

# **The Silence of Sound: Acoustic Localization in Bubble Water**

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(Dated: December 14, 2009)

## **Abstract**

In this essay classical localization is discussed within the context of air bubbles scattering sound waves in water. Analytical theory, computational calculations, and conclusive properties of the localized state are discussed as well as future avenues of research.

## I. INTRODUCTION

In 1958, at Bell Labs, Philip Anderson conceived of the process of Localization in which a transition occurs from a conducting state to an insulating state by adjusting the density of impurities in a material. Anderson localization, even after 50 years, is still essentially conjecture. It has yet to be experimentally proven [5]. Many attempts have been made to verify localization but are met with two types of complications [9]. In quantum systems the difficulties arise due to the Coulomb interaction dominating the impurities. In classical systems it is shear (transverse) effects which lead to absorption and obscure results.

One major goal of current research is unambiguous observation of localization, and to do this research has now turned to classical systems. Classical localization refers to a wave scattering effect where waves become trapped in regions of space with a high enough density of scattering sites. Early attempts to model acoustic localization met with success [1] warranting further investigation. A typical experimental setup includes a medium with controllable density of impurities (e.g. bubbles of air in water), a wave source (e.g. ultrasonic sound waves), and detectors to capture the transmitted waves [4]. In order to observe localization one must find that the transmitted and reflected waves do not account for all of the energy released into the system. This brings us to the major complication in classical experiments: absorption effects.

When attempting to observe localization one must take great care to distinguish between energy trapped in the form of waves, and energy absorbed by the system as heat, shear effects, etc [9]. These difficulties are so insurmountable that even after so many decades definitive evidence for localization has not been observed. In terms of theory, however, many interesting results have been published. In the interest of working towards experimental verification of localization we shall focus on one particular theoretical application: localization of acoustic waves in bubble-water.

## II. THEORY

This model comes from a specific series of articles [3,4,5,6] through which we can understand both how to realize localization, and how to interpret the ordering. In this system the air bubbles act as hard sphere scattering sites. The following assumptions simplify calculations without eliminating the important features of classical localization. In this model we will assume that the bubbles are randomly oriented in space (they are, after all, interpreted as representing disorder). Complicated calculations and edge effects can be eliminated as

follows: First let the system be spherical and have the bubbles be all of the same radius; Then have the source be a wave emanating spherically from the center of the system. The latter choice helps eliminate reflected waves and edge (non-bubble) scattering.

Interestingly, the following definitions closely follow some of the lecture material from class. Lets begin by defining the amplitude and phase at a point in the system  $A \equiv A(\vec{r})$ . Let the source wave be denoted  $A(0) \equiv A_o$ . Normalize this such that the transmitted amplitude has simple form

$$\mathcal{A} \equiv \frac{A}{A_o}$$

$$\Rightarrow T = |\mathcal{A}|^2$$

Now we can of course rewrite this in terms of amplitude and phase

$$A(\vec{r}) \equiv A e^{i\theta}$$

With which we can now write down the form for the wave current

$$\vec{J} \equiv \text{Re}(A^*(-i\vec{\nabla})A)$$

$$\Rightarrow \vec{J} = |A|^2 \vec{\nabla}\theta$$

Giving us the phase gradient reminiscent of the lectures.

This is a good time to analyze classical localization in terms of the given parameters above. A source wave is formed and then propagates outward spherically. Each time it encounters an air bubble a 'secondary' spherical wave source is created at that location. This happens often and frequently and after many such instances we find the normal state to be one in which several overlapping waves constructively and destructively interfere. Now in terms of the phase field localization occurs when there is a region of space with the phase at a constant value, and surrounding that region the phase is abruptly (i.e. with exponentially decaying boundary) 180 degrees out of phase with the localized region. In this way the wave can only propagate on itself in that limited region and is always destructively interfered when exiting the same region [5]. Also note that this must occur with the amplitude not vanishing, otherwise we only get a trivial system.

In order to clarify the above analysis lets write the phase field as a plane unit vector. Taking the x-axis to be zero phase we write

$$\vec{n}_i = \cos(\theta_i)\hat{x} + \sin(\theta_i)\hat{y}$$

We will also choose the phase such that when a bubble begins emitting waves we set the phase value to zero. For simplicity we only write arrows at each bubble site. Finally in these figures the volume fraction of bubbles is  $\delta = 0.001$  and 100 averages were computed [5]. Now in Fig 1 we find a series of states with just these pictures. The localization effect only occurs in a small range of wave frequencies. In order to relate these calculations with our experimental realization goal we also give the transmission coefficient out of the sphere as a function of the reduced system size for each state in Fig 1.

