Herd Behavior and Phase Transition in Financial Market

Minghao Guo

December 13, 2009

Abstract

In this paper, I brief reviewed the herd behavior in financial market. Benerjee model and EZ model are introduced. Phase transition behavior just like in physical systems is found in EZ herding model. Power law distribution of returns with exponential cutoff of bump are studied in EZ model.

1 Introduction

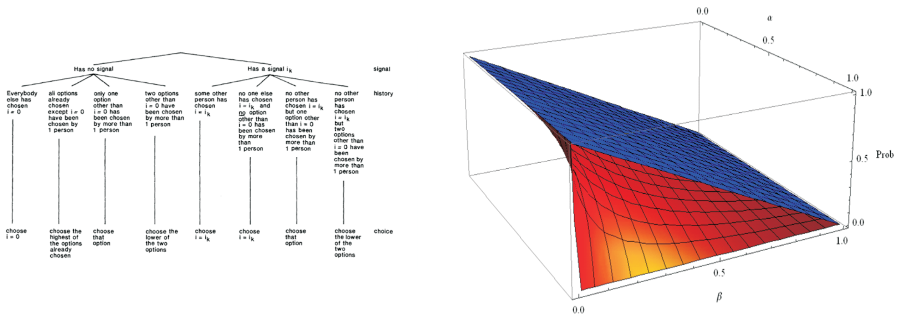
Physicists are attracted by the applications of physical ideas and methods in biological, social and economical sciences in the past decades[1, 2, 3]. Financial market as a complex system with characteristics similar to physical system is one of the hottest areas that have been studied[1]. It has been found that, for example, the distribution of returns shows distinct tails larger than the Gaussian distribution[4]. Several models have be developed to explain such fat-tail phenomenon including the "dynamic multi-agent model" by Lux and Marchesi[4] and "herd behavior" by Eguiluz and Zimmermann[5]. This paper will focus on the herd behavior and explain some phenomena in financial market with the herding model.

In this paper, the original Benerjee model herd behavior is introduced in section 2 to illustrate why people get herded in social activities. The EZ herding model describing the transmission of information is explained in section 3. The single-double peak phase transition is found. In section 4, some more details of EZ model are discussed including the finite size limit and the interacting EZ model.

2 Benerjee Herding Model

Herd behavior was first proposed by Abhijit V. Benerjee in 1992[2]. In Benerjee's herding model of decision making, people are inclined to mimic others' actions. It is obviously understandable when one has no preference and follows others. However, it is possible that one accepts others' idea even though his own information tells him to do something else.

The postulate of Benerjee's herding model is quite simple[2]. Suppose there is a set of assets indexed by i in $[0, 1]$. Each asset has its return denoted by $z(i)$. There is a unique "right decision" i^* such that $z(i^*) = z > 0$, while for all other i's, $z(i) = 0$. Everyone makes his decision in sequence. The later person is allowed to observe decisions of previous people. However, he only knows the decisions previosus people made, but does not know how they have been made. With a probability of α, he may have his own signal suggesting him what to do. However, there is another probability of $1 - \beta$ that it turns out to be a wrong signal which he does not know it. Then he makes his decision with others' information and his own signal, if he has got one, but does not know whether he should believe it or not. Three basic assumptions



A B

Figure 1: Benerjee's herding model. (a) The k-th decision maker's choice ($k > 2$). Adapted from Ref. [2]. (b) The probability that no one has chosen the right option for (red) herding model and (blue) non-herding model.

of this model are made[2]:

(1) If a decision maker has no signal, and all others have chosen $i = 0$, then he follows others by choosing $i = 0$.

(2) When a decision maker is indifferent between his own signal and others' choice, he follows his own signal.

(3) When a decision maker is indifferent between others' different choices, he chooses the one with highest value of i.

It is easy to find out how the first person makes decision. If he has a signal, he just follows it. If not, he chooses $i = 0$. The second person will always follow the first person if he has not a signal, or follow his signal if he has one. Figure 1(a) shows the law how following people make decision based on their own signal and decisions made by previous people.

