Deconfined Quantum Criticality

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Abstract

According to the Landau-Ginzburg-Wilson (LGW) paradigm, any direct transition between two distinct symmetries should generally be of first order. The recently proposed theory, so called Deconfined Quantum Criticality (DQC), however, suggests that a second-order phase transition would occur between two ordered phases in a spin-1/2 Heisenberg model on a square lattice. There are controversies over the properties of transition, whether it is of first order or second order. In this paper, we review theory of quantum phase transition in two-dimensional spin system which is conjectured not to fit in the framework of LGW paradigm and discuss recent numerical works that support or disprove the second order conjecture.

1 Introduction

1.1 Two dimensional electron systems

Since the discovery of the high temperature superconductivity in the cuprate compounds, the two-dimensional electron gas model has been of more importance for both theoretical and experimental grounds. When the cuprate materials like La_2CuO_4 are doped with mobile electrons or holes (Sr), for instance, the superconductivity is induced in the CuO_2 plane, which may be described with the superexchange model. It has been believed that understanding of two dimensional antiferromagnetic model would reveal the underlying mechanisms. For the similar ground, Mott materials have also been intensively investigated since they exhibit either antiferromagnetic order or quantum disordered ground state depending on the exchange coupling constants and the geometry of the effective 2D Hamiltonian. In these contexts, the studies on the phase transition depending on the control parameters such as the strength of exchange couplings and the doping, have been thought to be highly relevant, especially to 2D electron systems.

1.2 Landau-Ginzburg-Wilson paradigm

The modern theory of phase transition have been developed largely due to the Landau-Ginzburg framework combined with the Wilson's renormalization idea. In this framework, the high energy modes are integrated out to be absorbed into the effective relevant parameters. By this mechanism, long wavelength and long time scale behaviors of physical quantities are essentially determined by the fixed points and the flow diagram in the parameter space. This is so called Landau-Ginzburg-Wilson (LGW) paradigm.

A central idea of the LGW paradigm is encapsulated in the order parameter, which characterizes the different symmetries of the phases on either side of the critical point. At the critical point, the fluctuation of the order parameter diverges so that the system of interest performs a kind of averaging out process over all length scale, smaller than the divergent correlation length set by the fluctuation. Because of this integrating out procedure, nonanalyticity arises to the free energy of the system. On the other hand, the order parameter chooses the optimal quantity to minimize the coarse-grained free energy so that phase transition occurs across the nonanalyticity. When the order parameter jumps in a discontinuous fashion, it is often said to be of a first order transition, while a continuity in changes called a second order phase transition [2].

So far the LGW framework has been successful in studying various phase transitions and critical behaviors. However, there is a question of its extent, especially to the transition inherent to quantum fluctuation.

1.3 Quantum Phase Transition

Generically, the quantum phase transitions are relevant at extremely low temperature, especially at zero temperature. In principle, quantum mechanics is dominant to a phase transition, when the temperature of the system becomes lower than some characteristic energy scale. Exactly at zero temperature, a transition occurs because of the changes in non-thermal parameter. Even though quantum mechanics seems to be restricted in the zero temperature regime, it can also affect to the some extent, competing with thermal fluctuation.

On the basis of the LGW paradigm, the critical phenomena of a quantum system at $T = 0$ can be mapped into the classical system with the dimensionality $d \to d + z$. Here the extra dimension z is a dynamical exponent, which tells how the space and the time are connected in phase transition. This exponent characterizes an temporal length scale, in addition to the divergent length scale in the classical system. This equivalence can be clearly understood by looking at the quantum partition function. By mapping the temperature of the system to the imaginary time, the quantum partition function transforms into the classical partition function. Naïvely speaking, such process should be nothing but adding one more dimensionality, which is not necessarily true. For instance, the dynamical exponent z at the critical point is not unity in case of the incommensurate Bose-Hubbard model. So, the asymmetry between the time and the space at criticality becomes apparent through the dynamical exponent z [5].

