THE EMERGENCE OF SOCIAL HIERARCHIES

OR

"HOW TO WIN FIGHTS AND INFLUENCE PEOPLE"

Term Essay
Physics 569, Fall 2006
Jonathan K. Whitmer

ABSTRACT

The Bonabeau model attempts to explain the origin of hierarchical structure in society on the basis of random aggressive interactions between agents in the society. Agents which win most of their fights rise higher in the hierarchy, while those who lose most fights occupy the lower tiers. This model grew out of ethological studies on eusocial wasps, but has been subsequently modified to represent broader social structures. The model exhibits a transition from an egalitarian to a hierarchical structure as a function of the population density, the importance value of the interactions, and the rate at which past results are forgotten. This paper presents the background for and assumptions in the model, discusses its phase behavior, and the extensions that have been made to the model.

I. INTRODUCTION

Hierarchical structure is evident in many animal and human societies, and it plays a very important role in the everyday life of these organisms. For agents in a tiered society, the rank one takes inside the group is important in determining what type of resources are available [1-3]. Higher-ranking agents tend to have better access to food, and an increased chance to reproduce. Hierarchies are also important in the division of labor among agents in a society, by defining priority orders by which entire group tasks are undertaken [4]. Many factors contribute to the organization of hierarchies, such as differences in size or aptitude among the agents of a particular society [4, 5].

The hierarchies that emerge in these systems can often be related to dominance relationships among the agents of the society [3, 4]. The formation of a dominance hierarchy can be thought of as a spontaneous symmetry breaking in the strength of individuals, which have identical status in the absence of the hierarchy [6]. Conflicts between the agents occur through aggressive displays, or fights, and memory of these determines which agents are dominant over others. The relationships between pairs of individuals are stable, and often form a linear hierarchy. A hierarchy is called linear if an agent's rank is uniquely determined by the number of agents that are subordinate to it [2]. There exists one agent who is dominant above all others in the group, the second ranking agent in the hierarchy dominates over all individuals except that one, and so on, with the lowest ranking individual dominating no other agents. Non-linear hierarchies can form, however, if the interactions between agents are not transitive [2]. The simplest such system requires at least three individuals: individual A dominates B, and B dominates C, but C dominates A. The structure need not be so simple, however, and higher order loops can exist.

One question that comes to mind about the organization of these hierarchies relates to the fundamental properties a society must have in order to self-organize into a dominance hierarchy. A vast array of social systems exhibit such hierarchies [3-6], leading one to inquire if there is a simple model which extracts the general trends of dominance that are observed. In addition, it would be very interesting to examine if intrinsic or extrinsic behaviors determine the structure of the hierarchies that form. Many models have been proposed to this end [2-15]. In the following paper, I will examine the biological motivations and assumptions behind the Bonabeau social model [6], which is firmly rooted in the assumption that these hierarchies can self-organize via randomized interactions. This model has many modifications, some of which will also be discussed.

II. BACKGROUND

Models of self-organizing hierarchies have their basis in observation of various animals and humans [2, 4]. The dominance behavior of many animal societies, such as cows, chickens, and many social insects has been documented [3-6]. As one example, Theraulaz, et al conducted ethological examinations of eusocial *Polistes Dominulus* wasps [3, 5]. These wasps form colonies of about 20 individuals, and there are few intrinsic differences between individual wasps. Hierarchical interactions are very important in this species, consisting of fights or aggressive acts between pairs of individuals. The end result of these is that the pairs of individuals adopt a dominant or submissive posture. Eusocial societies are characterized by the presence of reproductively specialized individuals. Indeed, there is a reproductive hierarchy in the females of this species, which may have connections to the outcome of these dominance interactions [3-5].

The number of these interactions varies depending on an individual's rank in the society. Individuals residing near the top of the hierarchy are most often engaged in fights with the other inhabitants of the colony, while the most subordinate individuals are very passive, rarely engaging in fights. The resulting hierarchical structure is highly linear. As such, this species provides an ideal place to begin developing a model of hierarchy formation, since complicated dominance loops are absent. The ranking of individual female wasps within the hierarchy relates closely to their reproductive status, and division of labor with regard to caring for the brood [3-5].