One of the amazing results of this herding model is the probability of herding at a wrong option. Without herding model, if decision makers cannot see others' choices, the probability of no one has made right choice is obviously $1 - \beta$ when everyone's signal is wrong, if he has one. However, in this herding model, it can be calculated that [2] this probability is

$$\frac{(1-\alpha)(1-\beta)}{1-\alpha(1-\beta)} \tag{1}$$

Figure 1(b) shows the differences between these herding and non-herding models. In the herding model, there is a sharp drop as β goes away from 0 when almost everybody as a signal ($\alpha \to 1$). It indicates that the right option can easily be found when most people have signals in the herding model, even though their signals are highly probable to be wrong. In non-herding model, we cannot find such sharp drop as the probability of everyone is wrong is just proportional to $1 - \beta$.

3 EZ Model of Herd Behavior

3.1 EZ herding model of information transmission

The EZ model, named after Victor M. Eguiluz and Martin G. Zimmermann, is an important application of herding model in information transmission[5]. The idea of the model is quite simple. It involves a system consists of N agents, which can be pictured as N vertices in a network. Some agents may connect with each other to form a cluster. Every single agent in a same cluster shares same information. According to Benerjee's model, they should make a same decision: to buy or to sell. As time goes, some other agents may merge into a cluster, which indicates that information spreads out and more people get to know the information. The mathematical expression of the model is that[5]:

(1) Initially, all agents are isolated, i.e. there are N clusters in the system each of size 1.

(2) At time t, a random agent is selected.

(3) With probability a, this agent becomes active, and hence all agents in the same cluster. The size of this cluster at this time is $s(t)$. They make a decision between buying and selling (1 or -1). After that, the cluster breaks into isolated agents, i.e. s clusters of size 1 form.

(4) With probability $1 - a$, this agent stays inactive. Another agent is selected randomly. If these two agents are of different clusters, then combine these two clusters to form a larger one.

(5) Repeat steps (2)-(4).

In step (3), all agents in a same cluster share information and behave in a same manner. The dissolving of the cluster after acting is necessary so that other new clusters can form indicating new information spreads. Step (4) shows a way of information transmission. The parameter a controls the speed of the transmission. When a is large, there is a small probability of dispersion of information. When a is relatively small, it gets more chances for information spreading. Eguiluz and Zimmermann presented a herding

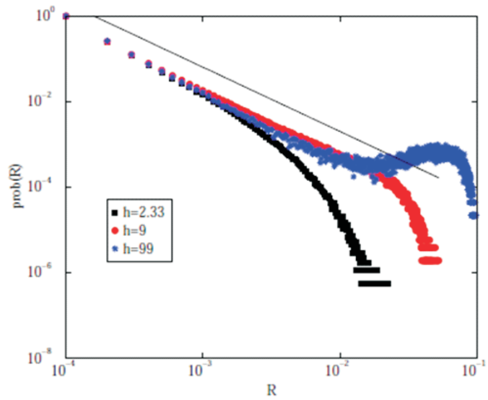


Figure 2: Log-log plot of the probability of returns R for herding parameters h=2.33, 9, 99 (a=0.30, 0.10, 0.01). Adapted from Ref. [5].

parameter $h = 1/a - 1$. It is more interesting for the case that information spreads fast enough such that herding behavior dominates, i.e. $a \to 0$, or $h \to \infty$.

Price index dynamics is introduced. At step i, price index P evolves as $P(t_{i+1}) = P(t_i)e^{s_i/\lambda}$. Here s_i is the size of the cluster that becomes active. λ is a parameter that controls the size of time evolution of price index. The price return which is defined as $R(t_i) = \ln[P(t_i)] - \ln[P(t_{i-1})]$ is a linear function of the cluster size s_i.