We now analyze those three given regimes. The first corresponds to frequencies below the localization. In this regime we can see the phase field values seem as random as the spacial location of the bubbles. In particular we find a very high rate of transmission of the source energy for this state. The situation is similar for frequencies above the localization range. Let  $k$  be the wavenumber of the source and  $a$  be the radius of the air bubbles. For values between

$$0.014 \geq ka \leq 0.060$$

we find our desired ordered state. The most striking features are the (nearly) complete coherence of the phases for each randomly located scatterer. Recalling that this phase is directly out of phase with the source itself we turn to the transmission coefficient plot. Here we find a straight line (on a log plot) which drops by over four orders of magnitude. This signifies the exponential decay of the transmission due to the localized wave becoming confined within its initial formation location. To see the region of frequencies in which our ordered state exists we have given Fig 2.

Now although we have shown that localization corresponds to long range order (in this case the order is in the phases of the scattered waves) and have found exponential decay of the transmission amplitude in that state we have avoided addressing the issue of broken symmetry. The reason for this can be found in the nature of the waves themselves. In this classical, hard sphere scattering we have been focusing on longitudinal waves. In fact we mentioned that it is exactly transverse wave effects which detract from clear experimental observations. As discussed in class the longitudinal components can not result in the formation of Goldstone bosons (although the transverse direction can) even upon symmetry breaking. It is a common misconception that longitudinal components can form Goldstone modes and this article [3] is no exception. Ye et al. claims that the localization of longitudinal waves results in the formation of a Goldstone boson in direct contradiction to what has been mentioned here. This is not to say that a symmetry has not been broken. In particular we see from Fig 1 that the ordered state of our system results in broken rotational symmetry of the phase. This continuous symmetry breaking is tempting to

label with Goldstone modes, but we must take care in the analysis of ordered phases.

### III. EXPERIMENT

An important requirement of theory is its connection to experiment. As stated earlier there are difficulties in realizing the ideal conditions necessary to observe classical localization. In the case of water we find the flow of air bubbles to be the largest obstacle. In our simple model we assumed stationary scattering sites (or instantaneous wave propagation) which is not a good approximation with a low viscosity fluid. On the other hand, bubble defects in solids are so tightly constrained that they do not have the hard-core scattering frequency range necessary for these experiments [8]. One series of articles has recently begun studying a modified version of the above simple model where water is replaced by media that hold stationary bubbles (i.e. are elastic like solids), but are also compressible with very low shear stiffness [8]. Under these so called 'waterlike' conditions it is thought that both theory and experiment can come together in an attempt to answer the question of localization with low ambiguity.

In particular one article goes beyond the localization calculation to compare with experimental results for nonlinear wave propagation in soft materials permeated by bubbles [9]. They find that in the case of vanishing surface tension the results for our simple model extended to soft media agree with the results of the effective medium method (EMM). Our simple model does not account for surface tension, which is responsible for absorption effects as mentioned in the introduction of this topic. An example of this method is given in [10].

### IV. FUTURE

Up to now we have focused on what has been done to explore classical localization. Our simplified model has also been shown to suffer from some important limitations especially where experimental realization is concerned. The extension of this model to soft media and the use of other methods such as EMM have helped bridge the theory-experiment gap, but the evidence for localization at the classical level is mediocre at best. Thus this subject is a very active field and we can speculate where it might go in the coming years.

First no one has attempted to use a model where transverse waves are localized (and thus allow for the formation of Goldstone modes). Not only would this be an interesting model to evaluate, but it allows for more experimental

observations to be made (since one of the principle losses of energy is due to transverse wave formation). That said, it would be difficult if not impossible to distinguish between transverse localization and absorption effects. This can be seen from the simple picture that if a material can form longitudinal and transverse waves, then the dispersive effect of breaking up energy over and over into small out of phase components is exactly what we were trying to avoid. Even with this complication, it would be interesting to search for localized transverse wave effects perhaps in models (or materials) where longitudinal waves do not propagate very effectively.

