Fractional Quantum Hall Effect: the Extended Hamiltonian Theory(EHT) approach

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Abstract

Phenomenology of Fractional Quantum Hall Effect(FQHE), especially charge fractization and quantization of Hall conductance, is presented here in a brief manner. Focus is concentrated on Extended Hamiltonian Theory(EHT) approach to FQHE. Natural appearance of fractional charge of composite particles, quantized Hall conductance in the expanded Hilbert space introduced in EHT is explored at mean field level. The effect of charge localization is also briefly discussed which accounts for the existence of Hall conductance plateau.

1 Introduction

That systems with interactions would quite often display bizarre and surprising effects under extreme external conditions is no news for modern condensed matter physicists. A great number of those effects account for the emergent properties of systems under the breaking of an intrinsic symmetry (spontaneous symmetry breaking). Fractional Quantum Hall Effect(FQHE), which is essentially the collective behavior of interacting electrons in low dimension and high external fields, counts as one typical example among many others. It shows that, electrons with interactions in low dimensions (D=2), when acting in concert, can respond to external magnetic field in a manner of composite particles with a charge even "smaller" than the individual electron charge. By balancing between interactions with external magnetic field and interactions within the system(Coulombic), the factorization of charge in two-dimensional electron systems (2DES) forms not only single phenomenon but actually a hierarchical series, characterized generally by the filling proportion of the Lowest Landau Level(LLL) of the electrons, or *filling factor*$\nu = p/(2ps+1), (p = 1, 2, \cdots, s = 0, 1, \cdots)$.

To understand the fractional charge and quantized Hall conductance of 2DES, various theoretical models have been developed over the past two decades. Most of which fall into two branches. Wave-function approach:

Laughlin and later Jain[4] successfully constructed the ground state wave functions of a 2DES in a perpendicularly directed external magnetic field in first quantization form for a series of filling factors, which are now known as Laughlin factors $\nu = 1/(2s+1)$ and Jain filling factors $\nu = p/(2p+1)$. The fractional charge of collective electrons turned out to be a requirement on the trial wave function constrained by gauge invariance of the electron Hamiltonian, so does the quantum Hall conductance[4]; Chern-Simon Field approach: Starting from macroscopic Hamiltonian of 2DES, Zhang, Hansson and Kivelson[8] took a different route to FQHE by introducing an extra "virtual" gauge field into the Hamiltonian, and virtually eliminated the effect of the external fields by transforming the Hamiltonian into one that describes no longer a system of strongly interacting electrons, but a system of Chern-Simon(CS) particles(fermions or bosons)which sees virtually no external field and interacts only weakly among themselves.Those CS particles can be viewed pictorially as attaching certain numbers(depending on the filling factor)of magnetic flux quanta ($\phi_0 = \frac{2\pi\hbar}{e}$) to each electron in original system (known as "flux attaching"). Each flux carries a fractional charge $e* = e/(2p+1)$ and thus gives rise to the fractional charge of the CS particles. Different as they are in many aspects, both theoretical branches clarified one physical picture in common, that fractional charge present in FQHE originates from a new composite particle created by excitations of electrons in LLL.

Extended Hamiltonian Theory (EHT), developed by Murthy and Shankar[6][5][7], grasped this idea of composite particle (CP). Essentially an extension of conventional Chern-Simon Field Theory, EHT introduces, instead of one gauge field, a canonical pair of fields into the electron Hamiltonian. The carefully chosen fields enlarge the electron Hilbert space to one that includes not only electron states but also oscillatory modes from the introduced fields. By proposing extra constraints on the introduced fields that it doesn't reflect "real physical effect", Murthy and Shankar was able to transform the original electron system into a pair of decoupled systems: one belongs to the composite particle and one the introduced virtual fields. Relations between the fractional charge of composite particles present in either Laughlin's theory or Chern-Simons field theory and the charge of the oscillatory modes arise naturally as well. My report focuses just on the buildup of EHT approach and the birth of these relations. In section II, a digress into the experimental discovery of FQHE and fractional charge is conducted. Section III concentrates on the buildup of EHT approach and derivation of FQHE from it at mean field level. Charge Localization, another indispensable factor that accounts for the appearance of Hall conductance plateau over finite variance of magnetic field, will be briefly explained at the end of the section.

