December 10 2008 David Zeb Rocklin

The Physics of Traffic Jams: Emergent Properties of Vehicular Congestion

The application of methodology from statistical physics to the flow of vehicles on public roadways challenges the standard physical notion of a “particle”. However, while the individual driver, is macroscopic, unique, and possessing free will, the character of the overall flow of traffic can be captured by ”microscopic” models similar to those standard in physics. “Car following” models and others such as the cellular automata (CA) and hydrodynamic models can capture both the phenomena observed by traffic engineers and the experience of being in traffic. These models explore the observed phase transition between free-flowing and congested states of traffic.

Introduction

Whenever one travels on a public road one enters into an enormous, complex system which labors to facilitate and coordinate the journeys of millions of independent, unpredictable drivers. The roadways have evolved through a mix of uncoordinated evolution and central planning and while they perform adequately, they are neither fully optimized nor fully understood. The casual driver is left to ponder many of the mysteries of the highway, not the least of which is the origin of the traffic jam that so inconveniences him.

Highways are altered every day, augmented by new lanes and ramps. Traffic researchers note that transportation costs account for around 15% of the United States Gross National Product [1], implying a substantial economic benefit to even minimal improvements in highway planning. Determining the manner in which to improve throughput (as well as other factors, such as safety) is a worthy and practical problem. It is also a fascinating one.

Traffic systems can include vast networks of interlocking roads controlled by traffic lights and other time-dependent effects. It is easy to imagine complex traffic flow patterns found on such roads. However, as we shall see, even highways far from any interchanges or obstacles can exhibit complicated emergent behavior. We shall see that as more cars enter the roadway traffic flow undergoes a phase shift between distinct modes. Traffic jams occur spontaneously, blossoming into existence and evolving in time and space before gradually dying out. The underlying order and structure common to all highways shines through the individual peculiarities of traffic conditions.

Experimental Studies

Techniques

The study of traffic patterns is an observational, rather than an experimental science. Researchers cannot control the traffic flow, they can merely choose when and where to observe it.

Aerial Photography

Aerial photographs can be used to plot the actual trajectories of cars. The following diagram (modified) is owed to Trieterer [2]:

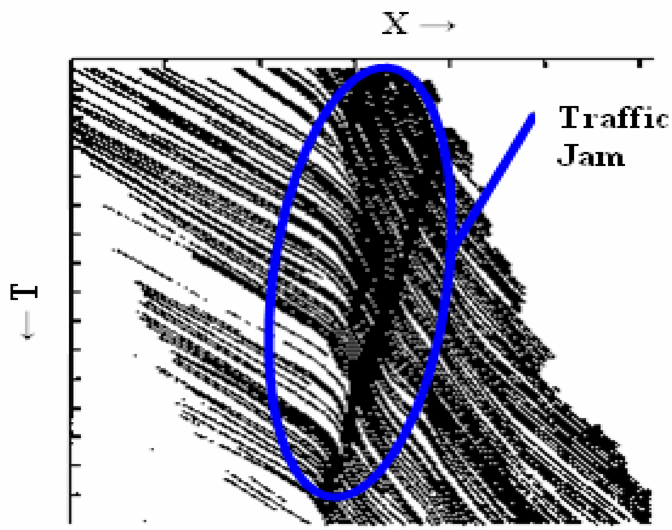

Figure 1
Trajectories of cars traveling along a highway, taken via aerial photography. Each line represents the path of a single car. Steep vertical lines indicate the slow movement characteristic of traffic jams.

Lines trace the path of single vehicles through space and time. Steep lines correspond to low speeds. We can see a collection of such low speeds in the middle of the diagram, indicating a traffic jam. We use the phrase "traffic jam" here to indicate a state of traffic in which vehicles travel very slowly due to congestion. We see that after developing spontaneously, the jam rapidly widens as many vehicles slow suddenly. It gradually narrows as it progresses backwards or "upstream" (that is, against the flow of traffic) along the highway, eventually dissipating altogether. Trieterer's study was the first to demonstrate that traffic jams occurred spontaneously.