The object of interest in these experiments is the origin of the hierarchy. To examine this, they started each colony with thirteen newly emerged female wasps who had not previously interacted with each other, and marked them so that observations were facilitated. If a wasp died they were promptly replaced with a new wasp, so that the number of wasps per colony was kept at thirteen. In addition, to perturb the hierarchy and observe the resulting structure, the alpha individual would be removed from the hierarchy and replaced with a newly emerged wasp. During the experiments, the total number of dominances and submissions of each wasp were recorded. These observations are used to create the dominance index X_i, defined by

$$X_i = \frac{\#\,\text{dominances}}{\#\,\text{total encounters}} = \frac{\#\,\text{dominances}}{\#\,\text{dominances} + \#\,\text{submissions}} = \frac{D_i}{D_i + S_i}$$

These experiments lead to three general conclusions [5]:

1) Removal of alpha individuals results in a sharp increase in the number of dominances in the colonies while the new hierarchy is settled. 45% of these interactions were dominances by the new alpha agent, while the next two places in the hierarchy accounted for 35% of the remaining dominances.

2) An individual's probability of dominance in an interaction is highly dependent on the individuals rank. This, in turn, can be indicated by an agent's dominance index X_i.

3) Individuals with an increased tendency to dominate an interaction will interact more often, while subordinate wasps interacted with each other much less frequently.

This lead Theraulaz, et al. [3] to propose a model which relies on individual learning, with reinforcement taking place at two levels. First, an individual's dominance probability over another individual is influenced by its past experience in dominance interactions. Second, the probability of an agent to partake in these interactions should be determined by their place in the hierarchical structure, thus also on their past dominance experience. This model is the precursor to the Bonabeau model [6] discussed later in this paper.

III. BASIC MODEL

The basis of the model is a positive feedback mechanism proposed by Hogeweg and Hesper [7]. Individuals who have won more fights have a higher probability of winning future fights than agents who have not won. An agent's rank in society is thus hypothesized to not depend on intrinsic factors (though these certainly play some role). A society that is examined begins as a completely egalitarian one, and any hierarchy which forms is a function of its past experience [3-6].

Each individual i has a characteristic 'force' F_i, which determines its ability to win an aggressive interaction, and is a function of the agent's past winning experience. This term is often stated to be a resource holding potential, which in animals means greater food supply and chances of finding a mate, and in human societies can be likened to wealth or political influence. An agent's status is increased through winning competitions with other individuals, or projecting dominance in a hierarchical interaction. Likewise it is decreased through losses, or subordinate behaviors. When a contest is won, the force increases by a constant value, according to [6]

$$F_i = F_i + \delta^+ .$$

Likewise, a loss results in a decrease of force,

$$F_i = F_i - \delta^- .$$

For simplicity, it is usually assumed that both δ^+ and δ^- are equal to one, so that the number of wins minus the number of losses gives the force for a particular agent. It is assumed that the outcome of fights is probabilistic, and that the probability of agent i beating agent j is given by a strictly increasing function which gives equal probability for equally matched agents to win. The Fermi function [3, 5, 6]

$$Q_{ij}^+ = \frac{1}{1 + e^{-\eta(F_i - F_j)}}$$

satisfies these conditions. In addition, this function is properly symmetric in the sense that the conjugate probability of i losing to j is the same as the probability of j beating i. The term η which appears in this expression is an adjustable strength of force parameter.

The last part of the model relates to the frequency of interaction. Each agent is assumed to have a probability of interaction that is an increasing function of their societal rank, and the probability of two individuals fighting in a unit time is the product of these probabilities. Again, a Fermi function was chosen [5],

$$P_i = \frac{1}{1 + e^{-F_i/\theta}}$$

with θ an adjustable interaction threshold. This model was seen to reproduce exactly the linear hierarchies seen in experiment, and reproduces features of the experimental data that do not immediately follow from the assumptions [4, 5]. For instance, the hierarchical profile, defined as a function f satisfying

$$X_i = f(R_i)$$

where R_i is the rank of individual i, is exactly reproduced with experimental error by the model.

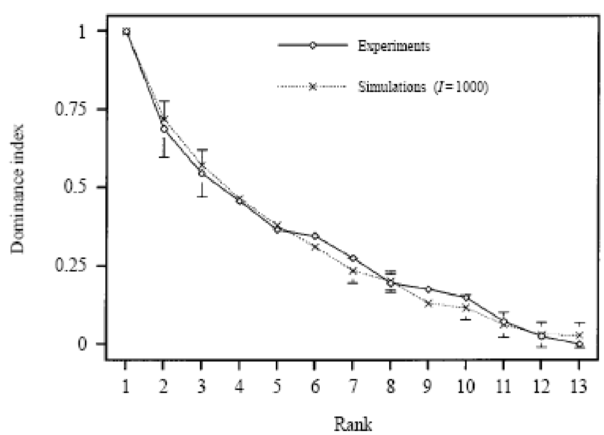

Figure 1: Hierarchical Profile of a colony of 13 *P. Dominulus* wasps. From [5].

