

# Current Studies of the BCS-BEC Crossover via Feshbach Resonance

Philip Powell  
Physics 569

December 13, 2006

## **Abstract**

The past decade has seen marked advances in cooling techniques, making possible the realization of Bose-Einstein condensates and degenerate Fermi gases, most notably in the alkali gases. In this paper we review the properties of these systems and the experimental techniques which have made them possible. We then discuss the recent use of a Feshbach resonance in Fermi systems, which allows one to move continuously from a loosely bound Bardeen-Cooper-Schrieffer (BCS) limit to a molecular Bose-Einstein condensate (BEC) limit. Finally, we review studies that have demonstrated the emergence of a condensate in both limits, consider the properties of these two limits, and discuss future work which may elucidate the nature of the BCS-BEC crossover.

# 1 Introduction

The past twenty years has seen a remarkable advance in experimental techniques in condensed matter physics, not the least of which relate to the preparation of ultracold systems. The ability to routinely achieve temperatures in the nanokelvin range has greatly increased our capacity to study both fermionic and bosonic systems in their non-classical limits. Of course, degenerate Fermi gasses and Bose-Einstein condensates possess vastly different properties, which follow directly from their respective statistics. However, in the last ten years it has become possible to connect these two systems and convert directly between them by using a Feshbach resonance [1, 2].

The ability to continuously convert a degenerate Fermi gas into a Bose-Einstein condensate is a triumph of modern experimental physics and a theoretical physicist's dream. What was henceforth a theoretical ideal is now realizable in the laboratory and it opens the door for a plethora of experiments which may test current theories of many-body systems. In particular, the capacity to tune the two-body interaction strength arbitrarily allows one to study the phase transition in much greater detail than one could do with a finite number of data points.

This paper is organized as follows. In Sec. II we present the properties of the free Fermi and Bose gasses as well as the qualitative effects of two-particle interactions. We also discuss the principles of the transition from the Bardeen-Cooper-Schrieffer (BCS) limit to the Bose-Einstein condensation (BEC) limit. In Sec. III we survey the advancements made in preparing and studying the alkali gases at ultra-low temperatures and in particular their use as BECs and degenerate Fermi gases. In Sec. IV we present the technique pioneered by Inouye and Ketterle, which utilizes a Feshbach resonance to arbitrarily tune the two-particle interaction. We describe the thermodynamic properties of the resonance and survey current experimental work utilizing the technique, discussing the new information which it has given us regarding the behavior of very low temperature systems. Finally, in Sec. V we discuss the future of the technique and its potential for use in elucidating the properties of many-body systems.

## 2 Principles of the BCS-BEC Crossover

### 2.1 The Free Fermi and Bose Gasses

The properties of the free Fermi and Bose gasses follow directly from the statistics of each, which are given by the well-known distribution functions

$$f(\epsilon, \mu) = \frac{1}{e^{(\epsilon-\mu)/k_B T} \mp 1} \quad (1)$$

where the top sign is for Bosons and the bottom for Fermions. If we consider a free Fermi gas of  $N_F$  spin 1/2 particles the Fermi energy is given by  $\epsilon_F = \pi^2 \hbar^2 n_F^2 / 2m_F L^2$  where  $n_F$  is defined by

$$N = 2 \times \frac{1}{8} \times \int_0^{n_F} 4\pi n^2 dn = \frac{\pi n_F^3}{3} \quad (2)$$

The Fermi temperature ( $\epsilon_F = k_B T_c$ ) is therefore

$$T_F = \frac{\hbar^2}{2mk_B} (3\pi^2 n_F)^{2/3} \quad (3)$$

where  $n_F = N_F/V$  now denotes the density of Fermions (not the ‘‘Fermi number’’  $n_F = (3N/\pi)^{1/3}$ ).