Induction Loop Detectors

Trajectories are useful for following the flow of individual cars, but do not characterize the system as a whole. Researchers characterize traffic by its behavior at a given point on the road, rather than following cars. There are a variety of measurements which could be used to describe the highway. For example, an individual driver might well be most aware of his velocity and the distance between his vehicle and the next, his headway. A number of valid choices exist, but researchers generally make use of traffic's "fundamental diagram":

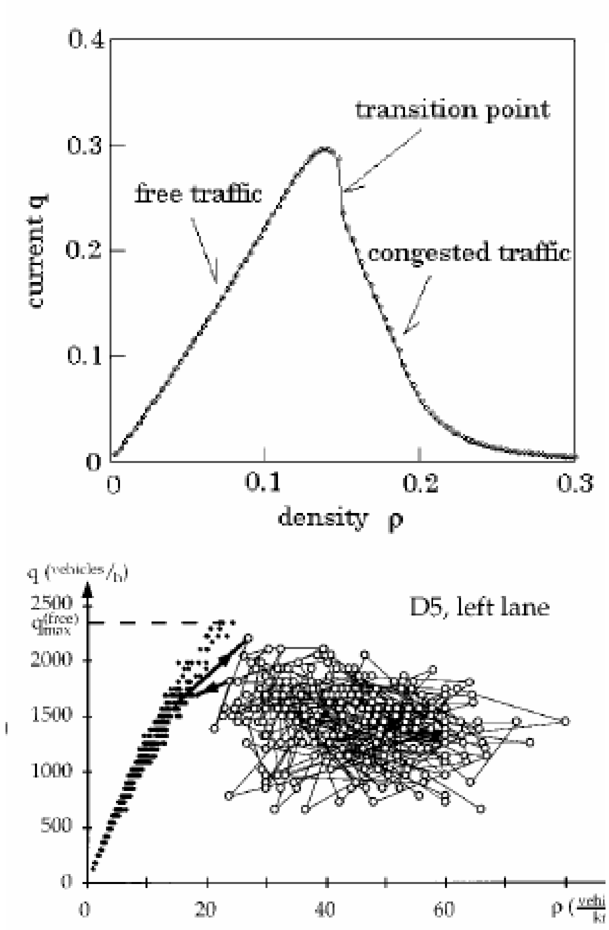

Figure 2
Data from a simulation [3] depict the transition from free to congested traffic. The simulation predicts a sharp transition to a congested phase. Real world data (bottom) drawn from induction loops placed on German highways [4] realize a similar fundamental diagram. The black lines show time evolution between congested points, each of which represents a time average over a few minutes. Note that the spread of values in the congested range is owed not to measurement error, but to a wide range of observed fluxes at a given density. This spread, characteristic of real world fundamental diagrams obscures the nature of the phase transition observed.

The density is the number of vehicles per length of highway. The flux is the number of vehicles which pass a given point on the highway per unit time. The points describe the state of a section of highway at subsequent times. At low vehicle densities, we have the "free traffic" regime, in which vehicles do not interact, and flux increases linearly with density. At high densities, vehicles reduce speed significantly as it becomes impossible to coordinate a smooth flow of traffic, leading to a reduction in flux even as density increases. This "congested traffic" region is significantly more complicated than free flow, reflected in a much greater spread of values in the experimental measurements in this region. In order to analyze the behavior of the transition between the free-flowing and congested phases, it becomes necessary to develop a theoretical model. Experimental data in the transition region is unclear and often seems to depend heavily on the particular highway observed. As we shall see, indirect observations do provide persuasive evidence for the characterization of the flow.

In generating fundamental diagrams and other tools of analysis, physicists use data taken from inductance loop detectors. These detectors can detect single cars as they pass, although typically data is subsequently time-averaged. Induction loops can measure both the time of crossing and the vehicle's speed.

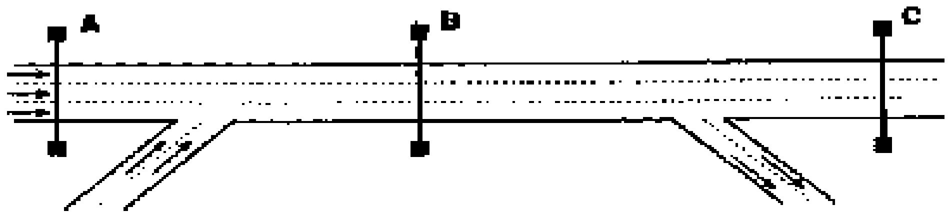

Figure 3 This figure originally appeared in a paper by F. L. Hall [5], collected in [6]. It shows the placement of induction loop detectors placed around freeway on- and off-ramps. Loop detectors count cars as they pass, as well as measuring their speeds.

Observed Phases of Traffic

Experimental observations show three distinct phases of traffic on highways: free flow, synchronized flow, and traffic jams. Experimental observations allow the characterization of each of these states, as well as the phase transitions between them.