2 Experimental Observation

- $\nu = 1/3$ **FQHE**

FQHE was first experimentally observed by Stormer *et.al*[1] in 1981. Stormer and his coworkers formulated a 2DES at the interface of two semiconductors:GaAs and AlGaAs. Conducting electrons congregate at GaAs side of the interface and form a thin layer with μm thickness, a result of the matching of lattice structure and constants between two materials and a slightly different surface electron energies (GaAs has energy level about $300meV$ lower than AlGaAs). The sample is prepared by modulation doping with an elaborate low electron density and high electron mobility [1] and then exposed to Hall measurement of the resistivity tensor of the specimen. A typical result is shown in (Figure 1).

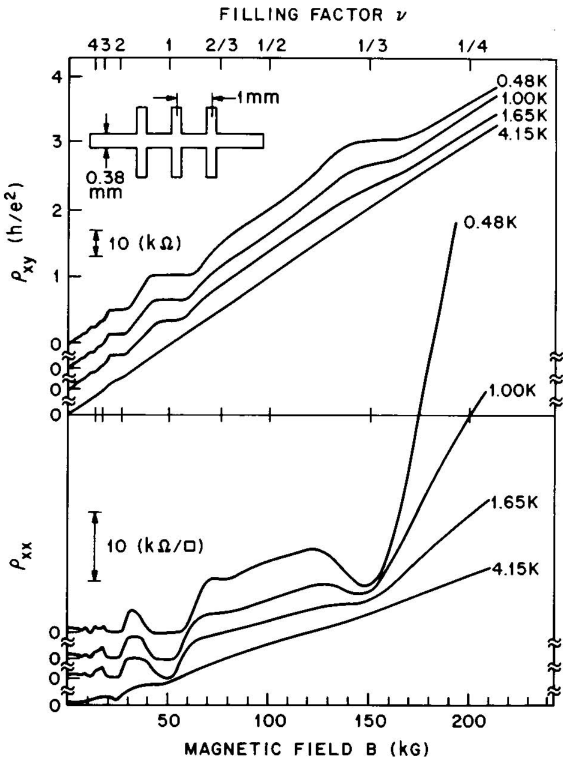

Figure 1: Experimental Observation of 1/3 FQHE: ρ_{xy} Hall resistivity, ρ_{xx} Magnetoresistivity. The sample is modulated doped GaAs/AlGaAs[1]

Regular quantum Hall conductance plateaus appear at integral filling factors ($\nu = 1, 2, 3, \cdots$), as predicted by Integral Quantum Hall Effect(IQHE), which was well awared of at that time in 2DES in magnetic field. A remarkable deviation from IQHE occurs at $B = 15T$, where Hall resistivity data forms another plateau, rather than a straight line predicted by IQHE. Stormer *et.al* identified this new plateau with fractional filling factor $\nu = 1/3$. Hall resistivity was measured three times high as that of IQHE at $\nu = 1$, indicating the appearance of a fractional charge $q = \phi_0/(6\pi\hbar/e^2) = e/3$.

- **Observation of $e/3$ fractional charge and FQHE at other filling factors**

Enormous experiments following Stormer on Hall measurement of 2DES sample disclosed FQHE at other non-integral filling factors. Figure 2 shows a typical sample of FQHE at various rational fraction filling factors[4].

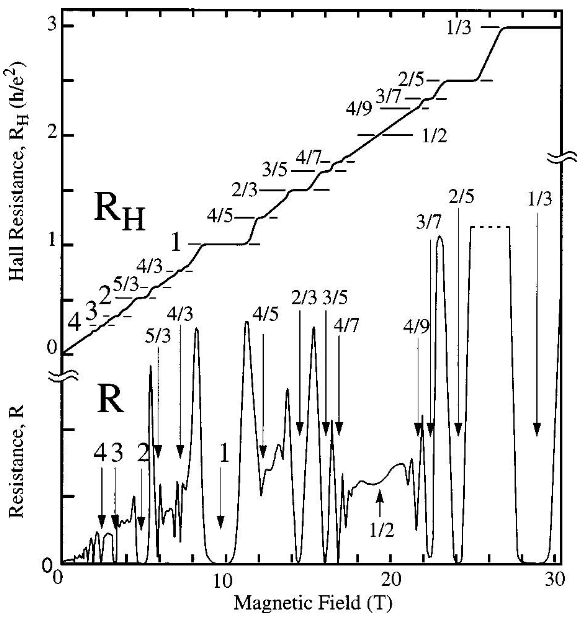

Figure 2: Typical IQHE and FQHE at various filling factors, quoted from [3]