The model's biological basis is the subject of debate [4, 5]. This is in part because it involves quantities, such as the force, which are hidden variables, unable to be probed by experiment. Also, the assumptions raise a few issues. The assumption that all wasps start the colony on equal footing is not accurate in most cases. Also, some differentiation between wasps occurs after the colony has formed, and this may not be taken into account by mere learning [3, 5]. Nevertheless, the model has success in describing the experimental data, and has also found application to bumblebees, primates, and crayfish [4].

IV. BONABEAU'S MODEL AND OTHER MODIFICATIONS

The Bonabeau model takes the above model and changes two essential parts, to examine how this affects the hierarchies that form [6]. First, the probability of interaction is not a constant function, rather the individuals involved in the simulation are taken to diffuse via random walk on a two dimensional square lattice. When an agent attempts to enter a location occupied by another individual, the two fight, and their random walks are then continued. The rules of the simulation are fixed so that n-agent interactions do not occur for $n > 2$. When i enters j's space and wins, the two switch places [16]. If i loses, their locations remain the same. The second modification is that a forgetting function is introduced. In the absence of a contest, an agent's force relaxes to zero in accordance with the rule

$$F_i = F_i - \mu \tanh(F_i).$$

Hence the relaxation rate is greater the greater the value of F is, asymptotically approaching μ. The inclusion of forgetting is motivated by observations of cockroaches, which show that agents removed from the competition do not immediately regain their previous rank [6].

The probability of relaxation is proportional to the probability of not encountering another agent on the lattice, and so is directly proportional to the density $\rho = N/(L^2)$. Hence an interesting interplay should develop between the strength of the force (as mediated by η) and the density of individuals on the lattice. The above model was examined through mean field theory and Monte Carlo simulation to determine what types of hierarchies developed at different values of η and ρ. The position of each individual within the hierarchy is determined by its probability P_i of a win in a random contest at some time, given instantaneously by the average over Q_{ij}, $i \neq j$. A measure of the structure of the hierarchy is better made by using the dominance index X_i, which loosely corresponds to the probability P_i averaged over all times. If a stable hierarchy is formed at some time during the simulation, the two are equal in the asymptotic limit $t \to \infty$.

Two distinct structures were noted to appear in this examination. When the value of the population density was too low, the society was egalitarian, as averages of P_i over the length of the simulation resulted in a value of 0.5 for each of the agents. The hierarchical profile is thus uniform, though instantaneously it fluctuates. Above a critical population density, this egalitarian structure vanishes. The time averaged values of P_i are no longer equal for all agents. The fluctuations in instantaneous rank giving rise to the egalitarian society are suppressed, and all agents attain a stable value of P_i after some time. The rankings are all different, and the corresponding structure is hierarchical.

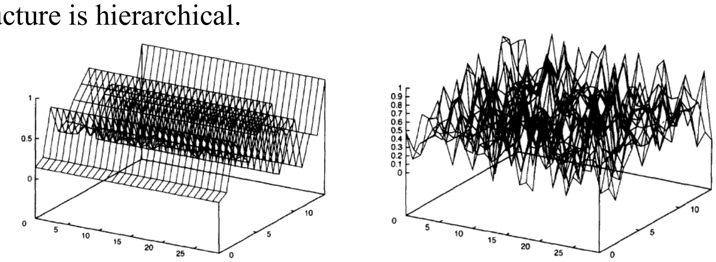

Figure 2: Time evolution of P_i for densities above (left) and below (right) the transition. From [6].

An order parameter relating the degree of hierarchical structure for this model is the variance σ^2 of the time averaged dominance indices with regard to their mean value [6]. This is 0.5 in all cases, since the initial conditions of the system are for agents to be equal. In the egalitarian society, all dominance indices are equal, thus this order parameter is zero, while it reaches a finite value when the society has a stable defined structure. For the evolution and relaxation rules defined above, the variance exhibits interesting behavior. For values of η much greater than one, the phase transition from disordered to hierarchical phases is continuous. The variance just beyond the critical density for transition is small. If η is much less than one, a jump is observed, meaning that the phase transition is discontinuous. This can be seen in figure 3.