This guiding principle, the LGW framework, however, does not always work for the various quantum system. One famous case, by now old, but still challenging, is the continuous phase transitions between distinct quantum Hall states, which can not be described in terms of the local Landau order parameter. Another example, we will review, is the transition between the Néel state to a Valence Bond Solid (VBS) state in a 2D Mott insulator. The LGW paradigm, in general, predict the transition between two distinct symmetries such as the Néel state and a VBS state, to be either of a first order transition, or intermediate disordered state which breaks both symmetries, or another intermediate disordered state with neither order. However, a recently proposed scenario[3, 4] argues that the phase transition between two different broken symmetry state can be continuous, which is inconsistent with the LGW paradigm.

Before we review the phases of insulating quantum magnets in the following section, it should be emphasized the difficulties in this study. In real physical system, it is hard to construct the deconfined quantum critical system as was argued in [3, 4], though the VBS state was reported to be shown in some organic compounds such as $Zn_xCu_{4-x}(OD)_6Cl_2$. Furthermore, the numerical simulations of microscopic model via Quantum Monte Carlo involves infamous fermion sign problem. However, this issue is still of significance since it can shed light on the mechanism of the superconductivity as well as superfluidity in systems of trapped ultracold atoms.

2 Theory of Deconfined Quantum Criticality

In this section, we first review the phases in the model of two-dimensional Heisenberg Hamiltonian, and then discuss the connected field theories [1]. The key concept in the Deconfined Quantum Criticality (DQC) lies in the use of new degrees freedom in describing the critical point, instead of order parameter fields for the either phases.

2.1 Insulating Quantum Magnets

Let us consider a system of spin half moments on a two dimensional square lattice with the effective Hamiltonian.

$$H = \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} J_{\mathbf{r}\mathbf{r}'} \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}'} + \cdots = H_0 + \cdots \tag{1}$$

Here $\langle \mathbf{r}\mathbf{r}' \rangle$ denotes nearest neighbor pairs, and $J_{\mathbf{r}\mathbf{r}'}$ is the exchange coupling. We will first consider the case of isotropic coupling $J_{\mathbf{r}\mathbf{r}'} = J_0 > 0$ which leads antiferromagnetic ordering, and will later generalize with additional interactions between spins.

2.1.1 Néel State

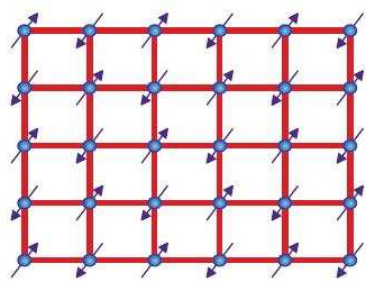

Figure 1: Néel ground state of the spin half antiferromagnet Hamiltonian H_0 with positive isotropic exchange couplings $J_{\mathbf{r}\mathbf{r}'} = J_0$. Even in the presence of fluctuations in spins $\mathbf{S}_{\mathbf{r}}$, the Néel state displays nonzero magnetic moment. Figure taken from [6]

In case of isotropic exchange coupling, the system develops a single quantum phase, the Néel order as illustrated in Fig 1. In the ground state, the spin rotation symmetry is spontaneously broken, while the lattice translation symmetry is still preserved.

$$\langle \mathbf{S}_{\mathbf{r}} \rangle = \eta_r \mathbf{N} \ , \qquad \eta_r = (-1)^{x+y} \tag{2}$$

In the Néel state, $\mathbf{N}$ slowly varies compared to the scale set by the lattice size, and the low lying excitation shows linear gapless dispersion. In other words, this AF states shows a long-range correlations between spins.

In describing the low energy spectrum, it is useful to take the Feynman path integral for the spacetime trajectories of all the spins in the system. In the long wavelength limit, the effective action can be obtained in terms of the smooth order parameter field $N(\mathbf{r}, \tau)$, by averaging over the square lattice spins.

$$S_n = \frac{1}{2g} \int d\tau \int d^2\mathbf{r} \left[(\partial_\tau \mathbf{N})^2 + v^2 (\nabla_\mathbf{r} \mathbf{N})^2 + s\mathbf{N}^2 + u(\mathbf{N}^2)^2 \right] + iS \sum_\mathbf{r} \eta_r \, A_r \tag{3}$$

