What the flock is: emergent collective motion

Christopher Maffeo

December 19, 2008

Abstract

Many species of fish, birds and insects exhibit collective motion in the form of schools, flocks and swarms. Attempts to understand the mechanism of their collective motion suggest a many-bodied, local interaction, leading most studies to use a computational approach to study a discrete model of interaction between members of the flock. Over the past twenty years, simulations have demonstrated that collective motion may emerge from very simple behavior of each entity in flock, where the entity chooses its direction of motion from the average of its local environment. A model, inspired by studies of granular systems, shows collective motion of self-propelled particles that collide inelastically. Here, the essential features from these minimalist models are described, and the resulting implications for biological flocks are discussed.

Introduction

The emergence of a consensus arises from the interactions between the individuals and is known as collective behavior. Collective behavior encompasses a plethora of phenomena in the social sciences, economics, and animal behavior, such as the emergence of a common language in primitive societies [1], a belief in a price system [2], and the collective motion of animals, which is the focus of this paper.

The biological and evolutionary implications of flocking are compelling. For animals, flocking is emergent in an evolutionary sense. Simulations suggest that flocking emerges as a means of predator evasion (as opposed to a foraging tactic) [3], in accordance with sonar imaging studies that recently demonstrated waves in anchovy schools in response to predation [4]. While the biological role of flocking behavior is a tantalizing topic worthy of mention, this paper concerns the emergence of flocking behavior as a statistical, rather than biological, phenomenon.

There are a plethora of biological examples of flocking across many scales, ranging from animals to protozoa, to filaments within the cell [5]. Outside of biology, researchers are trying to apply collective motion to autonomous robots [5]. Moreover, Humans exhibit flocking behavior in crowd situations. A series of scale bridging images of collective motion are shown in Fig. 1. The presence of collective motion with similar features across a wide variety of scales indicates that the patterns that emerge have more with how the entities interact than the complexities of the individual entities.

Most models of flocking behavior describe the behavior of each animal in relation the position and velocity of that animal's neighbors. When certain parameters in the model are tuned, collective motion emerges spontaneously. The details of a minimalist model of collective motion are explained in the next section, followed by a qualitative description of basic extensions of this model. A second minimalist model is introduced, and the similar underlying features of both models are discussed. Finally, efforts to collect 3-D data of actual flocks are discussed.

Modeling approaches

The study of flocking behavior largely consists of simulations of a flock of autonomous animals with a defined set of behavior. Hereafter I will refer to