Most of the features of the symmetric model can be described and understood using a mean field theory. To do this, we need to understand how the forces evolve in time. The relevant quantities are the number of dominances, submissions, and the force of each agent. If the probability of interaction is given by the density of N individuals on the lattice, then these equations evolve as

$$\frac{dD_i}{dt} = \frac{\rho}{N-1}\sum_{j\neq i} Q_{ij}^{+}$$

$$\frac{dS_i}{dt} = \frac{\rho}{N-1}\sum_{j\neq i} Q_{ij}^{-}$$

$$\frac{dF_i}{dt} = \frac{dD_i}{dt} - \frac{dS_i}{dt} - \mu g(F_i)$$

where $g(F_i)$ is a generic forgetting function with $g(0) = 0$. These equations can be numerically integrated with uniform initial conditions $F_i = 0$, $D_i = S_i = 1$ to obtain a hierarchical profile at late times.

These initial conditions form a fixed point of the system, since the probabilities Q_{ij} are then equal for all agents initially, so that the sums defining dS/dt and dD/dt are equal. Since the difference of those two quantities determines the evolution of the force, and the force starts at zero (so that $g(F_i) = 0$), the force does not evolve. To test the stability of this fixed point, a Gaussian random force is added to the integration scheme in order to crate noise. If the system is at an unstable fixed point, the fluctuations will destroy the state, and the integration will lead to a different late time solution.

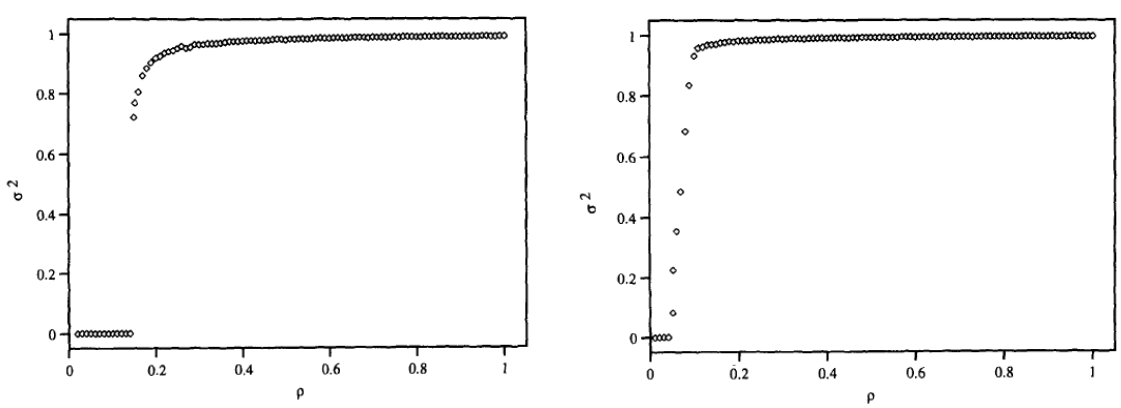

Figure 3: Plots of the order parameter as a function of density for the discontinuous transition (left) and continuous transition (right). These correspond to the behaviors of the system approaching a subcritical bifurcation (for the discontinuous) and a supercritical bifurcation (for the continuous). Values of simulation parameters are μ = 0.1, N = 10, η = 1.5 (left), 4 (right). Plots from [6].

If the forgetting factor is always zero, the results of the original paper [3] are obtained. In this limit, also, the force is seen to increase linearly with time, proportional to the density of the system. In the presence of forgetting, a bifurcation occurs when the population density ρ is increased, taking place at $\rho_c = \mu$ [6]. There is no hierarchical structure for lower values of ρ. The nature of the bifurcation depends, in turn, on the value of the strength of force, η. If this is less than the critical value given by $\eta_c = 2N/(1+N)$, the bifurcation is subcritical, and in the complementary case it is supercritical. This corresponds to the location of the tricritical point at ρ_c, η_c. In the case $\eta < \eta_c$, the egalitarian state is linearly stable, and small fluctuations will not cause the state to be unstable, even for $\rho > \rho_c$, though strong enough fluctuations when ρ grows beyond the critical value can make the hierarchy appear abruptly. This gives the discontinuous transition for small values of η. The hierarchy in this case is linearly structured, with each agent's dominance factor equally spaced in the interval [1, 0]. In case $\eta > \eta_c$, the continuous transition appears, driven by fluctuations in the hierarchical profile [6]. With this in mind, re-examination of figure 3 shows us examples of transitions in the system that occur in the subcritical region (left graph), and the supercritical region (right graph).