The prediction of appearance of fractional charge by Stormer in experiment and later on by Laughlin in his trial wave function for $\nu = 1/3$ FQHE was also convinced by following experiments of various methods. Shot-noise measurements by Saminadayar *et al.* are one of the most widely cited. Saminadayar and his collaborators successfully measured the shot noise associated with tunnelling in the fractional quantum Hall regime of a 2DES sample with filling factor $\nu = 1/3$ [2], the experimental proof is illustrated in Figure 3.

Despite the difference in the filling factor of various FQHE, the gen-

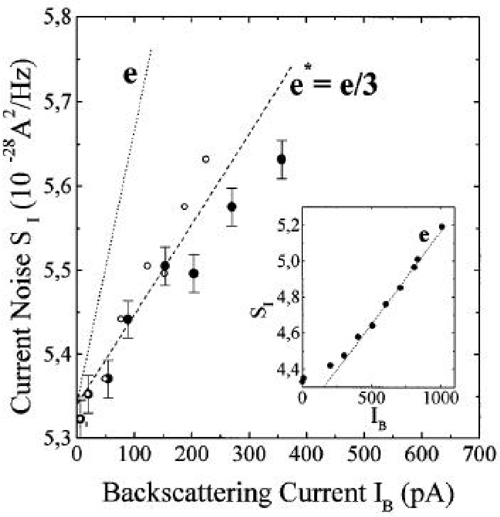

Figure 3: Tunneling noise of 2DES sample at $\nu = 1/3$; Schottky formula of shot noise predicts $S = 2qI_B$, where q is the charge of the current carrier. Experimental data reveals the desired linear relationship, but the slope is 1/3 as expected if the carrier is electron, indicating the carrier charge is only a fraction of electron charge[2]

eral phenomenon inherits several generic characteristics: (i) the filling factors are all rational number of form

$$\nu = p/(2ps + 1), (p = 1, 2, \cdots, s = 0, 1, \cdots, \tag{1}$$

and the corresponding fractional charge to appear is of form $e^* = e/(2ps+1)$; (ii) FQHE happens with the signature of a constant quantized Hall resistivity (ratio of the electric potential transverse the current over the current)

$$\rho_{xy} = \nu^{-1} h/e^2, \tag{2}$$

over a finite variance of magnetic field and a nearly vanishing magnetoresistivity ($\rho_{xx} \simeq 0$, inverse ratio of current over potential along the current) at the turning point of Hall conductance plateau. Any theory of FQHE must correctly reproduce these basic features, and EHT accomplishes this in a natural way, which we turn to explore now.

3 Theoretical Hamiltonian Model

In our construction of Extended Hamiltonian Theory, we confine the strength of magnetic field exactly at the value required by filling factor ν, i.e.$B = 2\pi n/(e\nu)$, where n is the number density of electrons in the 2DES. Also, we confine ourselves for simplicity to the case when $\nu = p/(2p + 1) < 1$, i.e. all the electrons are filled up in the lowest Landau level(LLL) of the system and assumption that their spins are all uniformly polarized is hence implied. We would also take a bold step to ignore Coulombic interactions between electrons in the derivation. Over-simplified as it may seem, it will be clear soon that EHT, even without introduction of interactions, has already incorporated basic features of FQHE. Introduction of interactions will refine our detailed results such as Chern-Simon wave function ψ_{CS}, but the derivation would no longer be intuitive and concise. A brief discussion of the effect of interactions will be included at the end for completeness. Natural units ($\hbar = c = 1$)are applied except stated otherwise. A "cyclotron length" $l_0 = (eB)^{-1/2}$ is also defined in natural units, which will show up extensively in our derivation.

