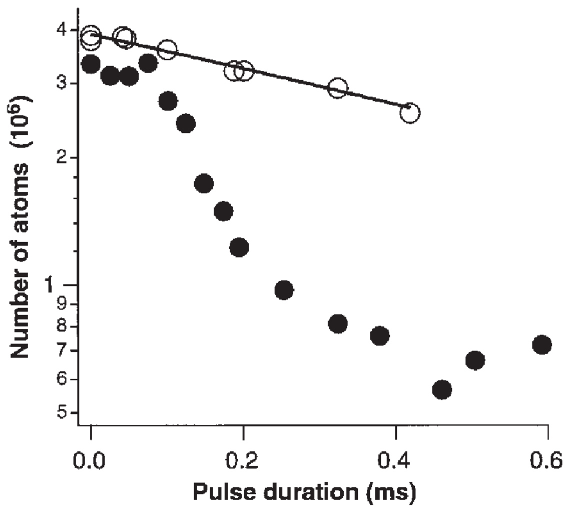

Figure 5: The number of condensate atoms at rest as a function of pulse duration [4]. Faster decay rate in this figure can equivalently be thought as faster spontaneous emission rate. Normal Rayleigh scattering with parallel polarization (○) and superradiant Rayleigh scattering with perpendicular polarization (●) are shown.

4 Ensemble of quantum dots

Quantum dots (QDs) are semi-conductors where electrons are confined to mathematical points [7], and they are often considered to be independent systems [8]. But since they have many potential applications in various fields [1] such as computing, biology, and electronics, it would be fascinating to scale these small individual QDs to a larger composite device. Unfortunately, this coupling has been so far limited to a few nm (Ref.[8] and citations within). In Ref.[8], Scheibner *et al.* study the possibility of such realization.

In this essay QDs are approximately treated as two-level systems, e.g. two-level atoms with the ground state and the excited state [8]. When exciting light is applied to an ensemble of QDs, it is observed that the decay rate[5] from the excited state by non-resonant (nr)

[5] *Decay rate* in QDs can be interpreted as *spontaneous emission rate* in atoms.

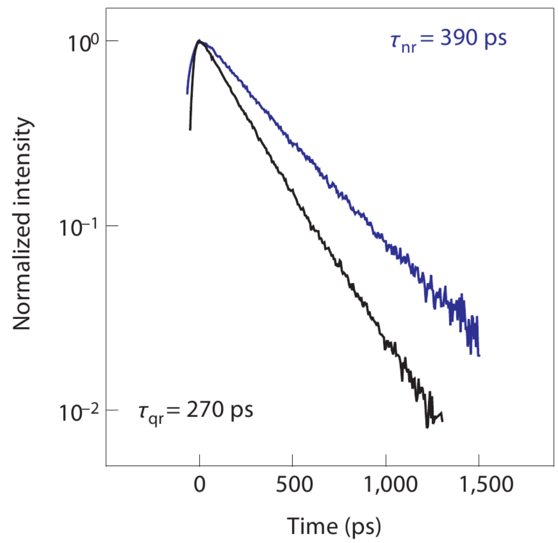

Figure 6: Decay signal from non-resonant (blue) and quasi-resonant (black) excitations [8]. τ is decay lifetime.

excitation is slower than that by quasi-resonant (qr) excitation (see Fig.6). According to Ref.[8], this spectral dependence is actually in contrast to the expected faster decay rate by non-resonant excitation, which is a characteristic of independent QDs and is caused by multi-excition generation.

With a more intuitive argument, if there exists interaction between QDs, one can expect its coupling strength to scale as proportional to the number of QDs, N, and inversely proportional to the separation between QDs, R. Then, connected by this scaling relation one can derive the strength difference, Δ, between in uncoupling and coupling regimes [8] as a function of two experimentally observable quantities, the emitted radiation intensity I and the decay lifetime τ:

$$\begin{aligned} \Delta = \frac{N(\lambda)}{R(\lambda)} &\propto I(\lambda) \cdot \sqrt{I(\lambda)} \\ &\propto \frac{\tau_{\mathrm{nr}}(\lambda)}{\tau_{\mathrm{qr}}(\lambda)} - 1 \end{aligned} \tag{2}$$

where λ is the wavelength of radiation. The experimental result shown in Fig.7 is in agreement with Eq.2 and this implicitly indicates the existence of interaction between QDs.