It is noted by Sousa and Stauffer [16] that this analysis does not really correspond to an equilibrium situation, as the forces F_i do not stabilize at long times for most non-egalitarian cases. This is due to the fact that the hyperbolic tangent forgetting function does not act fast enough to curb the growth of F_i at large values, since the hyperbolic tangent makes the forgetting function roughly just the constant μ. This is not necessarily a problem, since the biologically relevant variable is the dominance index X_i, which does become stable at late times, and not the force F_i. In any event, they were lead to propose modified forgetting rule

$$F_i = F_i(1-\mu)$$

which ensured that the forces reached equilibrium. This rule makes the spontaneous hierarchy formation characterized in the Bonabeau model disappear [6, 16]. The variance of the system is slowly increasing as a function of density, but there is some hierarchical structure evident at all concentrations. A sharp phase transition can be re-imposed on the system by changing the feedback mechanism so that the strength of force parameter η is fixed as a constant for the first ten or so steps to allow fluctuations to build, and then replaced by the variance of the system for later evolutions [16], though none of the bifurcation structure observed before is noted.

In another paper, Stauffer and Martins [10] examined the case where the winning and losing reinforcement is asymmetric, with the loss being more detrimental than the win is beneficial. This amounts to the choosing of $\delta^+ \neq \delta^-$ in the original Bonabeau model [6]. If this is the case, the phase transition has an onset at much lower densities, due to the fact that the value of an agent's force feeds back into the calculation of the probability for them to win another competition. Since the resulting force is negatively skewed toward losses, losers will tend to keep losing, and it is less likely that rank-changing fluctuations will occur.

Mean field theory for a related version of the model was performed by Lacasa and co-workers [17]. This proceeded in much the same way as the mean field theory by Bonabeau et al., with the inclusion of the multiplicative rule due to Sousa and Stauffer [16]. Thus agents have a probability of ρ to fight, and complementary probability (1- ρ) not to fight. In the latter case, they will relax, as defined by the model. One difference is that, instead of the dominance index, the hierarchy is assumed to be determined by the differences in force parameters. The increase in a successful fight is set to 1, and the decrease set to a constant f. This allows the equation for evolution of the status to be written as [17]

$$\frac{dh_i(t)}{dt} = (1-\rho)(1-\mu)h_i(t) + \frac{\rho(1-\mu)}{N-1}\sum_{j\neq i}\left[Q_{ij}^{+}(h(t)+1) + Q_{ij}^{-}(h(t)-f)\right].$$

The egalitarian phase corresponds to $h_i = h_j$ for all i and j, though this is not a stable fixed point for all values of ρ and f. In studying the linear stability of the above equation it becomes clear that there is a critical density

$$\rho_c = \frac{4\mu(N-1)}{\eta(1-\mu)(1+f)}$$

when the egalitarian state becomes linearly unstable, at which point a supercritical bifurcation emerges. A sequence of these bifurcation diagrams is given in figure 4 for increasing values of N in the symmetric $f = 1$ case, and the special case of $N = 2$ with an in the asymmetric case. Since N is fixed, the path of increasing ρ implies that the system size is being lowered. Note that in the symmetric graphs, the bifurcation diagram is tiered. The full differentiation between individuals does not become apparent until density is almost 1. The differentiation between individuals in the multiplicative model is thus very complicated. The hierarchical structure not only develops at densities greater than the critical density, but it changes structure as evidenced by the evolution of the fixed points of the system with density. This has not been noted in the Bonabeau model, though the authors give a plot for values of η = 0.001 and μ = 0.0001, placing the system in the subcritical region (figure 5). Hence the structure of the bifurcations cannot be easily compared.