Synchronized flow

A study by Rehborn and Kerner [4] shows that as traffic density increases traffic enters a state of synchronized flow. Even in the absence of outright traffic jams, regions of synchronized flow are sufficient to decrease the flux as density increases. Along with free traffic flow and jams, states of synchronized flow suffice to characterize all commonly observed traffic patterns.

In synchronized flow, vehicles move substantially slower than free flow. In Kerner and Rehborn's observations, synchronized flow occurred at around 50 km/hr, while the same highway saw 90 km/hr average speeds in free flow. Another feature of synchronized flow is the ambiguous effects on flux of greater vehicular density. This is reflected in Figure 2, where the synchronized flow region displays a much greater spread of data points than does the free flowing phase. Based upon their observations, Kerner and Rehborn identified three distinct types of synchronized flow:

1) Stationary, homogenous flow. These states displayed uniform flow which remained for as long as a few minutes.

2) Density waves. In these states, vehicle velocity was nearly constant, but waves in flux (and therefore density) were observed propagating through the line of cars. Distinct, often uncorrelated waves were observed in separate lanes.

3) Essentially nonstationary and inhomogenous flow. In these states no clear pattern emerges, although there is no reason to believe that density waves are not present.

All three types of synchronized flow were associated with sets of data points that covered a wide spread over the fundamental diagram. That is, no type of synchronized flow was found which showed a deterministic relationship between density and flux. This seems to indicate that the fundamental diagram is not in fact capable of characterizing traffic, but rather that the flux of vehicles depends upon their history and/or random processes, in addition to the traffic structure's density.

Traffic Jams

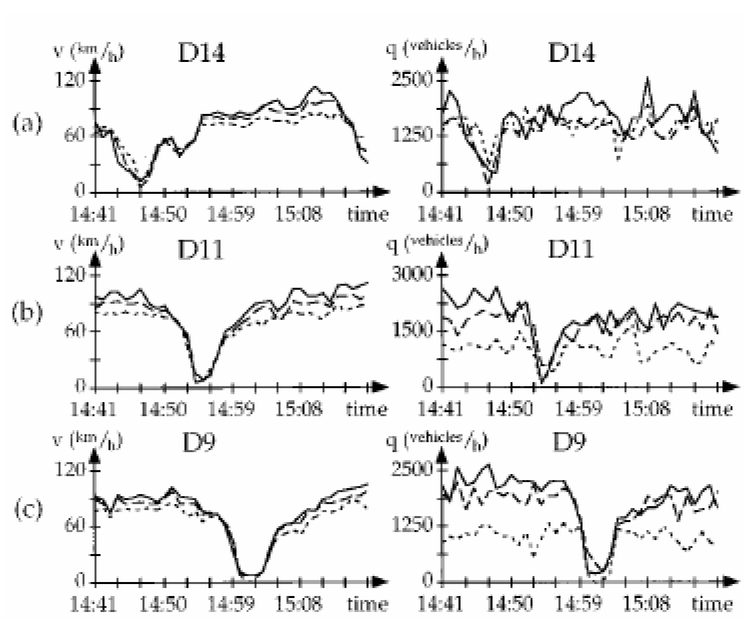

Figure 4
Kerner and Rehborn show the time evolution of velocity and flux through three detectors on a German highway. The severe dips indicate a traffic jam, moving upstream through the three detectors as time progresses.

In addition to their characterization of synchronized flow, Kerner and Rehborn observed many traffic jams. Traffic jams constitute a third phase of traffic, whose chief identifying feature is a drastically reduced velocity. While jams can occur due to some external effect, such as a narrowed roadway or an increase in flux surrounding an on-ramp, jams are observed appearing and disappearing spontaneously on open highway. They are generated by effects from within the traffic flow itself.

Above, velocity and flux are plotted as functions of time at three different detectors. The different lines on a graph correspond to the three different lanes. D9 is “upstream” from D11 and D11 is upstream from D14. We see that the jam, characterized by drastically reduced speeds, broadens and moves backwards, against the flow of traffic. It also leads to a synchronization of velocities between lanes.

Phase Transitions

In a separate study, Kerner and Rehborn [7] analyzed the transitions between the phases of traffic they observed. Using induction loop detectors, generally placed near highway on- and off-ramps, they found that free flowing traffic could transition to either synchronized flow or jams, depending on “small peculiarities” of the particular free flow configuration that they were unable to isolate.