Eguiluz and Zimmermann performed numerical simulations of this model[5]. They chose the agent population to be $N = 10^4$, herding parameter h varies from 2.33, 9 to 99 ($a = 0.30, 0.10, 0.01$, respectively). Figure 2 shows the distribution returns of the different herding parameters. For small R, probability of returns approach the solid line, which shows a power law relation $prob(R) \propto R^{-\alpha}$, where $\alpha = 1.5$. For larger R, there exists a critical herding parameter h^*. When $h < h^*$, the distributions display a continuous cross over to an exponential cutoff. When $h > h^*$, there is a bump in the probability of high returns. This qualitative change leads to the creation of “financial crashes”.

3.2 Phase transition in information transmission

Plerou, Gopikrishnan and Stanley[6] first found that in buying and selling activities in financial market, there exists a phenomenon just like phase transition in other physical systems. Trade and Quote database was used to analyze each and every transaction of the 116 most actively traded stocks in the two year period 1994-1995. They calculated the conditional probability distribution of demand of different local noise intensity Σ versus volume imbalance, which interprets the net demand in the market. It is found that for $\Sigma < \Sigma_c$, the most probable value of demand is near zero, called "equilibrium phase", indicating that buying and selling are equally important in this case. For $\Sigma > \Sigma_c$, there are two peaks symmetrical distributed around zero demand, which is called "out-of-equilibrium phase" meaning that either buying or selling dominates in the market.

Similar phase transition can be found in EZ model. In figure 2, the exponential cutoff for small h and bump in high returns for large h show different phases in herd behavior. There is a critical value h^* in this transition. More detailed study shows that[7] there is single-double peak two-phase phenomenon in EZ model. An order parameter r is introduced to describe the fluctuation in a time t:

$$r(t) = \langle |R(t') - \langle R(t') \rangle_t| \rangle_t \tag{2}$$

Here $\langle \cdots \rangle_t$ means the average over t' in a time interval t. Define $Z(t)$ as the change in return $R(t)$ (equivalent to coordinate translation):

$$Z(t) = R(t = 0) - R(t) \tag{3}$$

Probability distribution of return Z is shown in figure 3, with herding parameter $h = 19$ ($a = 0.05$), agent population $N = 10000$, time interval $t = 10$ and 100, and fluctuation parameter r varies from 0 to 60. The herding parameter h is chosen such that it is near the critical value in figure 2. For small r, there shows a single peak in return, while double peaks for larger r. The peaks become narrower and farer away from origin because of the absence of long-range correlation of the magnitude of the returns[7].

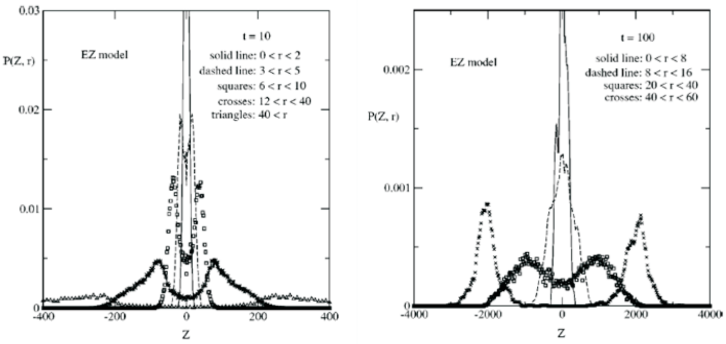


Figure 3: Probability Distribution of returns with $N = 10000$, $a = 0.05$ for different values of r. (a) time steps=10; (b) time steps=100. Adapted from Ref. [7].