Meanwhile, for a spin-0 Bose gas (1) is clearly an increasing function of  $\mu$  so that the total number of particles satisfies the inequality [3]

$$N_B \leq \int_0^\infty d\epsilon D(\epsilon) f(\epsilon, 0) \quad (4)$$

where  $D(\epsilon)$  is the density of states. Bose-Einstein condensation will roughly occur at a temperature  $T_c$  when (4) is satisfied with equality. Thus, we have

$$N_B = \frac{V}{4\pi^2} \left( \frac{2m_B k_B T_c}{\hbar^2} \right)^{3/2} \int_0^\infty dx \frac{\sqrt{x}}{e^x - 1} \quad (5)$$

The integral evaluates to  $\sqrt{\pi}\zeta(3/2)/2$  so that we obtain the BEC temperature

$$T_c = \frac{2\pi\hbar^2}{m_B k_B} \left( \frac{n_B}{\zeta(3/2)} \right)^{3/2} \quad (6)$$

Comparing (3) and (6) we see that

$$\frac{T_c}{T_F} = \left( \frac{64}{9\pi\zeta(3/2)^2} \right)^{1/3} \frac{m_F}{m_B} \left( \frac{N_B}{N_F} \right)^{2/3} \quad (7)$$

In the case of a Feshbach resonance in which a BCS-BEC transition is achieved by forming composite Bosons from pairs of Fermions (so that  $m_B = 2m_F$  and  $N_B = N_F/2$ ) we have

$$\frac{T_c}{T_F} = \left( \frac{2}{9\pi\zeta(3/2)} \right)^{1/3} \approx 0.30 \quad (8)$$

Thus, we find that the BEC temperature is on the same order as the Fermi temperature. This is not surprising, since both temperatures are defined so that the number of states below  $k_B T_{c,F}$  is roughly equal to the number of particles  $N$ . However, it is important, for it suggests that if we begin with a degenerate Fermi gas at low enough temperature, it may be possible to convert it into a gas of composite bosons cold enough to Bose condense.

## 2.2 Effects of Two-Particle Interactions

Having now studied the free Fermi and Bose gases, we consider the effects of two-particle interactions, first on a Bose gas and then on a Fermi gas. The first thing we note is that including interactions, a Bose-Einstein condensate is stable only for  $a > 0$  [4]. That is, perhaps counter-intuitively, only for repulsive interactions will a condensate form. As we shall see then, one use of a Feshbach resonance is to tune the scattering length from its ‘‘default’’ value (i.e. in the absence of an external magnetic field) of  $a < 0$

across the resonance to  $a > 0$ . Thus, an atomic cloud which would not condense in the absence of a magnetic field may be induced to condense via a Feshbach resonance.

Turning now to Fermi systems, one of the major steps in the development of the BCS theory of superconductivity was Cooper's demonstration in 1956 that a Fermi gas in its ground state was unstable to *any* attractive potential. In particular, under such a potential the gas would form "Cooper pairs," loosely bound particles which are highly correlated in momentum-space, but with a separation in position-space larger than the inter-particle separation [5, 6]. The energy of this two-particle bound state is given by [7]

$$E \approx 2\epsilon_F - 2\hbar\omega_c e^{-2/D(\epsilon_F)U_0} \quad (9)$$

where  $\omega_c$  is a cut-off frequency and  $U_0$  is the strength of the inter-particle potential. For a low density gas ( $kn^{1/3} \ll 1$ ) s-wave scattering dominates the interactions and the two-particle interaction can be modeled as  $U(\mathbf{r}) = U_0\delta(\mathbf{r})$  where  $U_0$  is related to the s-wave scattering length by [8]

$$U_0 = \frac{4\pi\hbar^2 a}{m_F} \quad (10)$$

Thus, the effect of an attractive two-particle interaction is to perturb the Fermi sphere by the formation of Cooper pairs. This pairing in the *BCS limit* is a strictly many-body effect, as an arbitrarily weak attraction does not produce a bound state in the three dimensional two-body problem.

If, on the other hand, the two-particle interaction is sufficiently strong then even the two-body problem has a bound state, and the existence of the Fermi sea is not required for its formation. In this case, referred to as the *BEC limit* the pair will be bound as a bosonic "molecule" even in the absence of other particles. Thus, as the interaction strength is tuned from arbitrarily weak to sufficiently strong the Fermi gas will undergo a continuous transition from the BCS to the BEC limit. It is this transition, previously only a theoretical ideal, which the Feshbach resonance makes possible, and it is this technique and its applications which we will presently discuss.