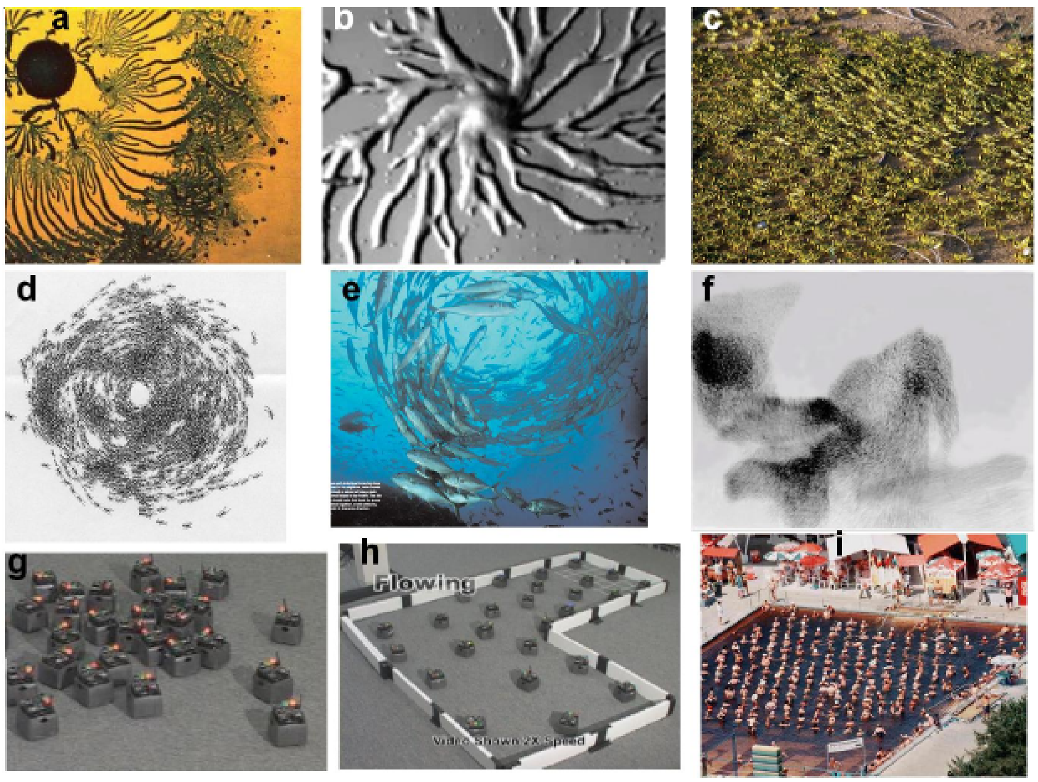

Figure 1: Images of collective motion indicating general behavioral patterns, taken from Vicsek [5]. (a) Bacterial colony grow out from a central colony, leaving a trace of bacterial behind. (b) Starving slime molds aggregate into a central rotating region. (c) Wingless locusts march in a field. (d) A rotating colony of South American fire ants. (e) A school of fish forming a vortex. (f) A flock of thousands of starlings. (g) A group of interacting robots about to disperse. (h) The same robots, instructed to keep a distance from one another. (i) People dispersed in a triangular lattice while sitting for hours in a Hungarian hot spring pool.

such models of animals as particles.

There are two trends for modeling of flock behavior. Within the physicist community, minimalist models of pairwise interaction between particles are sought that give rise to new modes of collective motion. Conversely, much of the work in the biological community seeks to accurately model the behavior of actual flocks to a precise degree, in an attempt to uncover the underlying individual behavior. I will focus on the first type of model, and the modes of collective motion that emerge from the models.

The Vicsek model

One of the simplest models of flocking behavior, the Vicsek model (VM) [5, 6, 7], uses particles that move a constant velocity v_0 in a two-dimensional, finite unit cell. At each timestep the particle looks at all the other particles within a distance r_0, and updates its direction of motion to coincide with the other particles, with some error white noise. More precisely, in two dimensions,

$$\theta_j(t+\delta t) = \arg\left[\sum_{k\neq j}^{N} e^{i\theta_k(t)}\right] + \eta\,\mathrm{rand}[-\pi,\pi], \tag{1}$$

where η is a coupling parameter that represents the amount of error for each particle, and the sum runs over all particles within some r_0 of particle j. When $\eta = 1$, the particles follow a random walk, and when $\eta = 0$, they rapidly become oriented in the same direction. The order parameter at a given instant for the system is the average direction of motion of all the particles,

$$\langle\phi\rangle_t = \frac{1}{N}\arg\left[\sum_{k}^{N} e^{i\theta_k}\right]. \tag{2}$$

As η is varied, a second order phase transition [5] in the order parameter occurs, and the rotational invariance of the system is spontaneously broken. The critical noise level, η_c, at which the system's behavior transitions from disorder to order depends on the average density of particles in the system; systems with a larger average density of particles are able to transition at higher noise levels. These results are summarized in Fig. 2

Models of flocking behavior incorporate error in two essentially different ways: the particle can either make mistakes when moving after perfectly

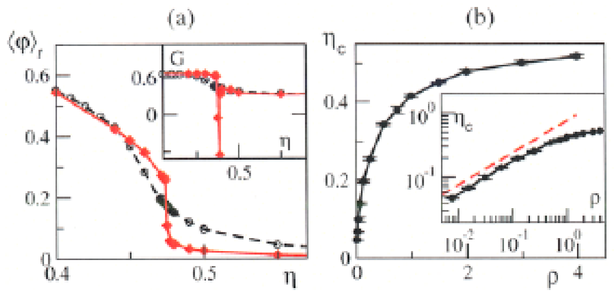

Figure 2: Phase transition in the Vicsek model. (a) Time average order parameter $\langle\phi\rangle_t$ vs. noise strength η for two VM systems with the same density, but different sized periodic unit cells ($L = 64$ black circles; $L = 256$ for red diamonds). The other parameters are the same. (b) The phase diagram for onset of collective motion. Kindly ignore the insets. Figure taken from Chaté et al. [7].

evaluating its neighbor's positions and directions (as in the VM), or the particle can make perceptual mistakes when evaluating each neighbor's position and direction. The distinction appears minor at first glance, however, when error is incorporated in the second manner, a first order phase transition is observed as the noise parameter η is varied. This is because with the second type of error, groups of particles that are locally oriented experience weaker noise than disordered groups. In the latter case, the correlated groups will not occur as readily, and when they do, the system will transition rapidly to an ordered state since they are affected less by the noise.

While the VM captures interesting features of emergence, it fails to capture one of the most basic properties of animal behavior: cohesion. That is, when VM particles are simulated in an infinite space with $\eta > 0$, the flock disperses [6]. A cohesive extension of the VM includes an interaction force that is attractive at r_0, but repulsive at shorter distances [6]. In addition to the emergence of orientation, the cohesive VM exhibits emergent cohesion as the strength of cohesive forces is increased. Essentially, the addition of cohesive forces extend the results of the VM to unbounded simulations cells.