We start by writing down the Chern-Simon Hamiltonian(density) without interaction in second quantized form.

$$H_{CS} = \psi_{CS}^{\dagger} \frac{|-i\nabla + e\mathbf{A}^* + \mathbf{a}|^2}{2m} \psi_{CS} \tag{3}$$

where ψ_{CS} is the Chern-Simon wave function describing the composite particle of electron attached with magnetic flux quanta. $\mathbf{a}$ is the Chern-Simon gauge field that satisfies the constraint

$$\frac{\nabla \times \mathbf{a}}{2\pi l} = \psi_{CS}^{\dagger}\psi_{CS} = \psi_e^{\dagger}\psi_e = \rho$$

(One could well start with electron Hamiltonian and get (3) by Chern-Simon approach, we bypass this derivation here for concision, standard reference can be found in [6] and [8])

The effect of the gauge field is to cancel the magnetic field *on the average*, so that for example at filling factors $\nu = 1/(2p+1)$, Chern-Simon particle sees an effective zero-field $A^* \simeq 0$

Using the constraints on $\mathbf{a}$, we transform (3) symbolically as

$$H_{CS} = \psi_{CS}^{\dagger} \frac{|-i\nabla + e\mathbf{A}^* + (\nabla\times)^{-1}2\pi l\rho|^2}{2m} \psi_{CS} \tag{4}$$

EHT enlarges the electron Hilbert Space in the following fashion: in momentum space, for each electron momentum q, EHT associate a canonical pair of vector fields

$$\mathbf{P}(q) = i\hat{q}P(q), \qquad \mathbf{a}(q) = -i\hat{z} \times \hat{\mathbf{q}}a(q)$$

and canonical commutator between the two fields,

$$[a(\mathbf{q}), P(\mathbf{q}')] = (2\pi)^2\delta(\mathbf{q} + \mathbf{q}') \tag{5}$$

where the Dirac function is evaluated in two dimensions. Hamiltonian (4) is totally equivalent to

$$H_{CS} = \psi_{CS}^{\dagger} \frac{|-i\nabla + e\mathbf{A}^{*} + \mathbf{a} + (\nabla\times)^{-1} 2\pi l \rho|^{2}}{2m} \psi_{CS} \tag{6}$$

provided that we restrain

$$a(q)|physical\rangle = 0$$

, or

$$[a, H] = 0$$

This enables us to find simultaneous eigenvalues of H and a, and that corresponds to $a = 0$ solves the original problem. To set up further equivalence between this EHT model and original Chern-Simon model, EHT introduces a projection operator that connects the wavefunctions of these two models:

$$\wp = \int dx da |x\ a\rangle\langle x\ a| \frac{\delta(a)}{\delta(0)} \tag{7}$$

The physical meaning of (7) is clear if one consider its action on a wave function of the expanded Hilbert Space $\Psi(x, a)$ (x labels the position quantum number of the original particle while a labels the non-physical field degrees of freedom)

$$\Psi_{\wp}(x, a) = \wp\Psi(x, a) = \frac{\delta(a)}{\delta(0)} \Psi(x, a) = \frac{\delta(a)}{\delta(0)} \Psi(x, 0) = \frac{\delta(a)}{\delta(0)} \Psi_{CS}(x) \tag{8}$$

EHT then introduces a unitary transformation to get rid of the "inverse of curl" term shown in (6),

$$U = \exp[\int d^{2}q i P(-q) \frac{2\pi l}{q} \rho(q)] \tag{9}$$

under which the Hamiltonian transforms into

$$H = \frac{1}{2m} \psi_{CP}^{\dagger} (-i\nabla + e\mathbf{A}^{*} + \mathbf{a} + 2\pi l \mathbf{P})^{2} \psi_{CP} \tag{10}$$

$$\psi_{CS}(x) = \psi_{CP} \exp[\int d^{2}q i P(-q) \frac{2\pi l}{q} e^{-iqx}] \tag{11}$$

$$0 = (a - \frac{2\pi l \rho}{q})|physical\rangle \tag{12}$$

Eq.(12) is just the constraint $0 = a(q)|physical\rangle$ written in terms of transformed a field.

Expand Eq.(10)(drop the subscript CP henceforth), we get

$$
\begin{aligned}
H &= \frac{1}{2m}|(-i\nabla + eA^*)\psi|^2 + \frac{n}{2m}(a^2 + 4\pi^2 l^2 P^2) \\
&\quad +(\mathbf{a} + 2\pi l P)\cdot \frac{1}{2m}\psi^\dagger(-i\overleftrightarrow{\nabla} + eA^*)\psi \\
&\quad +\frac{\delta(\psi^\dagger\psi)}{2m}(\mathbf{a} + 2\pi l\mathbf{P})^2 \\
&= H_0 + H_1 + H_2
\end{aligned}
\tag{13}
$$