### 3 Alkali BECs and Degenerate Fermi Gases

The technological demands of studying the BCS-BEC transition are quite formidable, and it is only in the past decade that sufficient advances have been made to make this study possible. The first accomplishment we note is the formation of BEC in the alkali gasses. Dilute alkali gases are a promising medium for condensed matter research and especially for BECs, for they lend themselves well to both laser and evaporative cooling. In addition, atomic cloud densities are easily characterized and, as we will see, the inter-atomic interactions can be arbitrarily tuned via a Feshbach resonance [10]. The cooling techniques necessary to achieve BEC temperatures in the alkali gases are themselves highly nontrivial and the source of the 1997 Nobel Prize (Steven Chu, Claude Cohen-Tannoudji, and William D. Phillips).

In 1995, through the use of evaporative cooling Anderson *et al.* were able to cool  $^{87}\text{Rb}$  atoms to 170 nK and observe quite starkly the emergence of a BEC [10]. Fig. 1 shows

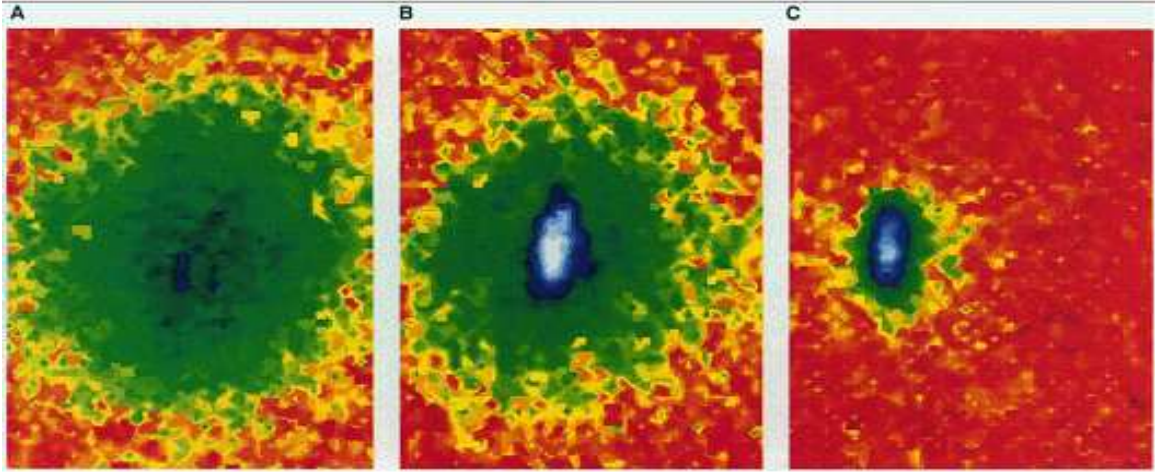


Figure 1: False-color images display the velocity distribution of the  $^{87}\text{Rb}$  atomic cloud (**A**) just before the appearance of the condensate, (**B**) just after the appearance of the condensate, and (**C**) after further evaporation has left a sample of nearly pure condensate. The circular pattern of the noncondensate fraction (mostly yellow and green) is an indication that the velocity distribution is isotropic, consistent with thermal equilibrium. The condensate fraction (mostly blue and white) is elliptical, indicative that it is a highly nonthermal distribution. The elliptical pattern is in fact an image of a single, macroscopically occupied quantum wave function. The field of view of each image is  $200\ \mu\text{m}$  by  $270\ \mu\text{m}$ . The observed horizontal width of the condensate is broadened by the experimental resolution. [10]

their results, with the cigar-shaped purple/white portion of the figure corresponding to the BEC. Despite the great achievement, however, there were still limitations to BEC studies. In particular, the magnetic trapping which was used traps only the weak-field seeking atomic states and is susceptible to trap loss through spin relaxation collisions [11]. Not long after, though, further advances were made in optical confinement which avoided many of the drawbacks of magnetic traps and increased both the breadth and depth of atomic BEC studies.