4 Discussion of EZ Herding Model

4.1 Exact solution of EZ model

The distribution of returns $prob(R)$ and the distribution of cluster size $prob(s)$ are related in that return R is linear to the size of cluster s[5]. On the other hand, the distribution of returns is equal to the distribution of cluster multiplied by the size of the cluster which represents the probability to choose the specific cluster:

$$prob(R) \sim R^{-\alpha} \sim s^{-\alpha} \sim s * prob(s) \sim s * s^{-\beta} \tag{4}$$

where $\alpha = \beta - 1$. We has concluded in previous part the $\alpha = 1.5$, so that here $\beta = 2.5$. The model can be solved starting from the equation[8]:

$$\begin{aligned}
\frac{dP[l_1,\ldots,l_N]}{dt} &= -\frac{1-a}{N(N-1)}\sum_{i=1}^{N} il_i i(l_i-1)P[l_1,\ldots,l_N] - \frac{1-a}{N(N-1)}\sum_{i<j} 2il_i jl_j P[l_1,\ldots,l_N] \\
&+ \frac{1-a}{N(N-1)}\sum_{i=1}^{N} i(l_i+2)i(l_i+1)P[l_1,\ldots,l_i+2,\ldots,l_{2i}-2,\ldots,l_N] \\
&+ \frac{1-a}{N(N-1)}\sum_{i<j} 2i(l_i+1)j(l_j+1)P[l_1,\ldots,l_i+1,\ldots,l_j+1,\ldots,l_{i+j}-1,\ldots,l_N] \\
&- \frac{a}{N}\sum_{i=2}^{N} il_i P[l_1,\ldots,l_N] + \frac{a}{N}\sum_{i=1}^{N} i(l_i+1)P[l_1-i,\ldots,l_i+1,\ldots,l_N]
\end{aligned} \tag{5}$$

Here l_i is the number of clusters of size i, which the constraint

$$\sum_{i=1}^{N} i l_i = N \tag{6}$$

$P[l_1, \ldots, l_N]$ is the probability of finding the system to be in the state $[l_1, \ldots, l_N]$. The first four terms on the right hand side of the equation describe the information dispersion in the system. Two clusters both of size i combined in the first and the third term, and of different sizes i and j combined in the second and the forth term. In the last two terms, the selected cluster of size i acts and dissolves into i clusters of size 1. The average number of clusters of size i is defined as

$$\langle n_i \rangle = \sum_{[l_1,\ldots,l_N]} P[l_1, \ldots, l_N] l_i \tag{7}$$

Xie etc.[8] solved this equation in the limit of $a \ll \frac{1}{\sqrt{N}}$. They got that

$$\frac{\partial \langle n_s \rangle}{\partial t} = -\frac{(2-a)s\langle n_s \rangle}{N} + \frac{1-a}{N^2} \sum_{r=1}^{s-1} r\langle n_r \rangle (s-r) \langle n_{s-r} \rangle \qquad \text{for } s \geq 2 \tag{8}$$

$$\frac{\partial \langle n_1 \rangle}{\partial t} = -\frac{2(1-\langle n_1 \rangle}{N} + \frac{a}{N} \sum_{r=2}^{\infty} r^2 \langle n_r \rangle \qquad \text{for } s = 1 \tag{9}$$

D'Hulst and Rodger[9] got the same equations using mean field approximation and solved them. Power law decay $\langle n_s \rangle \propto s^{-\alpha}$, where $\alpha = 2.5$, was found with an exponential cutoff for larger s. A numerical simulation of this model has been done by D'Hulst and Rodger with parameters $a = 0.01$ and $N = 10^4$. Though it does not satisfy the assumption $a \ll 1/\sqrt{N}$, they got a result of $\alpha = 2.7$, pretty well matches the theoretical result.

4.2 Finite size limit of EZ model

The limit of $a \ll 1/(N \ln N)$ was studied by Xie etc[8]. In this case, the information spreads so fast that it is highly possible that all agents in the system are combined into one single cluster before a cluster acts and breaks. It leads to the fact that the probability $P[0, 0, \ldots, 0, 1]$ approaches unity. While the probability that the system is in any other state