Inelastic collision models

A second very recent, minimalist model considers self-propelled particles that interact only through inelastic collisions [8]. The model seek to find connec-

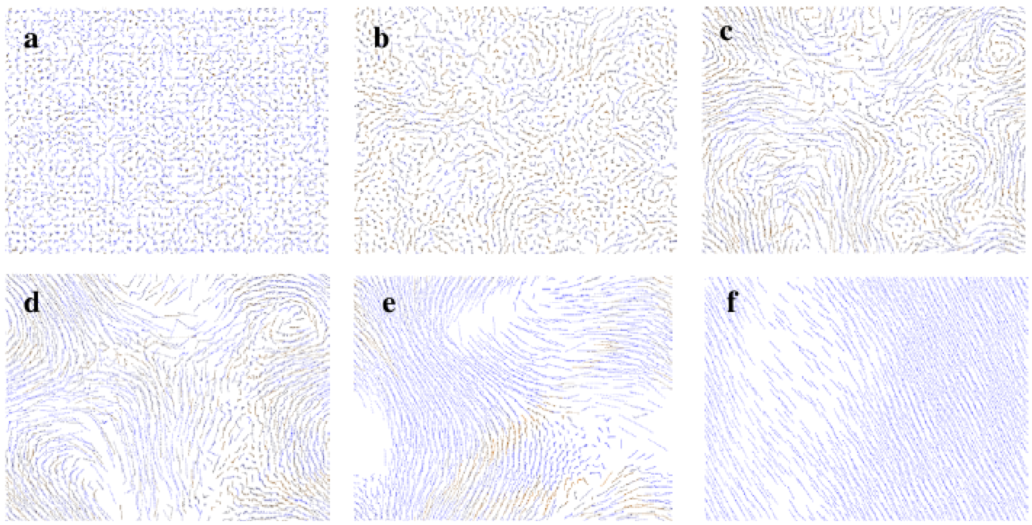

Figure 3: Time evolution of a system of 1200 inelastically colliding, self-propelled particles. Time increases in each frame. Figure taken from Grossman et al. [8]. For movies, see here.

tions between driven granular systems (such as rice on a vibrating plate), and minimalist flocking models such as the VM. The inelastic collision model results in transitions between disordered and ordered phases. Both migration (see Fig. 3) and vortex phases are observed, but the type of collective motion that emerges depends on the boundary conditions that are present. In the model, the underlying cause for the emergence of collective behavior is a gradual build-up of velocity correlation that occurs each time two passive particles collide, rather than active correlation of the velocities of two active, observant particles [8]. Nevertheless, the resultant collective motion displays considerable qualitative similarity to the collective motion in the VM model. I do not believe the models have been quantitatively compared.

Modes of collective motion

The known modes of collective motion exhibited by minimalist models of flocking behavior [5]:

1. a disordered phase
2. ordered planar motion

3. ordered rotational motion

4. a critical phase (flocks of different sizes moving in different direction)

The disordered, planar ordered and critical phases are all captured by the simplest VM, and. More complicated extensions of the VM with multiple tuning parameters have found 2-D phase diagrams for these states [3, 9]. Additionally, it was shown that when a flock is in an ordered state and the parameters controlling the phase of a flock are tuned, the flock exhibits hysteresis [9].

The critical phase, which occurs when the noise level η approaches η_c, may be most biologically relevant for studies of flocking birds or fish, because it is the phase in which maximal information is transferred between particles [5].

3-D images of a real flock

Studies of flocking behavior (especially 3-D) are primarily theoretical and computational, because data acquisition of flocks is difficult. However, the group StarFlag in Rome has been making strides towards 3-D data acquisition of starling flocks containing thousands of birds [10]. The group faces a variety of challenges, from capturing stereoscopic images of flocks, to tracking the trajectory of each bird in the images. Preliminary results show that each starling adjust its own direction to match its neighbors', as with the cohesive adaptation of the VM described above. Further, a splines behavior can be can be described by attraction to its neighbors at intermediate distances and repulsion at short distances. However, unlike most VM-like models, the starlings do not appear to have a fixed distance at which they pay attention to their neighbors. Instead, the groups state the birds watch their nearest six or seven neighbors regardless of distance, and suggests that this gives the flock enhanced cohesion.