These cooling techniques could be applied not only to Bose gasses, but were equally applicable to Fermi systems, though certain technical problems made their adaptation non-trivial [12]. Thus, in 1999 Demarco and Jin were able to cool  $^{40}\text{K}$  atoms (fermions) to  $T = 0.5T_F$  and thereby observe the first degenerate Fermi gas [13]. They were able to identify the degeneracy by measuring the internal energy of the gas and comparing to the classical limit. Fig. 2 shows the gas's energy as a function of temperature (or rather, its deviation from the classical value), which deviates significantly as  $T \rightarrow 0$ , and in accordance with the prediction for a non-interacting Fermi gas (black line).

Thus, by the turn of the millenium experimentalists were able to observe quantum mechanical gases in both Bose and Fermi systems. And while, as demonstrated in Fig. 2 the system studied by DeMarco and Jin exhibits nearly free behavior (i.e. no two-particle interactions), we have seen from (9) that a Fermi sphere is unstable against an arbitrarily small attractive potential. Thus, given an alkali Fermi gas with attractive two-particle potential, the atoms will form Cooper pairs in a manner precisely analagous to the pairing in superconductors [14]. We therefore expect that for a weak attractive potential an alkali Fermi gas will exhibit BCS behavior. In contrast, for a strong attractive potential we may expect the atoms to form Bosonic molecules which are themselves capable of forming a BEC. Remarkably, a mechanism exists by which the two-particle potential can be tuned arbitrarily so as to generate precisely this phase transition, and it is this technique to

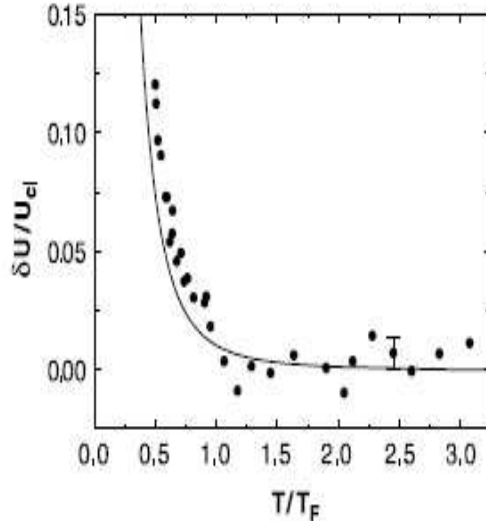


Figure 2: Fermi gas energy as a function of temperature. The vertical axis is  $\delta U = U - U_{cl}$  where  $U_{cl} = 3Nk_B T$  is the classical energy [13].

which we now turn.

## 4 BCS to BEC: The Feshbach Resonance

The prospect of using a Feshbach resonance to tune the two-particle interaction was first introduced by Herman Feshbach in 1958, in the context of nuclear physics [1]. However, due to the technical barriers already discussed, it was not until 1998 that the effect was observed in condensed matter systems by Shin Inouye *et al.* at MIT [2]. This method uses an applied d.c. magnetic field to manipulate atoms in different hyperfine states and exploit quasibound molecular states in order to vary the s-wave scattering length [9]. This works because the variable magnetic field allows for variable tuning of the molecular states' energies. The resonance occurs when the molecular state energy is equal to the energy of the two (free) scattering atoms, a possibility due to the different magnetic moments ( $\partial E / \partial B$ ) of the different states [15].

In his pioneering experiment, Inouye used  $^{23}\text{Na}$  prepared in the lowest hyperfine state  $|m_s = -1/2, m_l = 3/2\rangle$ , which at low fields is correlated with the  $|F = 1, m_F = 1\rangle$  state. The primary Feshbach resonance he studied is due to the existence of a quasibound molecular state  $|S = 1, m_s = 1, I = 3, m_I = 1\rangle$  (where  $S$ ,  $I$ , and  $F$  are the quantum numbers for electronic, nuclear, and total spin respectively).

As shown in Fig. 3, the identification of the Feshbach resonance was made by directly measuring the scattering length  $a$  as a function of the applied magnetic field. The scattering length near resonance is approximately described (theoretically) by [9]

$$a = a_0 \left( 1 - \frac{C}{B - B_0} \right) \quad (11)$$

where  $a_0$  is the scattering length far away from resonance and  $C$  is roughly independent of  $B$ . One can clearly see the resonant behavior in Fig. 3 as the scattering length varies by more than a factor of ten.