They conclude that phase transitions are initiated by “critical localized perturbations of finite amplitude” which occur spontaneously in free flowing traffic of sufficiently high density. To support this theory, which originates in an earlier study [8], they observe that a spike in the flux of vehicles in an on-ramp initiates a phase transition immediately downstream of the ramp. Deterministic effects such as these suggest that perturbations in open stretches of highway, when they arise, can lead similarly to phase transitions. Of course, it is virtually impossible to directly observe a perturbation, perhaps a single car lightly decelerating, with loop detectors.

Kerner and Rehborn’s research also addresses a central issue of traffic phase transition analysis: the order of the transition. They found that immediately following a transition to a congested phase of traffic that the density upstream became too great to support a return to free-flowing traffic. They concluded that phase transitions from free flow to either synchronized flow or traffic jams were first order.

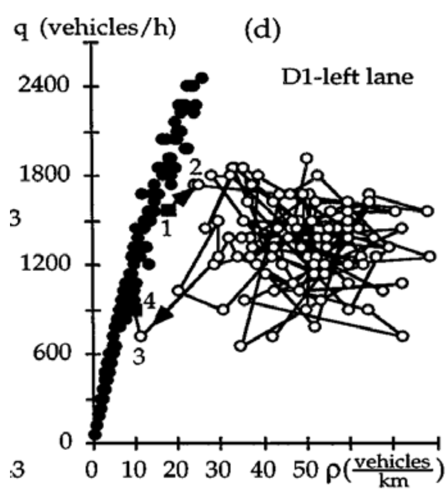

Figure 5
Kerner and Rehborn [7] show that phase transitions from free flowing to synchronized traffic are first order. The transition (points 1-2) does not reverse itself, but evolves broadly in the synchronized regime before returning much later to free flow (points 3-4). The black lines connecting points delineate time evolution.

Modeling and Simulation

Physics successfully describes the universe from length scales of galaxies to those of quarks, and beyond. However, the human scale possesses a factor unique in physics: intelligence. Applying physics methodology to describing traffic systems, whose constituent parts possess intelligence, is a new frontier.

Isaac Asimov, in one of his classic works of fiction [9], proposed psychohistory: a science of precise predictions of collective human behavior made through computer modeling of macroscopic effects, in direct analogy with the ability of thermodynamics to predict the macroscopic behaviors of systems without microscopically describing each constituent particle. Decades later, physicists have proven capable of predicting quite accurately the behavior of human beings (or at least those driving on a highway) by modeling them as particles, volumes of fluid, and harmonic oscillators.

Vehicular traffic is a complicated non-equilibrium statistical problem, and many different models have been proposed to study it. Microscopic models of traffic are those in which individual vehicles are treated as particles, each with their own rules of motion. For example, a Cellular Automata (CA) model might hold that vehicles travel on a discrete lattice of spatial points over discrete points in time. In the simplest model, each round vehicles merely advance a single site if the site in front of them is unoccupied, and remain stationary if it is not.

Simulation of Jams

There are a number of ways can be observed through simulations. We present a 1995 study from Nagel and Paczuski [1].

Nagel and Paczuski use a discrete time and space particle hopping model based on an earlier model proposed by Nagel and Schreckenberg [10]. Each section of highway is either unoccupied or occupied by a car traveling at an integer velocity between 0 and some v_{max}. Cars traveling at less than v_{max} and with more than v_{max} empty cells in front of them have room to accelerate, and increase their speed by one with probability ½, otherwise maintaining their speed. Cars with fewer empty cells in front of them than their velocity decelerate. Again with odds of ½ they decelerate more than necessary, occupying one of the cells between their current position and the cell directly behind the leading car. Additionally, even vehicles with no cars in front of them occasionally decelerate, initiating spontaneous jams as car after car behind them decelerates.

This model makes use of acceleration noise, in which some drivers over- or under-accelerate randomly to observe traffic jams. This is consistent with empirical observations of driver behavior, such as those of Kerner and Rehborn [7]. In this model, acceleration noise gives rise not only to jams, but to the emergence of a new critical point. The acceleration noise causes flux to drop off from its maximum at a density well below that at which a deterministic model would still be in the free flow regime.

In fact, the simulation reveals that this maximal flux is achieved by the cars exiting a long traffic jam, as they eventually accelerate and spread out. When traffic density is increased beyond the point achieved by cars exiting a long jam, traffic jams emerge which have an *infinite* lifetime. Thus, after these persistent jams flux is again that of the output from a long jam.