$$A = {\sum_{[l_1,\ldots,l_N]}}' P[l_1, \ldots, l_N] \tag{10}$$

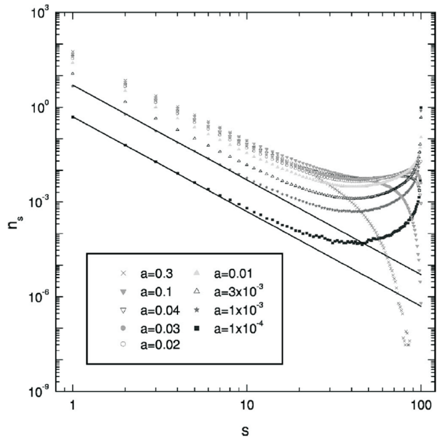


Figure 4: s dependence of $\langle n_s \rangle$ for different values of a in a system with $N = 100$. Solid lines show the power law relation $\langle n_s \rangle \propto s^{-\alpha}$, where $\alpha = -3$. Adapted from Ref. [8].

is very small and proportional to a. In mean field approximation, it can be found that $A = Na$. Then the s dependence of $\langle n_s \rangle$ can be derived:

$$\langle n_s \rangle = N^2 a \frac{(2s-2)!}{2^{2s-1} s!^2} \tag{11}$$

Using Stirling's approximation, one can find that $\langle n_s \rangle$ still obeys the similar power law $s^{-2.5}$. However, the fluctuation becomes more important in this finite size limit which makes mean field theory not valid for small a. More exactly solution shows that in finite size limit, for small s, $\langle n_s \rangle$ obeys

$$\langle n_s \rangle = \frac{N^2 a}{2s^3} + O(Na) \tag{12}$$

By numerical simulation, it can be proved that $A \propto \frac{1}{2} Na \ln N$, which is inconsistent with $A \propto Na$ in mean field theory. For small s, the first term dominates. $\langle n_s \rangle$ follows the power law with an exponent -3 instead of -2.5 given in mean field theory. However, the fact that the second term becomes considerable when s is large leads to $\langle n_s \rangle$ deviates from power law of s for large s shown in Figure 4. In figure 4, there is an exponential cutoff for large s when $a < 1/\sqrt{N}$. It turns to be a bump for large s when $a > 1/(N \ln N)$. The transition between the two phases occurs at a critical value a_c, where $1/(N \ln N) < a_c < 1/\sqrt{N}$.

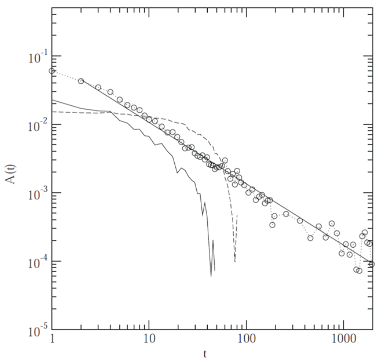


Figure 5: Volatility auto-correlation in log-log scale. Adapted from Ref. [7]. Dashed line: $-A(t)$ for $\delta = 0$, $c = 0$, $b = 0.05$, $N = 10000$; Solid line: $A(t)$ for $\delta = 0.8$, $c = 0.3$, $b = 0.0005$, $N = 20000$; Circled line: $A(t)$ for $\delta = 1.0$, $c = 0.6$, $b = 0.00025$, $N = 40000$;

4.3 Interacting EZ model

Herding parameter a or h in EZ model is the only but very important parameter of the model. In the original EZ model, the herding parameter is set to be constant. It makes the problem very simple, but lost a crucial aspect that the financial market is fluctuating. When the market is more fluctuating, information disperses quickly since people are sensitive to it and tend to spread information extensively, which makes the herding parameter a smaller, or h larger. However, parameter a should larger when the market becomes more stable because people are less fanatic to all kinds of rumors. Zheng etc.[10] suggested an interacting EZ model such that the parameter a is changeable:

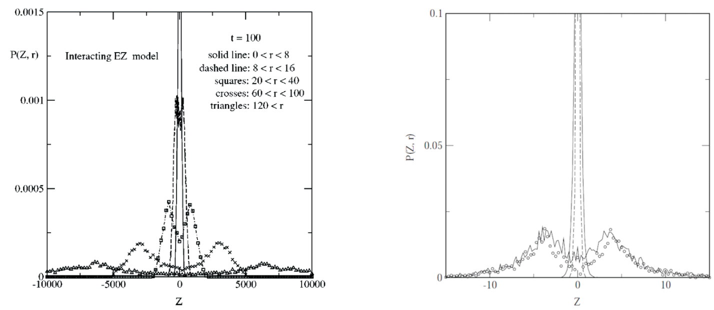
$$a_t = b + cs_{t-1}^{-\delta} \tag{13}$$

Here variable s is the size of the cluster, b, c and δ are all positive constants. Parameter a is smaller (h is larger) when a large cluster becomes active at a previous step. That means that the speed of information dispersion is larger if a large cluster forms and acts.

Figure 5 shows the volatility auto-correlation which is defined as:

$$A(t) = [\langle s(t')s(t+t')\rangle - (\langle s(t')\rangle)^2]/\sigma \tag{14}$$

For $\delta = 0$, $c = 0$ case, the herding parameter is a constant as in original EZ model. The volatility increases linearly as time goes. $s(t)$ is anti-correlated



A B

Figure 6: Probability distribution if returns in interacting EZ model. (a) Simulation from different fluctuation r with $N = 10000$, $b = 0.001$, $\delta = 1$, $c = 0.6$. Adapted from Ref. [7]. (b) Data from German DAX for 10 minutes compared with simulation. Solid lines: $P(Z, r < 0.1)$ and $P(Z, r > 0.5)$ of German DAX data. Dashed lines: simulation of $P(Z, r < 0.1)$ and $P(Z, r > 0.5)$ with $\delta = 1.0$, $b = 0.00025$, $c = 0.6$, $t = 100$. Adapted from Ref. [10].

in time direction, which can be seen that s becomes small after a large cluster acts and breaks. For $\delta > 0$ when the herding parameter is no longer a constant, $A(t)$ shows a time correlation relation. Especially for $\delta = 0$, a power law behavior occurs. This long-range time correlation indicated a critical point of herding phase transition.

The probability distribution of returns is plotted in figure 6(a). It is obviously shown that the single peak for small r and double peaks for large r. The transition happens when $8 < r < 16$. Figure 6(b) compares the simulation result with German DAX data using slightly different parameters. It shows very good coincidence between two if appropriate parameters are chosen.

5 Summary

In this paper, I have reviewed the herd behavior and its application in financial market. With the basic idea of herd behavior that people are more likely to follow others' decision, the EZ model describes how information spreads and people act based on the information. Several phase transition phenomena can be found in this model. Fluctuation r as an order parameter shows the single-double peak transition in the probability of returns. Herding parameter a as an order parameter shows different power law behavior in the distribution of cluster sizes.

References

[1] T. Lux, Appl. Financial Economics **6**, 463 (1996).

[2] A.V. Benerjee, The Quarterly Journal of Economics, **107(3)**, 797-817 (1992).

[3] J. Jiang, W. Li and X. Cai, Physica A **387**, 528-526, (2008).

[4] T. Lux, M. Marchesi, Nature **397**, 498 (1999).

[5] V.M. Eguiluz and M.G. Zimmermann, Phys. Rev. Lett. **85**, 5659 (2000).

[6] V. Plerou, P. Gopikrishnan and H.E. Stanley, Nature **421**, 130 (2003).

[7] B. Zheng, T. Qiu, F. Ren, Phys. Rev. E textbf69, 046115 (2004).

[8] Y.B. Xie, B.H. Wang, H.J. Quan, W.S. Yang and P.M. Hui, Phys. Rev. E **65**, 046130 (2002).

[9] R. D'Hulst and G.J. Rodgers, Eur. Phys. J. B **20**, 619-625 (2001).

[10] B. Zheng, F. Ren, S. Trimper, D.F. Zheng, Physica A **343**, 653-661 (2004).