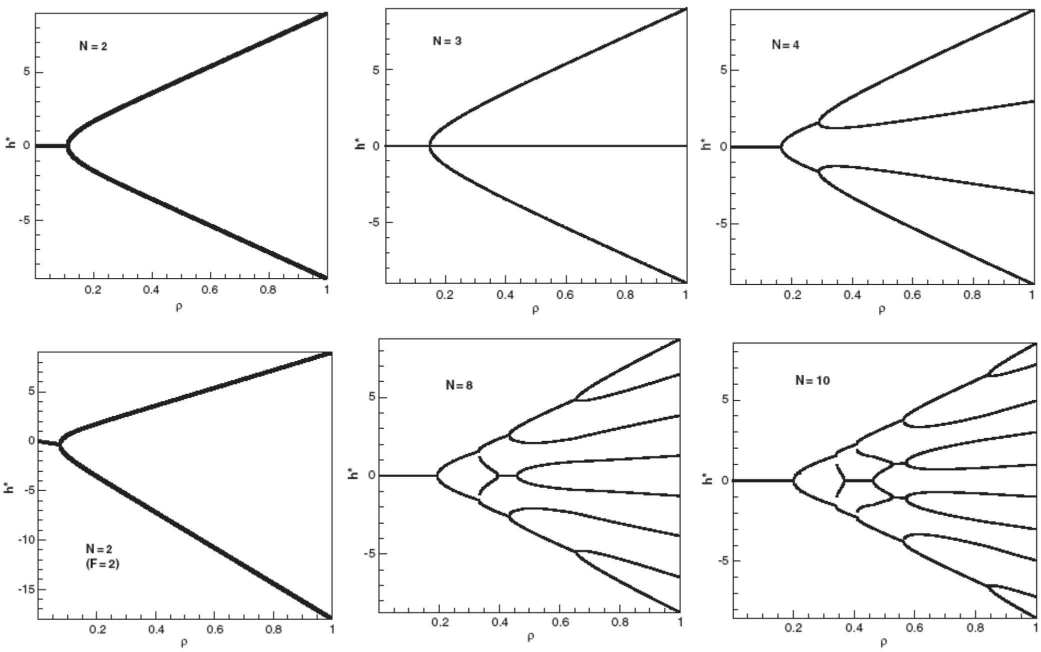

Figure 4: A sequence of bifurcations for varying N. The two on the left correspond to N = 2, with the top case being symmetric, and the bottom asymmetric, with f = 2. The other plots have varying N, but the same conditions as the top left graph otherwise. μ = 0.1, η = 1 in all cases. From [17].

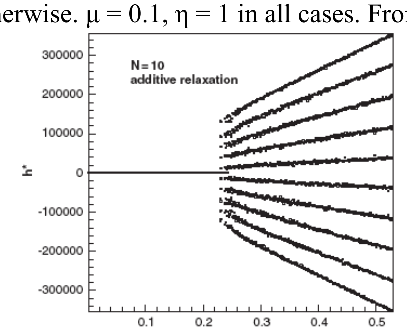

Figure 5: Subcritical bifurcation of Bonabeau model. μ = 0.0001, η = 0.001. From [17].

V. EXTENSIONS BEYOND THE BASIC MODEL

Odagaki and Tsujiguchi have studied modifications of the standard Bonabeau model to 'timid' [15] and 'challenging' [14] societies. The aim of these models is to reproduce systems where an overall societal trend influences the frequency of combat, and therefore the dominance structure. In the timid society [14] individuals always try to avoid fighting, so the standard algorithm is modified by a preference for individuals to move to vacant sites during their random walks, should they exist. If no such sites exist, the neighbor with the weakest force is chosen to fight against. The population profile can be monitored by looking at each person's dominance index. For $X_i < 1/3$, the individuals are classified as losers, $X_i > 2/3$ are classified as winners, and the rest are the middle class. Monte Carlo simulations of this system found that two types of hierarchical societies can develop, and three types of hierarchies are observed, with dependence on ρ and μ.

The first transition is from the egalitarian state to a middle class society where a few losers appear, but by and large all differentiation is within a tight range around the average. This transition is continuous in the order parameter, thus second order. No 'winners' were observed in the simulations. The second transition is from the middle-class society to a society with a greatly reduced middle class where most agents are winners or losers. The transition to this society is of a discontinuous character. The nature of these transitions makes sense when they are thought about in detail. Initially, any deviations which occur should be around the middle class because individuals will want to avoid fighting, so that any persons which are forced to fight will cause differentiation, but try to diffuse away quickly so that no winning accumulates. Losing, however, will accumulate since interactions with the weakest links are preferred if an interaction must take place. In contrast, the highly dense society forces interactions on all persons, so that winning and losing both accumulate, and will do so quickly at some critical density where avoiding random walks become impossible. Provided that the strength of force is large enough to facilitate large differentiations between individuals without a long string of wins or losses, the problem for determining the critical density is similar to a percolation problem, as it should happen at the density when a significant portion of the agents are surrounded by other agents in all directions. These behaviors are depicted in figure 6.