where $\rho = \psi^\dagger\psi = \langle\psi^\dagger\psi\rangle + \delta(\psi^\dagger\psi) = n + \delta(\psi^\dagger\psi)$, n stands for the average density of the composite particles(which is just the average electron density), and $\delta(\psi^\dagger\psi)$ its fluctuation. Also, we define

$$
A\overleftrightarrow{\nabla}B = A\nabla B - (\nabla A)B
$$

Now we are finally prepared to get to the "final representation" applied in EHT. the second term in the first line of (13), or H_0, resembles the ordinary Hamiltonian of quantum oscillators, with natural frequency $\omega_0 = \frac{2\pi l n}{m}$, and therefore can be written in second quantized form by introducing ladder operators, and the ψ operator can be replaced by creation and annihilation operators of composite particles in momentum space. We'll set filling factor explicitly as $\nu = p/(2p+1)$ such that $\omega_0 = \frac{2p}{2p+1}\frac{eB}{m} = \frac{2p}{2p+1}\omega_c$, $B^* = B/(2p+1)$(by attaching $2p+1$ flux quanta to each electron). And we ignore contribution from last term in (13) since we're working at mean field level, no fluctuation is considered to the leading order. Putting pieces together, (13) can be reformulated as

$$
H = \sum_j \frac{\Pi_j^2}{2m} + \int d^2q D^\dagger(q)D(q)\omega_0 + \frac{\sqrt{2\pi}}{m}\int d^2q (c^\dagger(q)D(q) + D^\dagger(q)c(q)) \equiv T + H^{osc} + H_1 \tag{14}
$$

where

$$
\Pi = \mathbf{P} + \mathbf{eA}^*
$$

$$
D(q) = \frac{1}{\sqrt{(8\pi)}}[a(q) + 4\pi i P(q)]
$$

$$
c(q) = \hat{q}_- \sum_j \Pi_{j+} e^{-iqx_j}
$$

$$
\Pi_\pm = \Pi_x \pm i\Pi_y
$$

$$
[D(q), D^\dagger(q\prime)] = (2\pi)^2\delta(q - q\prime)
$$

$$
[\Pi_-, \Pi_+] = -2eB^* = -2eB/(2p+1)
$$

$$
[c(q), c^\dagger(q')] = 2eB^* n(2\pi)^2\delta(q - q\prime),\ [c(q), c(q')] = [c^\dagger(q), c^\dagger(q')] = 0
$$

Now H_1 still appears as a coupling between the composite particle field and the introduced fields.To lift this coupling, [6] conducts a further unitary transformation,

$$U(\lambda_0) = e^{iS\lambda_0} = \exp[\frac{\sqrt{2\pi}}{4\pi n}\lambda_0 \int d^2q(c^\dagger(q)D(q) - D^\dagger(q)c(q))] \tag{15}$$

with λ to be cleverly chosen as solution to $tan(\lambda_0/\sqrt{2p}) = \frac{1}{\sqrt{2p}} = \mu$ and the final representation of the EHT Hamiltonian reads as

$$H^{FR} = \sum_j \frac{\Pi_{j-}\Pi_{j+}}{2m} + \sum_j \frac{eB^*}{2m} - \frac{1}{2mn}\sum_{i,j}\int d^2q\Pi_{j-}e^{-iq(x_i-x_j)}\Pi_{j+} + \int d^2qD^\dagger(q)D(q)\frac{eB}{m} \tag{16}$$

Eq.(16) concludes the Hamiltonian for EHT approach, and the problem of electron systems in FQHE has been equivalently mapped to a systems of composite particles (in present case, composite fermions since odd number of flux quanta are attached to each electron to form the composite particle[3]) and a decoupled field of oscillators. The wave function of this Hamiltonian, when projected to the subspace where the oscillator quantum number is freezed at ground state(analogy to $a = 0$ in (8)), accounts correctly for the wave function of the physical composite fermions, and therefore the wave function of electron system in FQHE, at least at a mean field level. (One can actually build up Laughlin wave function solely from (13), which turns out to be more direct than working with (16)[6]). Note, however, the decoupling transformation (15) yields a third term in composite particle part of EHT Hamiltonian in (16), this can be shown to give rise to the desired renormalization of electron mass to the mass of the composite particle[6].
Now let's examine how the fractional charge and quantized Hall conductance are embedded in (16).

