Emergent Phenomenon in Congested Traffic Flow

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Abstract
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Introduction and Abstract

We don't need a study to tell us that traffic is getting worse everyday. But a recent article by Gordon T. Anderson tells us about just that. "Americans waste 5.7 billion gallons of fuel, and lose 3.5 billion hours of potential productivity by sitting in traffic" [4] every year. Even worse, the length of the rush hour commute has gotten worse when compared to other driving times. "Today, congestion means a rush hour trip takes 39 percent longer than an off-peak drive." [4] It seems as if it's time to invest some money in our transportation system. However, the United States Department of Transportation's budget is already $58.7 billion for fiscal year 2005. [5] So rather than proposing astronomical budgets and massive construction, perhaps we should look for a more efficient way to use our existing infrastructure. If we can understand how traffic, and particularly jams, comes about and function, then hopefully we can understand how to reduce or eliminate them, making travel much more efficient.

A number of researchers have been trying to do just this. Many approaches have been taken, but a particularly interesting, and promising, one is to look at traffic jams as an emergent phenomenon in the traffic system. The existence of phase transitions and other emergent phenomena in small particle systems has been studied for some time. The application of this field of study to traffic is a somewhat novel approach to the problem. Here the approach of three different groups using this method will be examined and discussed.

Each of these studies has some methods in common. Using numerical simulations on massive parallel computing systems, each automobile is treated as a simple object without a conscience goal of creating some overall order. The model treats a highway as a sort of one dimensional lattice. Every car, having some finite size, is placed on a lattice site, with empty lattice sites in between representing gaps between cars. The cars each have some characteristics such as desired maximum velocity and ability to accelerate. Distance and velocity are quantized into number of sites and number of sites per time respectively. These quantities usually have no units that translate directly to a real world application, but it would be a simple exercise to do so. The traffic begins with specific density and velocity characteristics, and has a small perturbation, such as one car suddenly stopping then accelerating to full speed again, or a change in maximum desired velocity, following distance, or highway width. For the most part, only interesting or key details of their exact model will be noted in the interest of brevity.

The result of this action is that a disturbance in the traffic flow forms, traveling upstream, manifesting itself as a traffic jam that moves through the cars. This is what's known as an emergent traffic jam with a phase transition associated with it. Vehicles in the jam have zero velocity, but since the wave travels through the traffic, vehicles at the front are able to leave once the jam has passed by. Usually a larger overall jam is broken up into many smaller jams within. It's been shown that these short wavelength jams, with time, will merge into larger wavelength jams with large gaps between. The mechanism which causes these smaller jams to dissolve into larger jams will also eventually doom the larger jam to dissolve itself, until traffic flows freely again.

Model by Kai Nagel and Maya Paczuski

The first model discussed is that of Kai Nagel and Maya Paczuski. Their study focused on single-lane traffic flow that is based in human driving behavior and found evidence supporting a phase transition between "low-density lamellar flow" [2] and the "high-density jammed behavior" [2]. Their model focused on random walk arguments and use of a cascade equation, with the goal being to predict the critical exponents for the transition, and to find an explanation for the "self-organizing behavior" [2].

Here the model is based again on the motion of particles on a one dimensional lattice moving in a forward motion. The three essential feature of this model are noted as, "a) hard-core particle dynamics b) an asymmetry between acceleration and deceleration which, in connection with a parallel update, leads to clumping behavior and jam formation rather than smooth density fluctuations c) a wide separation between the time scale for creating small perturbations in the system and the relaxational dynamics, or lifetime of the jams." [2] They did include both open and closed boundary conditions in this model.

The closed model uses a single lane freeway represented by a one-dimensional array of length L. Each site can be empty or occupied by a vehicle. If it is occupied, it can have any value of velocity between zero and what they label $v_{\max}$. This leads to $v_{\max}+2$ possible states. Velocity is again number of lattice sites per unit time. Crashes are not allowed in this model. Another interesting result that was found is that, while they used $v_{\max}=5$ for most of the model, it was noted that any value greater than or equal to 2 will have the same large scale behavior once rescaled by a short distance cutoff. "This short distance cutoff corresponds roughly to the typical distance required for a vehicle starting at rest to accelerate to maximum velocity." [2]

Jams that are generated will persist until the number of jammed cars in the model drops to zero. This model also deals with the time required for this to happen. They assume non-interacting jams, and every time a jam dissipates, the outflow is disturbed again. Their simulation then measured, "the lifetime distribution, P(t), the spatial extent *w* of the jam, the number of jammed vehicles *n*, and the overall space-time size *s* (mass) of the jam." [2]

They discovered that for large t (>100), the lifetime distribution followed a power law distribution $P(t) \sim t^{-(\delta+1)}$ where $(\delta+1)$=1.5+-0.01 for emergent jams generated by small perturbations far upstream. "This figure represents averaged results of more than 60,000 jams." [2]