As was shown by Haldane [7], the $O(3)$ nonlinear sigma model captures the fluctuation of the order parameter. Note the coarse grained action still exhibits the symmetries of the original Hamiltonian H_0. Here, the second term represents the Berry phase of all spins, while A_r denotes the area swept by the time trajectory of $\mathbf{N}$ on the surface of S_2 sphere in spin space. This is the result from the imaginary time mapping, which leads periodicity in spin configuration. In obtaining the Berry phase term, it is necessary to use the spin coherent state resolution in the imaginary time path integral. In the smooth configurations of the vector field $\mathbf{N}$, this term can be associated with the topological number, so called skyrmion number.

$$Q = \frac{1}{4\pi} \int d^2\mathbf{r} \; \mathbf{N} \cdot \partial_x \mathbf{N} \times \partial_y \mathbf{N} \tag{4}$$

Because of the ambiguity in the interior on the S_2 spin sphere, changes in Q by some integer amount are allowed (monopole event in spin configuration space). However, the rapid oscillation due to the factor η_r, the Berry phase term, or a Q changing event is largely suppressed unless Q changes by 4 (quadrupled) [7]. As will be discussed later, even this quadrupling changes is irrelevant at the quantum critical regime.

2.1.2 Valence Bond Solid State

The VBS state appears as an additional exchange coupling strengthens the bonds indicated by the ellipses in the Fig 2. In this state, spin rotation symmetry is preserved, while the lattice translation invariance is spontaneously broken. Either columnar or plaquette, the VBS state breaks Z_4 symmetry, as can be seen in Fig 2. On the other hand, the excitations from the VBS state can be created by breaking the spin singlet bonds, which is gapped $S = 1$ triplet quasiparticle. In fact, this triplet excitation is mobile, which makes difference from the Néel state.

Here, the four-spin interaction model [10] is useful to see how the states emerge depending on the control parameter.

$$H = J \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \mathbf{S}_\mathbf{r} \cdot \mathbf{S}_{\mathbf{r}'} - Q \sum_{\langle \mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\mathbf{r}_4 \rangle} \left(\mathbf{S}_{\mathbf{r}_1} \cdot \mathbf{S}_{\mathbf{r}_2} - \frac{1}{4} \right) \left(\mathbf{S}_{\mathbf{r}_3} \cdot \mathbf{S}_{\mathbf{r}_4} - \frac{1}{4} \right) \tag{5}$$

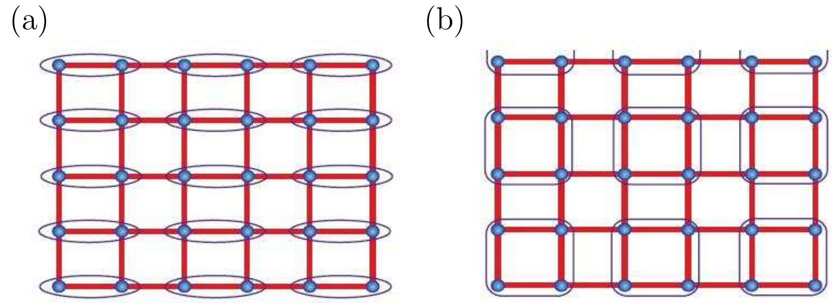

Figure 2: (a) Columnar VBS state of H_0+H_1 with $J_{\mathbf{rr}'} = J$. The spins are paired in singlet state. The ellipse in figure represents such spin singlet state $\frac{1}{\sqrt{2}}(|\uparrow\rangle_{\mathbf{r}}|\downarrow\rangle_{\mathbf{r}'} + |\downarrow\rangle_{\mathbf{r}}|\uparrow\rangle_{\mathbf{r}'})$. Rotations by $\frac{n\pi}{2}$ about a lattice site produces the four degenerate states. (b) Plaquette VBS state. Round square means S=0 combination of 4 spins. Figure taken from [6]

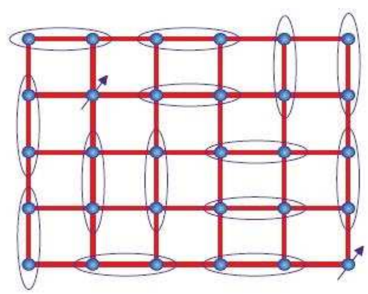

Figure 3: Spin liquid state. The two unpaired spin 1/2 spinons can move in the spin liquid background. Figure taken from [6]

According to the numerical computation [10], AF order appears for $g = J/Q \gtrsim 0.04$, and VBS order for $g = J/Q \lesssim 0.04$. Under the later condition, the VBS state can be characterized by a complex field order parameter, ψ.

$$\psi = \eta_{r+\hat{y}} \mathbf{S_r} \cdot \mathbf{S_{r+\hat{x}}} + i\eta_{r+\hat{x}} \mathbf{S_r} \cdot \mathbf{S_{r+\hat{y}}} \tag{6}$$

whose expectation is the measure of the lattice symmetry, or, Z_4 symmetry. In other words, $\langle\psi\rangle$ quantifies how far the state is from the VBS state.