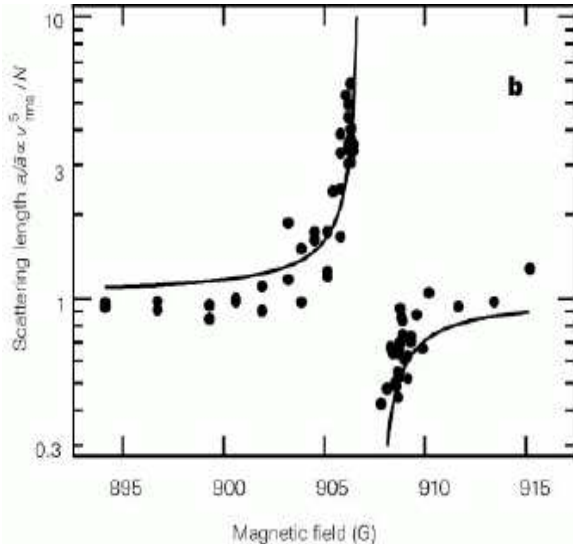


Figure 3: S. Inouye’s observation of the normalized scattering length of  $^{23}\text{Na}$  atoms as a function of applied magnetic field. [2]

Inouye’s work initiated an explosion in experimental study of the alkali gases and similar experiments were subsequently performed with  $^{85}\text{Rb}$ ,  $^{133}\text{Cs}$ , and  $^6\text{Li}$  [16, 17, 18]. Each of these species has been observed to Bose-Einstein condense when the scattering length is tuned appropriately ( $a > 0$ ). Of particular interest here is the isotope  $^6\text{Li}$ , because it is a Fermion, and can only be induced to Bose condense through pairing, either of a loosely bound Cooper state or a tightly bound molecular state.

The Bose condensation of Fermionic atoms was first observed nearly simultaneously in  $^6\text{Li}$  and  $^{40}\text{K}$  near the end of 2003 [19, 20, 22]. These experiments utilized the Feshbach resonance to adiabatically form Bosonic molecules from the Fermionic atoms. Greiner and Jochim observed BEC formation in  $^{40}\text{K}$  and  $^6\text{Li}$  for initial temperatures of  $T/T_F < 0.17$  and  $T/T_F < 0.2$  respectively, while Zwierlein found BECs (also in  $^6\text{Li}$ ) for  $T/T_F < 0.30$ . The precise limiting value for  $T$  may depend somewhat on specific system parameters, including the quality of the adiabatic approximation, but we may note that each of these values is in good agreement with the order of magnitude given in (8).

Fig. 4 (right) shows the optical density of the bosonic molecules in Greiner’s experiment following the magnetic field sweep through the resonance. The left portion ( $T = 0.19T_F$ ) shows the beginnings of BEC formation, with a slight deviation from the thermal Gaussian distribution, while the right portion ( $T = 0.06T_F$ ) shows the formation of a significant BEC fraction. At left, for the same experiment, is shown the condensate fraction as a function of the initial Fermi gas temperature. We can see that the BEC formation begins at approximately  $T = 0.17T_F$  as noted above, and increases steadily as  $T \rightarrow 0$ .

The condensates formed in the above experiments were formed on the BEC side of the Feshbach resonance, that is, the condensing particles were tightly bound Bosonic molecules formed from the Fermionic atoms. In a similar experiment performed by Regal *et al.* at JILA, the opposite regime was observed in which the condensate was formed from loosely bound Cooper pairs, similarly to the mechanism in superconductivity. Thus, BEC formation in both the BCS and BEC regimes has been observed, and notably the