The authors were surprised, as was I, that a very complicated system such as this can be described by a very simple exponent. "Numerically, the exponent $\delta+1$ is conspicuously close to 3/2, the first return time exponent for a one-dimensional random walk. In fact, for $v_{\max}$=1 this random walk picture is exact…"[2] To explain this the author asks us to consider a system with v_{max} =1. A queue of vehicles with velocity zero in the jam forms. To leave the jam forever, a vehicle at the front must accelerate to velocity=1. A probabilistic rule of acceleration will then determine the rate at which vehicles leave the jam. However, vehicles can still be added to the jam at the backside. The density and velocity of the cars behind the jam will determine the rate at which vehicles are added to the jam. Due to constraints set of acceleration, the number of cars in the jam will be equal to the spatial extent of the jam since the spacing between cars in the jam will be zero. This is also where the probability distribution for the lifetime of a jam of $P(t) \sim t^{-3/2}$.[2] "The argument shows that the outflow from an infinite jam is in fact self-organized critical. One can see this by noting that the outflow from a large jam occurs at the same rate as the outflow from an emergent jam created by a perturbation. Another consequence is that maximum throughput corresponds to the percolative transition for the traffic jams." [2] Also noted was, "Starting from random initial conditions in a closed system, the current at long times is determined by the outflow of the longest-lived jam in the system." [2]

This study also had some revealing results regarding the flow of vehicles out from a jam. It was found that the outflow of traffic from a jam will self organize, creating a critical state of maximum throughput. This state was achieved when the emergent traffic jams were just able to survive indefinitely. "This implies that the intrinsic flow rate for vehicles leaving a jam equals maximum throughput." [2] Results of this study show that maximum throughput is actually achieved when the left boundary condition is that of an infinitely large jam, and the right boundary condition is left open. This is explained by, "An intuitive explanation is that maximum throughput cannot be any higher than the intrinsic flow rate out of a jam. Otherwise the flow rate into a jam would be higher than the flow rate out, and the jam would be stable in the long time limit, thus reducing the overall current. By definition, of course, the maximum throughput cannot be lower than this intrinsic flow rate." [2] It is true that the maximum throughput selection is something which is intrinsic in driven diffusive systems. [2] This model differs though in that the left boundary condition is that of the front of the infinite jam drifting backward in time. "If the left boundary is fixed in space and vehicles are inserted at velocities less than v_{max}, then the outflow from a jam cannot reach maximum throughput".[2] This is particular notable since real world situations where one has a disturbance which cannot move, like onramps or reductions in lanes, lead to lower throughput downstream than the theory would predict. [2]

A closer investigation of the characteristics of the jams themselves reveals that a very large emergent jam at some point in time actually consists of many smaller dense regions of jammed cars, with gaps between in which vehicles move at maximum velocity. These regions are called "subjams" and "holes" respectively. [2] As mentioned earlier, longer lived jams will separate these smaller dense regions out, to form fewer larger jams with large gaps between. The mechanism that allows this to occur is the dissolution of these small subjams. "When one subjam dissolves because the cars in it accelerate to maximum velocity, the two holes on either side of it merge to form on larger hole. Holes at any large scale are created and destroyed by this same process."[2] It is proposed that this mechanism gives the largest contribution to large hole sizes. This, however, will also eventually doom the larger jams, as they too will dissipate. This leads to the following cascade equation for hole size.

$$\sum_{u=x+1}^{\infty} < h(x)h(u-x) >= \sum_{x'=1}^{x-2} < h(x')h(x-x'-1) > [2]$$

Another interesting result is that if the system is in a sense driven with frequent perturbations, the jams will interact with one another, which leads to a correlation length between jams. These systems were found to be sensitive to small perturbations due to the fact that the traffic in a complicated network is poised near the critical state determined by the largest jam. An interesting, and perhaps disturbing, result of this is that any system introduced to reduce the random fluctuations such as cruise control or the new radar based following distance maintenance systems, will actually push the system much closer to the critical point, which means more large jams on a road. Measuring this correlation length, however, was deemed outside the scope of their paper.

Model By Martin Treiber and Dirk Helbing

Martin Treiber and Dirk Helbing discuss possible mechanisms of the phase transition that occurs between free flowing and stop-and-go traffic. They focus in particular on the possible coexistence along the road of different traffic states caused by an inhomogeneity of traffic flow. This is the same sort of perturbation that has been discussed earlier, but they only list a segment where people start driving more carefully, in other words, increase following distance, or a region where their maximum desired speed drops, corresponding to a region of lower speed limit, or worse driving conditions. They identify three different states that appear along the stream of traffic. They label these states "'homogeneous congested traffic' (which, in a multilane model, is related to the observed synchronization among lane) → 'inhomogeneous congested traffic' (corresponding to the so-called 'pinch region') → 'stop-and-go traffic'". [1] The ordering is that of the first states listed being the most downsteam. Any more downstream or upstream and we have simply free flowing traffic either leaving or entering the system.