2.2 Deconfined Criticality

2.2.1 Emergent Gauge Field

A convenient, but crucial description is CP^1 parametrization for the both state. In this alternative description, the vector field $\mathbf{N}$ of the AF state is expressed in terms of spin-1/2 complex spinon field $z(\mathbf{r}, \tau) = (z_1, z_2)$.

$$N_a = z^\dagger_\alpha \sigma^{\alpha\beta}_a z_\beta \tag{7}$$

where σ_a's are the Pauli matrices. In this mapping, one can easily note the $U(1)$ gauge redundancy upon local phase rotation

$$z \rightarrow e^{i\theta(\mathbf{r},\tau)} z \tag{8}$$

Since the order parameter field $\mathbf{N}$ remains invariant under the rotation, the alternative action in the spinon language should be coupled with $U(1)$ gauge field a_μ. Now at the critical regime ($g = g_c$), the quantum field theory for z and a_μ becomes

$$S_z = \int d\tau \int d^2\mathbf{r} \left[\left|(\partial_\mu - i a_\mu) z_\alpha\right|^2 + s|z_\alpha|^2 + u\big(|z_\alpha|^2\big)^2 + \kappa(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 \right] \tag{9}$$

where, the spin wave velocity v and the coupling strength g in eq (3) have been rescaled, so that s and u are appropriately changed from eq (3). Here, the last term is obtained from short distance fluctuation of spinon field, which presents dynamics to the gauge field a_μ. Actually, the effective action in eq (3) does not provide a trivial triplet ground state (VBS) with large s. For this description to include the VBS state, there should be a supplementary term, which will brings a discrete Z_4 symmetry as the parameter s increases beyond the critical value s_c. In searching for such term, the Noether current associated with the "Maxwell" term does an important role.

$$j_\mu = \epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda = \partial_\mu \zeta \tag{10}$$

Since the Noether current should be a conserved quantity in 2+1 spacetime dimension, it tells us the gauge flux in the $s > s_c$ should be conserved. Here, the massless scalar field was introduced, whose constant shift is equivalent to the gauge transformation of a_μ field. Indeed, the shift symmetry in the context of the bosonic field ζ should correspond to the physical object [7], Z_4 rotation symmetry, while it should be connected with the changes in the total gauge flux by 2π. In consequence, a new variable can be introduced to the theory, which breaks a continuous symmetry down to a Z_4 rotation symmetry.

$$V = e^{i\pi\zeta/2} \tag{11}$$

$$S_V = \lambda \int d\tau \int d^2\mathbf{r} \left[V^4 + h.c\right] \tag{12}$$

Actually, it was shown this variable has consistent behavior with ψ, the complex field order parameter in the VBS state [7]. So the gauge field a_μ can be identified with the VBS order parameter. They both changes the topological number (the skyrmion number).

2.2.2 Suppression of the Berry Phase

The most crucial ideas in [3, 4] lies in the irrelevance of the Berry phase term in the effective low energy theory in eq (9) as well as relaxation of the condition $|z_\alpha|^2 = 1$. The original paper [3] considers easy-plane anisotropy in two component spinon field. This case needs additional simple term in the Lagrangian density.

$$\mathcal{L}_{epa} = w|z_1|^2|z_2|^2 \tag{13}$$

with $w < 0$. As is discussed in [4], this term contributes to stabilize critical points.