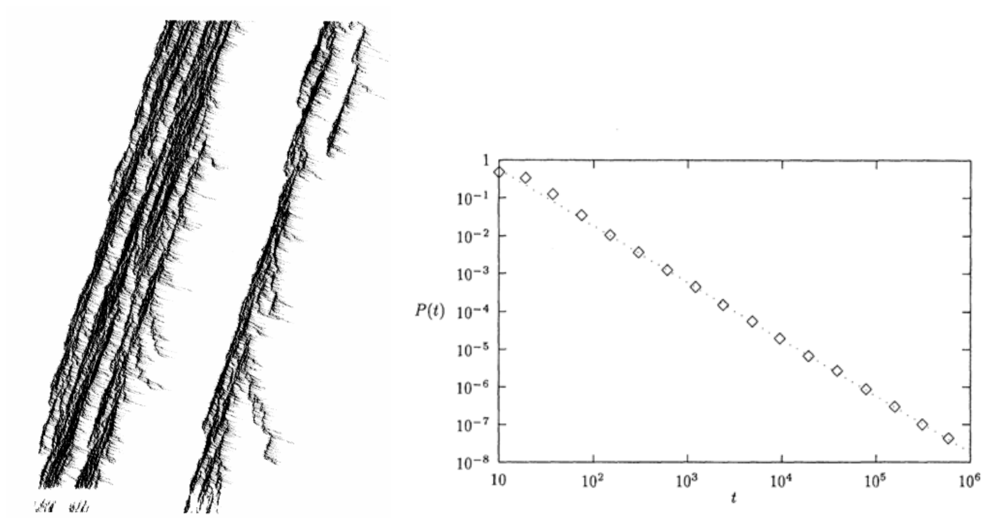

Figure 5 *Left*: Trajectories of cars in Nagel's model. Dark areas (steeply-sloped trajectories) correspond to traffic jams. Nagel's simulation captures traffic jam behavior consistent with experimental results: jams occur spontaneously and widen and drift "upstream" before dissipating.
Right: The probability distribution, P(t) of a traffic jam lifetime t follows a power law scaling, continuing to the times on the order of the length of the simulation. The exponent is measured from the simulation to be -1.5 +/- .01.

The authors conclude from their simulation that technology that improves driver control, such as cruise control (which automatically maintains a vehicle's speed) will not necessarily improve the flux along highways. While drivers will be able to maintain high speeds even at higher densities, these increased densities will eventually augment the effects of perturbations, leading to more large traffic jams which completely offset any advantages.

Optimal Velocity Models:

CA models satisfactorily recover many of the behaviors observed in traffic but rely on a somewhat obscure model of individual driver behavior. Optimal velocity models, first proposed by Newell in 1961 [11] begin from a more intuitive model of individual driver behavior.

In optimal velocity models, both space and time are continuous, and the velocity a vehicle chooses is based purely on the distance (headway) by which the vehicle in front exceeds it, or rather exceeded it some time ago:

$$\frac{d x_j (t+\tau)}{\mathbf{dt}} = V(\Delta \mathbf{x}_j(t))$$

That is, the velocity of the j^{th} vehicle is determined by a function V of its headway only. It is evaluated at a time retarded by τ, modeling a driver's finite reaction time. For τ sufficiently small, we may expand the governing equation to arrive at an acceleration equation:

$$\frac{d^2 x_j t}{\mathbf{dt}^2} = \frac{V(\Delta \mathbf{x}_j(t)) - \frac{d x_j t}{\mathbf{dt}}}{\tau}$$

The function V, which describes the velocity a driver chooses for a given headway, is somewhat arbitrary, but presumably vanishes at some nonnegative headway, and approaches the free velocity speed asymptotically as headway goes to infinity. The inverse delay time describes the sensitivity of the system.