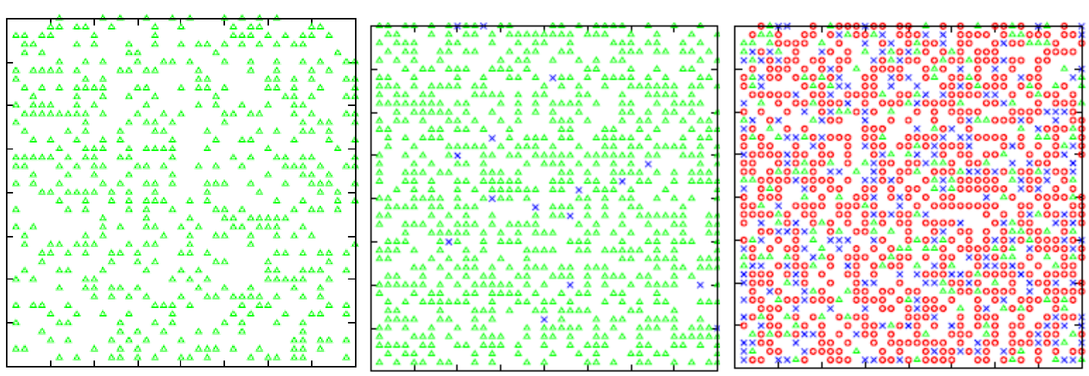

Figure 6: The sequence of phases in the timid society model. Green triangles are middle class, blue crosses are losers, and red circles are winners. In this sequence, $\mu = \eta = 0.1$, $\rho = 0.3, 0.5, 0.7$ successively. From [15].

A challenging society is modeled with an opposite diffusion condition [14]. Instead of avoiding cells that are occupied already, an individual is inclined to attract to them, and will always challenge the strongest of any of its neighbors. To reduce the tendency to stay in one place and keep fighting losing battles, individuals are forbidden to fight the same agent in successive turns. In this society, the transition to hierarchical structure occurs at a much lower value than in the standard Bonabeau model, as is to be expected for a system of individuals who prefer to be around each other, raising the effective local density (which determines the frequency of fights). In this model, a small number of winners appears, signifying the onset of the hierarchy. Since all other adjacent agents will prefer to fight with a winner over all others, but the probability of their winning is against them, there should be fewer winners than other classes. The losers and middle class occur in similar numbers. The new behavior that emerges in this model at moderate densities is a spatial correlation between agents, who attract each other, forming villages with a similar type of orientation. Winners are always positioned near the center of the villages, and lower class individuals on the outer edges. The authors liken these to medieval villages where a few wealthy dominated the village, and surrounded themselves with subordinate individuals. At higher densities, the villages span the entire lattice with a percolating cluster. This is depicted in figure 7.

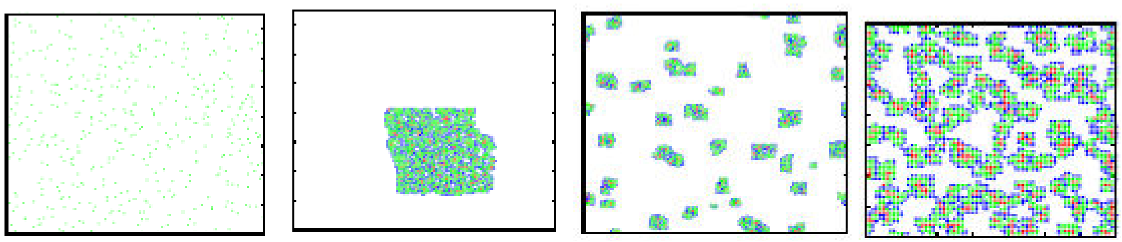

Figure 7: A sequence of images with increasing density for the challenging society with $\mu = 0.1$, $\eta = 0.05$. The densities are, from left to right, $\rho = 0.022, 0.056, 0.086, 0.714$. The spatial structure goes from diffuse, to one large village, to several smaller villages, to a percolating metropolis. From [14].