One of the more interesting applications of this research to the physics community is the possibility of extension into the quantum regime. If data supporting localization is found in soft materials with percolation then we can begin to look for analogous quantum models. It is possible that soft condensed matter systems could show similar effects experimentally. More generally if we can find a class of materials exhibiting localization then we can predict localization in quantum analogues. Among other interests this would allow for a more sophisticated search for true Anderson Localization.

## V. REFERENCES

- [1] T. R. Kirkpatrick, Phys. Rev. B 31 (1985) 5746.
- [2] Z. Ye, A. Alvarez, Phys. Rev. Lett. 80 (1998) 3503.
- [3] Z. Ye, H. Hsu (2000). 'Phase Transition and Acoustic Localization in Arrays of Air-Bubbles in Water'. arXiv:cond-mat/0006352v1
- [4] Z. Ye, H. Hsu, E. Hoskinson, Phys. Rev. Lett. A 275 (2000) 452.
- [5] C. Kuo, K. Wang, Z. Ye, App. Phys. Lett. 83 (2003) 4247.
- [6] K. Wang, Z. Ye, Phys. Rev. E 64 (2001) 056607.
- [7] B. Liang, X. Zou, J. Cheng, Chin. Phys. Lett. 26 (2009) 024301.
- [8] B. Liang, J. Cheng, Phys. Rev. E 75 (2007) 016605.
- [9] B. Liang, X. Zou, J. Cheng, J. Acoust. Soc. Am. 124 (2008) 1419.
- [10] A. Baird, F. Kerr, D. Townend, J. Acoust. Soc. Am. 105 (1999) 1527.

## VI. FIGURES

FIG. 1: On the left are the phase field values for each scattering site. On the right the transmission coefficient as a function of reduced system radius is given.

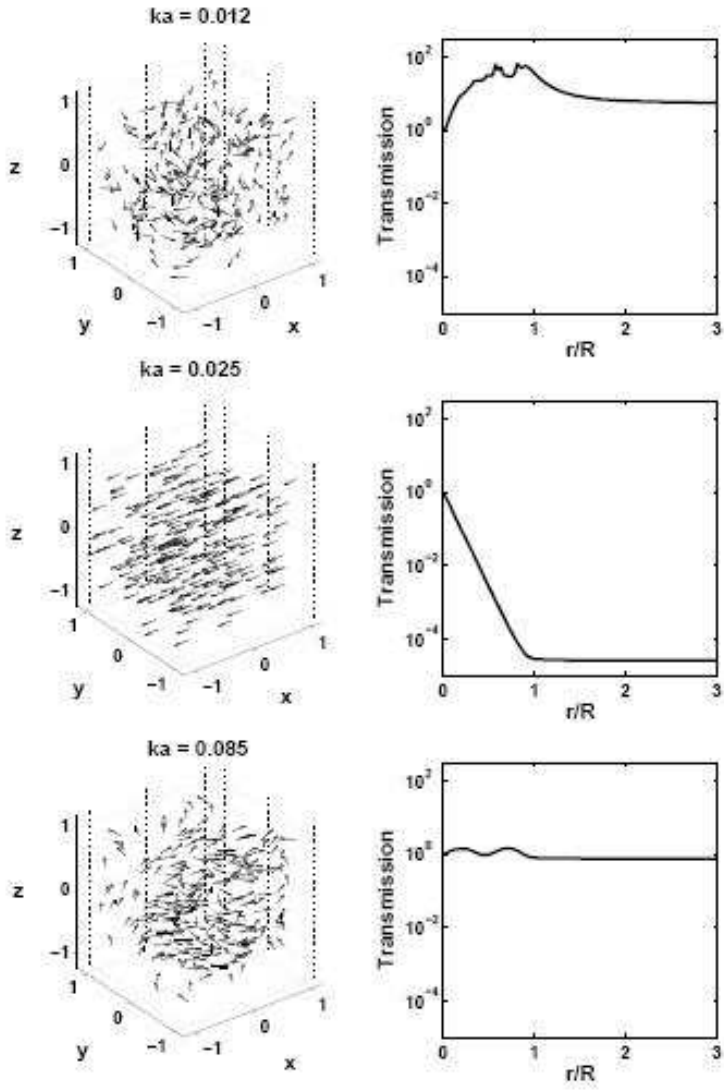


FIG. 2: The transmitted amplitude as a function of wavenumber. Note the reduced transmission denoting the Localization transition