Electron Charge Density: Using techniques analogous to the Heisenberg equation of motion for operators and take S present in Eq.(15) as the "effective Hamiltonian operator", one gets the equation connecting the charge density before and after transformation as

$$\frac{d\ \rho(q,\lambda)}{\lambda} = \frac{q}{\sqrt{8\pi}}(D(q,\lambda) + D^\dagger(q,\lambda)) \tag{17}$$

where $\rho(q,\lambda) = e^{-iS\lambda}\rho(q)e^{iS\lambda}$ and one sees immediately that $\rho(q,0)$ is the charge density before transformation to the final representation, and $\rho(q,\lambda_0)$ is the charge density desired in the final representation. Eq.(17) can be integrated as

$$\rho(q,0) = \rho(q,\lambda_0) + \frac{q}{\sqrt{8\pi}}(\frac{sin\mu\lambda_0}{\mu}(D(q)+D^\dagger(-q)) - \frac{\sqrt{2\pi}}{4\pi n\mu^2}(1-cos\mu\lambda_0)(c(q)+c^\dagger(q))) \tag{18}$$

Similar treatment with operator $a(q)$ brings about another relation as

$$\frac{qa(q,0)}{4\pi} = \frac{q}{\sqrt{8\pi}}(cos\mu\lambda_0(D(q)+D^\dagger(-q)) - \frac{\sqrt{2\pi}}{4\pi n\mu}sin\mu\lambda_0(c(q)+c^\dagger(q))) \quad (19)$$

(all the operators in Eq.(18)(19) are before unitary transformation)
Why bother writing down those lengthy expressions? The reason lies in that due to constraint condition $\rho(q,0) = \frac{qa(q,0)}{4\pi}$ as stated earlier, any combination

$$\gamma\rho(q,0) + (1-\gamma)\frac{qa(q,0)}{4\pi} \quad (20)$$

is physically equivalent and acceptable as a charge density operator of EHT Hamiltonian (16). EHT excludes this degeneracy of choice of γ by requiring that the charge density should satisfy magnetic translation algebra, proposed by Girvin, Jach and GMP[6], whose details are beyond our interest. What this algebra brings about is a unique choice of γ in (20) as $\gamma = 1/(2p+1)$ for the filling factor we're considering. And the charge density under this choice is, by working out explicitly expression for $c(q)$,

$$\rho(q,0) = \frac{q}{\sqrt{8\pi}}cos\mu\lambda_0(D(q)+D^\dagger(-q)) + \frac{\mathbf{1}}{\mathbf{2p+1}}\sum_j e^{-iqx_j} - il_0^2\sum_j(q\times\Pi_j)e^{-iqx_j} \quad (21)$$

The first term counts for the virtual charge of the oscillatory field, the third term is a dipolar term indicating possible nonzero dipole of the composite fermions in our system, and the second term, in great analogy to electron charge density $\rho_e(q) = e\sum_j e^{-iqx_j}$ for N charged particles sitting at $\{x_j\}|_{j=1}^N$, is our desired CHARGE of the composite fermion, and the fractional charge $e^* = e/(2p+1)$ is readily observed.

Quantized Hall Conductance: To find the Hall conductance ,we apply similar technique as in (18)(19) here to find the current operator

$$J(q,0) = \sum_j(\frac{\Pi_j}{m}e^{-iqx_j} + \frac{n}{m}\sqrt{8\pi}\hat{(q)}D(q)) \quad (22)$$

in final representation. Carry out the equation of motion for $J(q,\lambda)$, we have

$$J(q,0) = \frac{\hat{q}eBcos\mu\lambda_0}{\sqrt{2\pi m^2}}D(q) \quad (23)$$

Remarkably, the current is carried entirely by the oscillator! The cancellation of particle contribution to current leads eventually to a surprisingly simple derivation of the Hall conductance.
We recall that physical state of the EHT Hamiltonian has no contribution from the oscillatory field, i.e. it must be in the ground state of the oscillator part of the Hamiltonian. Calculation of Hall conductance σ_{xy}) in this case amounts to calculate the ground state average of $D(q)$ when the oscillator is

coupled with an external electric field. the coupling term is no surprise of the form $-\int d^2xe\rho(x)\Phi(x) = -\int d^2qe\rho(-q)\Phi(q)$. We apply Eq.(21) for $\rho(-q)$ and concentrate only on the oscillatory part of ρ, since no contribution of the current is from composite fermions, and get