A careful renormalization study on the scaling behavior of the coupling λ in eq (12) reveals that such coupling becomes dangerously irrelevant at the critical point. Instead of attacking the CP^1 description directly, it takes an alternative path by using the boson duality in 2+1 spacetime dimension. In this description, there is a correspondence between a spinon field z and a dual field Ψ: $z_\alpha \leftrightarrow \Psi_\alpha$. The perturbation term in eq (12), in this theory, then becomes

$$\mathcal{L}_V = -\lambda(\Psi_2^*\Psi_1)^4 \tag{14}$$

Near the fixed point $\lambda = 0$, the perturbative renormalization tells that λ becomes irrelevant when the scaling dimension of it is larger than $2+1$. Since the dual transformation implies the introduced bosonic field is a composite object out of the spinon field, it is expected to have larger dimensionality than that of XY model, which explains the irrelevance of the fourth power perturbation term.

In turn, this signifies that the skyrmion change event (monopole event) due to the Berry phase perturbation eq (12) becomes unimportant at the criticality. In more sophisticated language, the term that breaks the global $U(1)$ rotation symmetry of the spinon field down to a discrete Z_4 must be irrelevant in that regime. For this reason, the CP^1 parametrization is not a matter of choice in description, but an indeed valid model for low energy physics.

Before, we discuss the physical properties due to such suppression, it should be addressed the features of this description, in contrary to the LGW framework. The LGW procedure requires the gauge field a_μ to be a compact object. However, the irrelevance of the Berry phase term clarifies the gauge flux should be conserved at the critical regime. This means the gauge field a_μ becomes noncompact, from which the notion of "deconfinement" comes. Hence, the fractionalized spinon field (without charge) z and the emergent gauge field a_μ are natural degrees of freedom at the critical point.

2.2.3 Physical Properties near the Deconfined Critical Point

In this section, we will review the consequential physical properties in the deconfined criticality. As was argued in [3], the irrelevance of the magnetic monopole events implies there should be two distinct length scale from the VBS state to the critical point. One is the spin correlation length ξ and the other is a longer length scale ξ_{VBS}. The later length scale is a measure of the domain sizes in the VBS states. By the scaling analysis, the scenario predicts (i) large anomalous dimension η of the Néel order and (ii) the universal power law decays regardless of the state: the columnar VBS, the plaquette VBS, and staggered magnetic XY model in case of easy plane anisotropy.

Even the Gaussian approximation shows that η must be an order of unit, which does not fall into the LGW framework. Since the two point correlation function decays in power law with $D-2$ ($D = 2+1$), the correlation of the Néel order parameter, extracted from the spinon field theory, becomes

$$\langle \mathbf{N}(\mathbf{r}) \cdot \mathbf{N}(\mathbf{0}) \rangle \sim \langle z^*_\alpha(\mathbf{r})_\alpha(\mathbf{0}) \rangle^2 \sim \frac{1}{r^{2(D-2)}} = \frac{1}{r^{D-2+\eta}} \tag{15}$$

Some of recent numerical studies seem to support this prediction, which will be reviewed in the following section.

2.3 Numerical Studies

As to the numerical studies on the deconfined quantum criticality, there is no concrete conclusion so far. In this section, we will introduce a couple of recent works, which either suggests a continuous phase transition or a weak first order transition.

Recently, Sandvik's study upon the four-spin coupling seems to be strong in supporting the DQC. In that work, the probability distribution of the VBS order parameter was calculated depending on the number of lattices. Remarkably, circular symmetry was obtained, which seems to imply the Berry phase perturbation is irrelevant. This also means the bosonic field ζ, which is a dual to the emergent gauge field a_μ, develops a Goldstone boson. Another feature, emphasized in the paper [10], is unusually large anomalous dimension η. To get the numbers, the author adopted the ground-state projector quantum Monte Carlo method in the moderate size lattice system. Using the finite size scaling [2], the author succeeded to extract the fairly large exponent $\eta \sim 0.26 \pm 0.03$.

However, there are some issues that put some questions on the results. Most of all, Sandvik's study is limited on the finite size system, though it was enough to subtract the critical exponents. Also, it is restricted to exactly zero temperature. Recently Jiang *et al.*[11] performed a stochastic series expansion at finite temperature for larger system with the same four-spin interaction model. According to their results, the distribution of the VBS order parameter does not indicate either the columnar VBS order or the plaquette VBS state far from the criticality. This work disproves the Sandvik's claim, which implicitly assumes the VBS state should be developed as the control parameter $g = J/Q$ gets larger. So the emergent $U(1)$ symmetry may not be able to identify the VBS state. In addition, they demonstrated a weak first order transition.