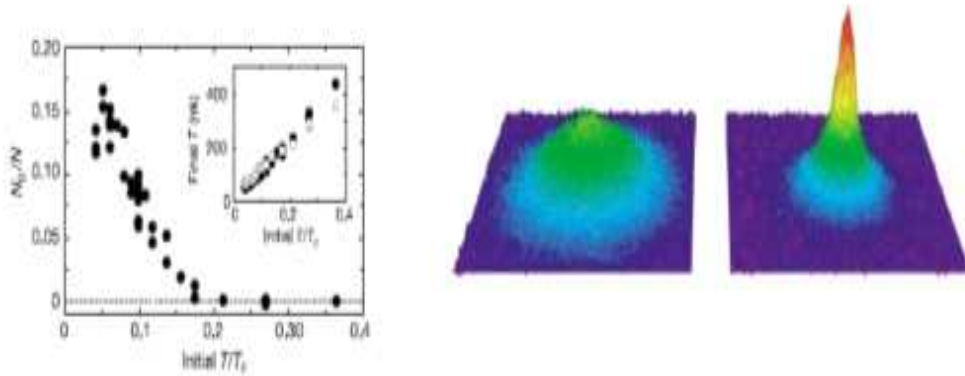


Figure 4: At left, Greiner's dependence of the condensate fraction and final temperature of the atom-molecule mixture on the initial temperature ( $T/T_F$ ) of the Fermi gas  $^{40}\text{K}$ . At right, a surface plot of the optical density for the (bosonic) molecule created by sweeping through the Feshbach resonance with an initial temperature of  $0.19T_F$  ( $0.06T_F$ ) for the left (right) image [20].

condensate lifetime is significantly longer on the BCS side of the resonance, probably due to Pauli blocking of the Fermi sphere [21].

The condensate fraction is limited by the energy of the initial Fermi gas and can be approximately computed by assuming an adiabatic conversion and equating the low-temperature expressions for the entropy of a Bose and Fermi gas [23]:

$$S_F = \frac{\pi^2}{2} N_F k_B \left( \frac{\beta_F}{\beta_0} \right) \quad (12)$$

$$S_B = \frac{5\zeta(5/2)}{2\zeta(3/2)} N_B k_B \left( \frac{\beta_c}{\beta_B} \right) \quad (13)$$

where  $N_F$  and  $N_B$  are the numbers of Fermions and Bosons,  $\beta = 1/k_B T$  and the subscripts refer to the Fermi temperature ( $F$ ), the Bose condensation temperature ( $c$ ), the final Bose gas temperature ( $B$ ) and the initial Fermi gas temperatures ( $0$ ). Equating these two entropies we find the relation

$$\frac{T_0}{T_F} = \frac{5}{2\pi^2} \frac{\zeta(5/2)}{\zeta(3/2)} \left( \frac{T_B}{T_c} \right)^{3/2} \approx 0.13 \left( \frac{T_B}{T_c} \right)^{3/2} \quad (14)$$

In order for a BEC to form we must have  $T_B < T_c$  or  $T_0/T_F < 0.13$ . This result is on the same order as the experimental results quoted above and does not take into account interactions or higher-order terms in  $T$ . From (14) we can also determine the final Bose gas temperature, which will determine the BEC fraction. The condensate fraction  $n_0(T)$  is given by [8]

$$n_0(T) = n_0(0) \left( 1 - \frac{m(k_B T)^2}{12n c \hbar^3} \right) \quad (15)$$

where  $n$  is the number density of atoms,  $c$  is the speed of sound, and  $n_0(0)$ , the condensate fraction at  $T = 0$  is

$$n_0(0) = n \left( 1 - \frac{8}{3V} \sqrt{\frac{na^3}{\pi}} \right) \quad (16)$$



Thus, using (14), (15), and (16) we can determine the BEC fraction as a function of  $T_0/T_F$ . However, without precisely computing the temperature dependence, we can see directly from (16) that the interactions place a limit on the condensate fraction. The value of this limit can be tuned along with  $a$ , and remarkably, condensate fractions in  ${}^6\text{Li}$  as high as 80% ( $3 \times 10^7$   ${}^6\text{Li}$  atoms) have been achieved [18].

## 5 Conclusions and Future Research

As we have seen throughout this survey, the major limiting factor in the study of the BCS-BEC crossover has historically been our ability to achieve the ultracold temperatures necessary to observe the transition. However, the vast improvements in evaporative and laser cooling, as well as optical confinement have given us an unprecedented capacity to prepare nanokelvin Bose and Fermi systems. In fact, the realization of 80% condensate fractions by Zwierlein, as noted above, suggests that the cooling techniques required are no longer the limiting factor in these studies.