$$\begin{aligned} H^{osc} &= \int d^2qeB/mD^\dagger(q)D(q) - e\int d^2q\rho(-q)\Phi(q) \\ &= \int d^2qeB/mD^\dagger(q)D(q) - e\int d^2q\Phi(q)\frac{q}{\sqrt{8\pi}}cos\mu\lambda_0(D(q)+D^\dagger(-q)) \\ &= \int d^2q\frac{eB}{m}[D^\dagger(q) - \frac{qmcos\mu\lambda_0}{\sqrt{8\pi}B}\Phi(-q)]\cdot[D(q) - \frac{qmcos\mu\lambda_0}{\sqrt{8\pi}B}\Phi(q)] \\ &\quad + \int d^2q\frac{eBq^2cos^2\mu\lambda_0}{8\pi m}\Phi(-q)\Phi(q) \end{aligned} \quad (24)$$

last term in (24) is just a constant and can be ignored, the first term indicates no more than a shift in the zero of $D(q)$, and therefore,

$$\langle D(q)\rangle = \frac{qmcos\mu\lambda_0}{\sqrt{8\pi}B}\Phi(q)$$

the electric current in momentum space of the physical state is thus

$$\langle(-e)J(q)\rangle = (-e)\frac{\hat{q}eBcos\mu\lambda_0}{\sqrt{2\pi m^2}}\langle D(q)\rangle = -\frac{e^2\nu}{h}q\Phi(q) = -\frac{e^2\nu}{h}E(q) \quad (25)$$

Eq.(25) yields no doubt the correct Hall conductance $\sigma_{xy} = \frac{e^2\nu}{h}$ (in consistency with Eq.(2)).

Charge localization and Hall conductance plateau: Point feature of FQHE in 2DES, namely fractional charge and quantization of Hall conductance, has been explored to details using EHT. Yet the existence of Hall conductance plateau involves more factors than what we have discussed here. It is believed[3][4] that existence of such a plateau is attributed solely to the complexity of the interaction of the electron system, in the presence of IMPURITIES in 2DES. The effect of impurities, which is inevitable in real 2DES sample, is to couple to the electron system a fluctuating or random potential. The influence of the potential can be formulated briefly as follows: due to random potential or potential fluctuation in the 2DES, the sharply seperated Landau levels of the electron system, or of the composite particle system are broadened into Landau bands. Yet the new energy levels generated by the broadening correspond mostly to localized particle states (the so-called Anderson Localization effect[3]) and does not contribute to long distance conducting property of the 2DES. Hence the excess of composite particles(fermions or bosons) created by slightly increasing magnetic field from its filling factor value are correlated and localized with the impurities in the sample in FQHE (In IQHE, it is the excess of electrons that are localized.), making no contribution to the conductivity of the system, and the Hall conductance remains constant at its quantized value, until the variation in the magnetic field is big enough for system to overcome energy gaps between FQHE at different filling factors and transit to FQHE at another preferred quantized state.

4 Summary and Conclusion

The construction of Extended Hamiltonian Theory, as an extension of conventional Chern-Simon field theory approach is reviewed at a non-interacting, mean field level. Features of FQHE such as fractional charge of the composite particle and quantized Hall conductance is verified using EHT Hamiltonian. Electron system in EHT is essentially expanded into two decoupled systems, one describing the physical composite particles which emerges through flux attachment process of electrons, the other non-physical oscillator Hamiltonian which has no explicit affect on real physical states, regulating the particle part of EHT Hamiltonian only implicitly. real FQHE involves not only correlations within electrons but also between electrons and impurities in the sample, which accounts heavily for the localization of low-excitation levels of electrons or composite particles(fermions or bosons) and the plateau of Hall conductance over a finite variance of magnetic field. Coulombic interactions of electrons, which is bypassed in the article, actually accounts for the renormalization of electron mass at low excited composite particle Landau level (beyond LLL), and the compressibility property of the 2DES system[6][7][8]. In general, FQHE emerges in electron systems with broken translational symmetry (low dimension and impurities) as a result of collective behavior of electrons. The fractional charge phenomenon and the success of gauge field theory approach in understanding FQHE implies surprisingly that "fractional quantum number and powerful gauge forces between these particles can arise spontaneously as emergent phenomena", as quoted Laughlin[4].

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