In addition to the numerical studies, Nogueira *et al.* [9] performed a renormalization group analysis for the model suggested in the DQC scenario. In this work, they argues that the result obtained from large spin should be able to explain the behavior of spin half spinon. In consistent with the numerical study in anisotropic easy-plane case in [8], their study presents a weak first order transition, inconsistent with the DQC framework. Even

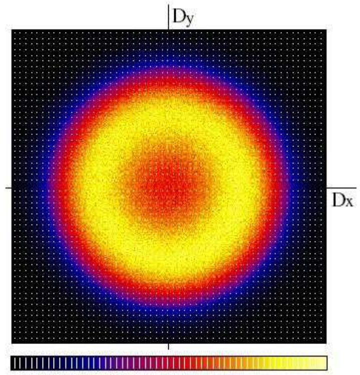

Figure 4: The probability distribution of the VBS order parameter ψ of the four spin interaction model for the 32×32 lattice system at $g = J/Q = 0$. The ring shapes seems to support the idea of an emergent $U(1)$ symmetry, irrelevant to the discrete Z_4 anisotropy of ψ. Figure taken from [10]

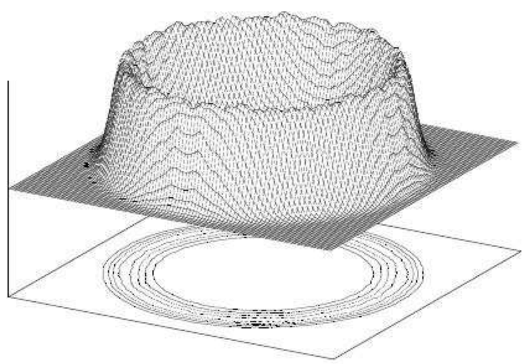

Figure 5: The probability distribution of the VBS order parameter ψ for the 96×96 lattice at finite temperature $\beta Q = 30$ and at $g = J/Q = 0$. The ring shape symmetry, *i.e.,* $U(1)$ rotation symmetry does not identify the VBS phase. Figure taken from [11]

though large scale Monte Carlo simulation seems to confirm the suppression of the Berry phase term so that spinons are confined at the critical regime, it does not provide any evidence of the continuous transition. In case of the isotropic $SU(N)$ antiferromagnet, however, deconfined spinon seems to govern the physics upon the onset of second order transition [9]. In this study, the crossover exponent[2] were examined to extract the scaling of the correlation length.

In sum, there is no conclusive result which verifies the validity of the DQC scenario.

3 Conclusion and Remarks

In this paper, we have shortly reviewed the recently proposed hypothesis, the Deconfined Quantum Criticality across the two distinct symmetry states. The key steps in that scenario are the adoption of the CP^1 fractional parametrization and associated $U(1)$ gauge field. The renormalization analysis proposes the Berry phase term in the LGW framework must be suppressed so that it removes the compactness of the gauge field at the critical point. From this mechanism, a continuous phase transition between the Néel order phase and a VBS state becomes possible, depending on the control parameter. This is the point that does not fit into the framework of the Landau-Ginzburg-Wilson. The term "deconfinement" means such transition does only survive at the critical regime, which will eventually be wound back to the either phases.

One of the salient features, predicted by this conjecture, is the large anomalous dimension in the correlation of the Néel order parameter. Some numerical works seems to support this expectation, but it is also argued to be plagued with limitations. Even if the emergent $U(1)$ gauge field do an important role at the criticality, it may not properly capture the VBS state. To resolve the controversies over this proposal, we need to investigate the role of the emergent gauge field in real physical system.

Even though it's difficulty to realize in the experimental ground, the analogy between the quantum magnets and the Mott insulators in the optical lattice system seems promising, in testing this scenario. Indeed, there are nice mapping from the 2D fermionic lattice systems to the optical lattice, in which filling fraction in the superfluity does the crucial role.[1] In theoretical sense, on the other hand, a construction of the effective low energy field theory may also be reexamined in the LGW framework. The proposed four-spin exchange model can be a first step to perform such work. Regardless of its validity, this proposed idea must be of great interest in searching the framework to explain the phenomena which are beyond the LGW paradigm.

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