Conclusions

Each minimalist model examined utilizes a local, noisy interaction between particles. When these particles interact, whether by active steering or by inelastic collision, their velocities receive minute correlations. The correlations build up because particles moving in the same direction are likely to

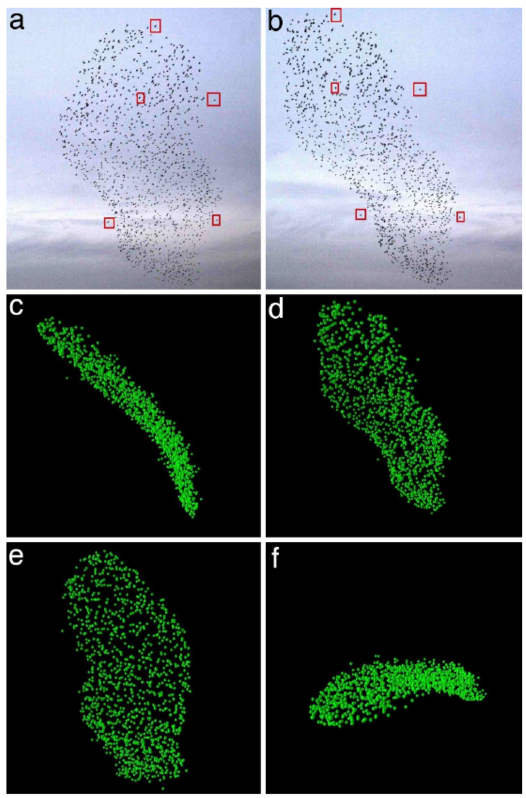

Figure 4: Capturing a flock in 3-D. (a,b) Stereoscopic photographs of a flock of 1,246 starlings. (c-f) 3-D reconstruction generated from a,b of the flock shown from four different perspectives. The perspective in b and d are the same. Figure take from Ballerini et al. [10]

interact repeatedly. If the particle density is sufficiently high, and the noise sufficiently low, the particles will transition from a disordered phase to an ordered phase, in which the velocity of the particles is aligned.

Information (conveyed through changing direction) is transferred through particles most rapidly in the critical phase, where different sized aggregates of correlated particles move in different directions [5]. Sonar observations of anchovy schools, which depict waves that propagate quickly (more than an order of magnitude faster than the average velocity of the school) through the school in response to sea-lion attacks [4]. Thus, critical phase may pertain to flocks that have formed to avoid predation.

Flocking stands out as a relatively simple example of collective behavior, that may shed some understanding on other more challenging, but more general issues in collective behavior. Human beings are extremely social animals that take part in networks of collective behavior that span the globe, typically without individual awareness. Understanding the nature of large scale collective behavior has applications in many fields, including economics, many branches of the social sciences, and of course in biological and physical sciences.

References

1. Cucker, F., S. Smale, and D. Zhou, 2004. Modeling Language Evolution. *Foundations of Computational Mathematics* 4:315–343.

2. Cucker, F., and S. Smale, 2007. On the mathematics of emergence. *Japanese Journal of Mathematics* 2:197–227.

3. Wood, A., and G. Ackland, 2007. Evolving the selfish herd: emergence of distinct aggregating strategies in an individual-based model. *Proceedings of the Royal Society B: Biological Sciences* 274:1637–1642.

4. Gerlotto, F., S. Bertrand, N. Bez, and M. Gutierrez, 2006. Waves of agitation inside anchovy schools observed with multibeam sonar: a way to transmit information in response to predation. *ICES Journal of Marine Science* 63:1405.

5. Vicsek, T., 2008. Universal Patterns of Collective Motion from Minimal Models of Flocking. *In* Self-Adaptive and Self-Organizing Systems, 2008. SASO'08. Second IEEE International Conference on. 3–11.

6. Grégoire, G., and H. Chaté, 2004. Onset of Collective and Cohesive Motion. *Physical Review Letters* 92:25702.

7. Chaté, H., F. Ginelli, G. Grégoire, F. Peruani, and F. Raynaud, 2008. Modeling collective motion: variations on the Vicsek model. *European Physical Journal B* 64:451–456.

8. Grossman, D., I. Aranson, and E. Jacob, 2008. Emergence of agent swarm migration and vortex formation through inelastic collisions. *New Journal of Physics* 10:023036.

9. COUZIN, I., J. KRAUSE, R. JAMES, G. RUXTON, and N. FRANKS, 2002. Collective Memory and Spatial Sorting in Animal Groups. *Journal of Theoretical Biology* 218:1–11.

10. Ballerini, M., N. Cabibbo, R. Candelier, A. Cavagna, E. Cisbani, I. Giardina, V. Lecomte, A. Orlandi, G. Parisi, A. Procaccini, et al., 2008. From the Cover: Interaction ruling animal collective behavior depends on topological rather than metric distance: Evidence from a field study. *Proceedings of the National Academy of Sciences* 105:1232.