Studies have shown the existence of a condensate on both sides of the resonance and have provided preliminary characterizations of the condensates (e.g. condensate fraction, lifetime, thermal background). However, the observation of a condensate in the BCS regime is technically challenging and advances in imaging techniques are needed to confirm and elaborate on existing results. Of particular interest is the observation that in  ${}^{40}\text{K}$  the appearance of a condensate in the BCS regime is accompanied by a significant ( $\sim 20\%$ ) decrease in the width of the the non-condensate fraction [21]. The origin of this phenomena is yet unknown and requires further study.

Another important aspect of the BCS-BEC transition which has not yet been sufficiently studied is the nature of the condensate formed as the system is continuously taken across the resonance. It is not known if the condensate in the intermediate region of the resonance is a linear combination of Cooper pairs and tightly bound molecules or if a more complex state occurs. The techniques necessary to study the intermediate regime are in place, though improvement of these techniques is inevitable, so it is only left for further experimental studies to be performed. Still, as noted above, imaging techniques needed to characterize BCS condensates may prove particularly important in gaining further insight into the nature of the BCS-BEC crossover.

## References

- [1] H. Feshbach, *Annals of Physics* **5**, 357 (1958).
- [2] S. Inouye *et al.*, *Nature (London)* **392**, 151 (1998).
- [3] A.J. Leggett, *Statistical Mechanics Lecture Notes* for a course at the University of Illinois (Urbana-Champaign), Lecture 13, Fall 2006.
- [4] S. L. Cornish, *et al.*, *Physical Review Letters* **85**, 1795 (2000).
- [5] L.N. Cooper, *Physical Review* **104**, 1189 (1956).

- [6] J. Bardeen, L.N. Cooper, and J.R. Schrieffer, *Physical Review* **108**, 1175 (1957).
- [7] Michael Tinkham. *Introduction to Superconductivity*. (Dover Publications, Inc., Mineola, NY, 2004), 45.
- [8] E.M. Lifshitz and L.P. Pitaevskii, *Statistical Physics, Part 2*, (Butterworth-Heinemann, Oxford, 1980), 21, 108.
- [9] A.J. Moerdijk *et al.*, *Physical Review A* **51**, 4852 (1995).
- [10] M.H. Anderson *et al.*, *Science* **269**, 198 (1995).
- [11] D.M. Stamper *et al.*, *Physics Review Letters* **80**, 2027 (1998).
- [12] D. Jin, *Physics World*, <http://physicsweb.org/articles/world/15/4/7> (2002).
- [13] B. Demarco and D.S. Jin, *Science* **285**, 1703 (1999).
- [14] H.T.C. Stoof and M. Houbiers, in *Bose-Einstein Condensation in Atomic Gases, Proceedings of the International School of Physics Enrico Fermi, Course CXL*, ed. M. Inguscio *et al.* (IOS Press, Amsterdam, 1999), 537.
- [15] T. Khöler and Krzysztof Góral, *Reviews of Modern Physics* **78**, 1311 (2006).
- [16] P. Courteille *et al.*, *Physics Review Letters* **81**, 69 (1998).
- [17] V. Vuletic *et al.*, *Physics Review Letters* **82**, 1406 (1998).
- [18] M.W. Zwierlein *et al.*, *Physics Review Letters* **92**, 12043 (2004).
- [19] M.W. Zwierlein *et al.*, *Physics Review Letters* **91**, 250401 (2003).
- [20] M. Greiner *et al.*, *Nature* **426**, 537 (2003).
- [21] C.A. Regal, *et al.*, *Physics Review Letters* **92**, 040403 (2004).
- [22] S. Jochim *et al.*, *Science* **302**, 2101 (2003).
- [23] L.D. Landau and E.M. Lifshitz, *Statistical Physics, 3rd Ed., Part 1*, (Butterworth-Heinemann, Oxford, 1980), 171, 182.
- [24] F. Dalfovo *et al.*, *Reviews of Modern Physics* **71**, 463 (1999).
- [25] A.J. Leggett, *Reviews of Modern Physics* **73**, 307 (2001).