This result shows that in the optimal velocity model vehicles behave as coupled harmonic oscillators. Extensions of the optimal velocity model which take into account not merely the position of the leading car but the next leading car or the following car increase the complexity with which car positions couple, and are commonly employed [3]. A numerical analysis of three cars with simple velocity profiles yields the following trajectories:

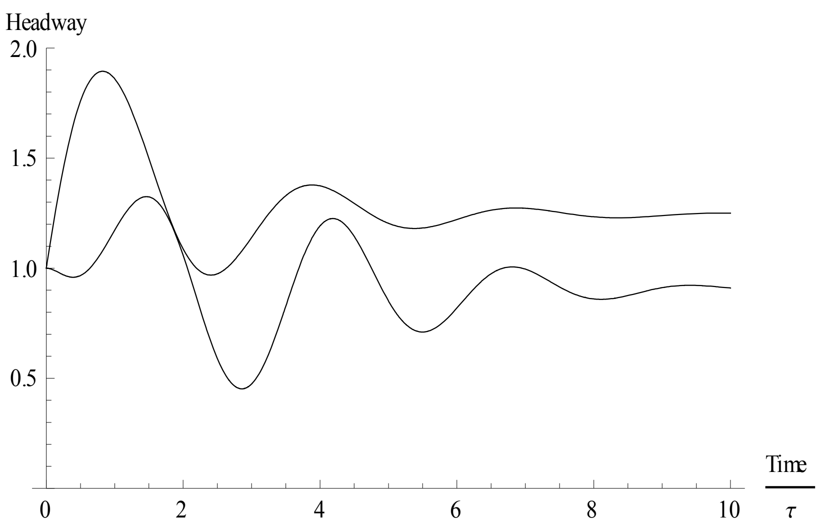

Figure 6 The headways of two cars in an optimal velocity model. The delay time leads to oscillatory behavior before an equilibrium is achieved.

Here, a car traveling at a constant velocity (as one with no car leading does in this model) is trailed by two cars with successively higher optimal velocities. These vehicles accelerate/decelerate until a delay time has passed since they have gone above or below their following distance. It is the delay time that leads to the oscillatory behavior for sufficiently close cars. It can also lead to collisions (headway = 0) if cars adopt sufficiently reckless velocity profiles. Eventually, oscillations are damped, and the cars proceed at the optimal velocity of the lead car. These cars plotted here have identical velocity profiles except for their optimal velocities. In real traffic, cars have substantially varying optimal velocity formulae and delay times, creating a complicated computer model.

Linear Stability Analysis

Beyond a stochastic distribution of driver profiles, noise enters into the system as vehicles fail to follow strict deterministic equations of motion. Drivers can temporarily misperceive their headway or see their delay time fluctuate stochastically. This noise can give rise to fluctuations in the vehicle flux even from a state of free traffic flow.

However, even in the absence of stochastic processes, linear stability analysis [3] reveals that uniform states (that is, evenly spaced vehicles traveling at a constant, uniform velocity) are unstable to density waves:

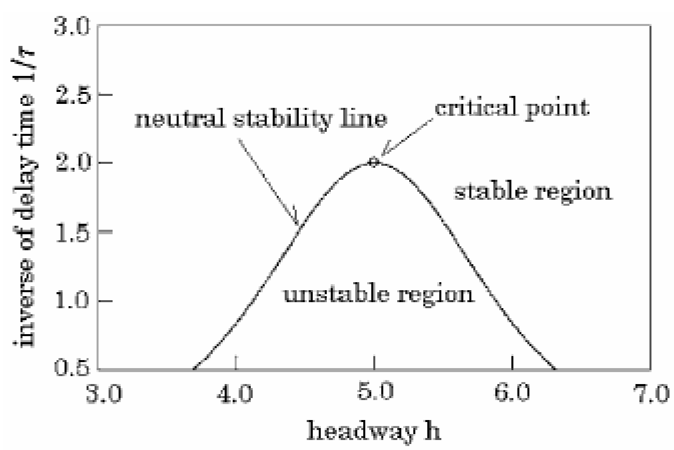

Figure 7:
The stability diagram for traffic flow in the optimal velocity model.
The critical point corresponds to the headway which maximizes flux.

The exact form of the neutral stability curve depends on the functional relationship between headway and velocity, but general features are apparent. Low delay times aid stability, and for sufficiently high delay times there exists a headway (which corresponds directly to a density) for which a state of uniform flow becomes unstable and density waves are produced. Nagatani shows that these density waves are back-propagating and composed of alternating regions of congested traffic and free traffic. This accounts for the reduction of flux around the critical density. Indeed, Nagatani's linear stability analysis recovers the same critical point as the flux-maximizing density.

Conclusion:

Describing highway traffic is a complicated, rich application of non-equilibrium statistical physics. Experimental observations and modeling have successfully characterized the critical behavior and self-ordering of vehicular traffic, including the emergence of spontaneous traffic jams, decisively demonstrating the relevance of physical technique to traffic systems. Traffic systems are a rich area of ongoing physical research.

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