VI. CONCLUSIONS, PROPOSALS, OPEN QUESTIONS

This essay presents a model for spontaneous hierarchy formation in agent based social systems, beginning from its roots in the ethology of wasps, and concluding with applications to realistic human social systems. The phase behavior of the model is very interesting, and non-trivial, the result of bifurcations in the fixed points of social rank from egalitarian society to a highly tiered structure. There are more variations of the model than those presented here. For instance, a related model with simplified rules, has been solved analytically by Ben-Naim and co-workers [11, 18, 19]. This model shows a sharp transition as the relative influence of competition is increased, generating an upwardly mobile middle class, whose status increases with time, and a lower class whose status remains essentially constant. This model has also been generalized to multiplayer games where more than two agents can interact at a time, and has found some success at explaining general behaviors of major team sports [20].

The Bonabeau model is not perfect, and in some ways can be improved to represent more realistic assumptions. One possible modification to this model that would yield more realistic behavior would be a response that places more importance on victories over stronger individuals, instead of incrementing by the same constant amount, which tends to perpetuate one's status even if they manage to be victorious against a strong opponent. This type of behavior is prevalent in human societies such as college sports rankings, which places more emphasis on wins over strong teams as opposed to weak ones. It would be interesting to see how feeding back the strength of victory would influence the hierarchies formed. Also, inclusion of different types of individuals, such as a few of the timid a few of the bellicose and a few of the indifferent, has the potential to generate interesting behavior in simulations. It is not clear to me whether this type of society would generate a spatially stable structure such as the challenging society does, a diffuse structure such as the timid society does, or something in between. The type of behavior could also be tied to the winning record, such as was the case in the original wasp model, through another feedback mechanism.

Another possible modification, which was initially done by Sousa [16], is that of changing the behavior of each agent so as to make the winning probability dependent on individual histories with each other agent. Such a model has the potential to give rise to intransitive triads, or other highly non-linear hierarchical structures, though so far no one has studied this. It would be interesting to see what types of assumptions are necessary to bring about a nonlinear hierarchy in these competition models, making the model more widely applicable to explain other animal and human societal interactions. In any event, the fact that such a simple model can create complex social environments is very interesting in its own right, and worthy of more study.

VII. REFERENCES

[1] G. S. van Doorn, G. M. Hengeveld and F. J. Weissing, Behaviour **140**, 1305 (2003)

[2] J. L. Beacham, Behaviour **140**, 1275 (2003).

[3] G. Theraulaz, E. Bonabeau and J.-L. Deneubourg, Journal of Theoretical Biology **174**, 313 (1995).

[4] S. Camazine, J.-L. Deneubourg, N. Franks, J. Sneyd, G. Theraulaz, and E. Bonabeau, *Self-Organization in Biological Systems* (Princeton University Press, 2001).

[5] E. Bonabeau, G. Theraulaz, J.-L. Deneubourg, Bulletin of Mathematical Biology **61**, 727 (1999).

[6] E. Bonabeau, G. Theraulaz, J.-L. Deneubourg, Physica A **217**, 373 (1995).

[7] P. Hogeweg and B. Hesper, Behavioral Ecology and Sociobiology **12**, 271 (1983).

[8] M. Kirley, in *Proceedings of the 8th annual conference on Genetic and evolutionary computation*, p. 159 (ACM press, 2006).

[9] D. Stauffer. International Journal of Modern Physics C **14**, 237 (2003).

[10] D. Stauffer and J.S. Sa Martins, arXiv:cond-mat/0308437 (2003).

[11] E. Ben-Naim, F. Vazquez, and S. Redner, European Physical Journal B **49**, 531 (2006).

[12] C. Schulze and D. Stauffer, arXiv:cond-mat/0405697 (2004).

[13] C. Hemelrijk, Animal Behaviour **59**, 1035 (2000).

[14] M. Tsujiguchi and T. Odagaki, arXiv:physics/060606 (2006).

[15] T. Odagaki and M. Tsujiguchi, Physica A **367**, 435 (2006).

[16] A. O. Sousa and D. Stauffer, International Journal of Modern Physics C **11**, 1063 (2000).

[17] L. Lacasa and B. Luque, Physica A **366**, 472 (2006).

[18] E. Ben-Naim and S. Redner, Journal of Statistical Mechanics, L11002 (2005).

[19] E. Ben-Naim, B. Kahng, and J. S. Kim, Journal of Statistical Mechanics, P07001 (2006).

[20] E. Ben-Naim, F. Vazquez, S. Redner. arXiv:physics/0512143 (2005).