In order to directly probe that the interaction depends on N, various mesa sizes are used to produce different densities of ensemble of QDs (Fig.8(a)). As the mesa size decreases, which is effectively removing QDs, decay lifetime is observed to increase in Fig.8(b), and therefore the interaction strength between QDs decreases.

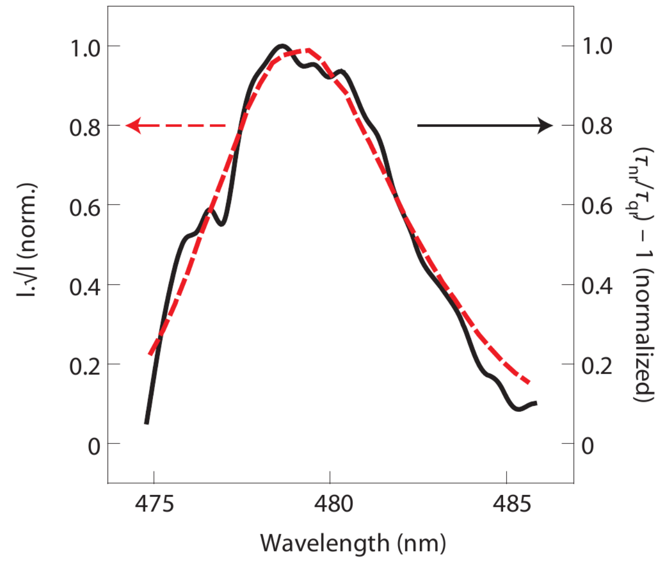

Figure 7: The change of emission rate due to coupling of QDs (Δ in Eq.2), obtained by two independent methods [8].

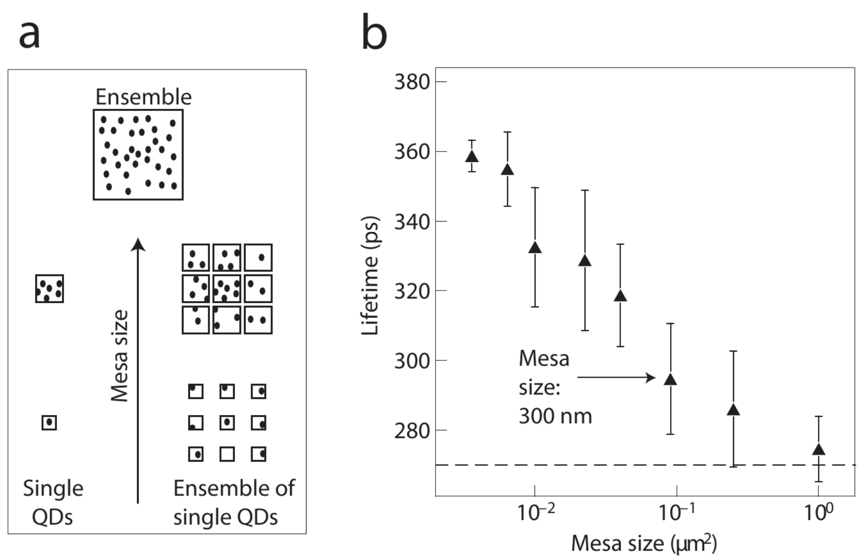

Figure 8: (a) Different mesa sizes produce effectively different densities of QDs. Smaller mesa size, more dilute ensemble of QDs is fabricated. (b) Decay lifetime with quasi-resonant excitation as a function of mesa size [8].

5 Summary

Through superradiance theory we have connected similarities from three differently prepared systems: Two laser-trapped atoms show a long distance (50 cm) optical interaction mediated via a mirror. When Bose-Einstein condensate is in coherent motion as a matter wave grating, highly directional radiation scattering is seen. An ensemble of QDs demonstrates the possibility in optical control over complex and scaled solid-state systems in the future.

References

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