Their model makes a few assumptions different from Kai Nagel and Maya Paczuski, but has similar results. These assumptions are first metastability of traffic flow. Second is a flow inside of the traffic jam which is of considerably smaller magnitude than that in synchronized congested traffic. Third is a regime of linearly unstable traffic flow that is sufficiently large and is connectively stable. "If 'synchronized' traffic is linearly unstable and free traffic upstream is metastable, upstream moving perturbations will grow and when their amplitudes become large enough, eventually form stop-and-go waves." [1]

The specific inhomogeneities tested are increasing the time gap from three halves of a second to seven quarters of a second, and changing desired velocity from one

hundred and twenty kilometers per hour to eighty kilometers per hour. They assumed, as initial conditions a homogeneous free flow of traffic of 1670 vehicles per hour.
It was found that as one observes behavior upstream of the inhomogeneity that small oscillations emerge and over time these oscillations travel upstream and grow to larger stop-and-go waves of fairly short wavelength (about 0.8km). [1] This is the same result as was found in the work of Kai Nagel and Maya Paczuski, but here the numeric results are more specific. These waves then go on to either dissolve or merge into even large jams which they label as "wide jams" [1] inside of which traffic stands still. Gaps between these larger jams have a typical distance of two kilometers to five kilometers. "Once the jams have formed, they persist and propagate upstream at a constant propagation velocity without further changes of their shape. No new clusters develop between the jams." [1]

Thankfully Martin Treiber and Dirk Helbing have included some wonderful diagrams of their results. Two of these do a great job of demonstrating the similarities and differences between the results they get, and the results of traffic density models done using the gas-kinetic-based traffic model. Both figures are from [1].

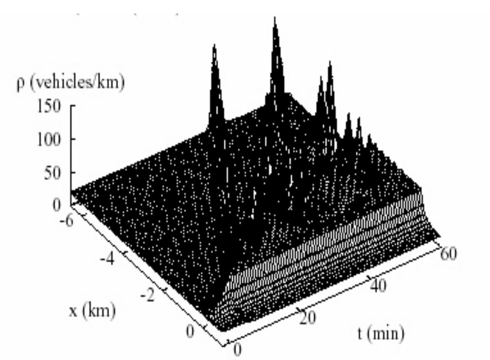

FIG. 1. Spatiotemporal density plot illustrating the breakdown to "synchronized" traffic (smooth region of high density) near an inhomogeneity, and showing stop-and-go waves emanating from this region. Traffic flows in positive x-direction. The inhomogeneity corresponds to an increased safe time headway T between $x = 0$ km and $x = 0.3$ km, reflecting more careful driving (see main text). Downstream of the inhomogeneity, vehicles accelerate into free traffic.

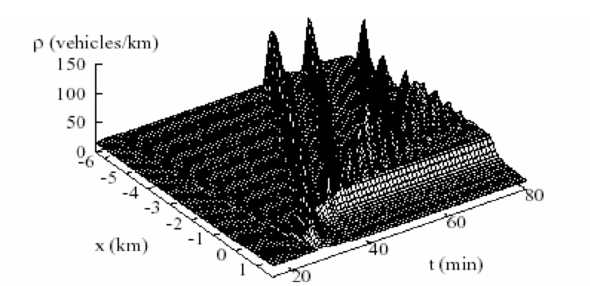

FIG. 4. Spatiotemporal evolution of the traffic density according to the gas-kinetic-based traffic model [21]. The assumed inhomogeneity of traffic flow comes from an on-ramp of length 200 m with an inflow of 220 vehicles per hour and freeway lane. The inflow to the main road is 1570 vehicles per hour and lane, and the breakdown of traffic flow is triggered by a perturbation $\Delta Q(t)$ of the inflow with a flow peak of 125 vehicles per hour and lane (see main text). The assumed model parameters are $V_0 = 120$ km/h, $T = 1.5$ s, $\tau = 30$ s, $\rho_{\max} = 120$ vehicles/km, and $\gamma = 1.2$, while the parameters for the variance prefactor [21] $A(\rho) = A_0 + \Delta A\{\tanh[(\rho - \rho_c)/\Delta\rho] + 1\}$ are $A_0 = 0.008$, $\Delta A = 0.02$, $\rho_c = 0.27\rho_{\max}$, and $\Delta\rho = 0.1\rho_{\max}$. In order to have a large region of linearly unstable but convectively stable traffic, we introduced a "resignation effect", i.e. a density-dependent reduction of the desired velocity V_0 to $V_0'(\rho) = V_0 - \Delta V/\{1 + \exp[(\rho_c' - \rho)/\Delta\rho']\}$ with $\Delta V = 0.9V_0$, $\rho_c' = 0.45\rho_{\max}$, and $\Delta\rho = 0.1\rho_{\max}$.

Also included is another diagram which clearly shows the results of the simulation with respect to the velocity of the vehicles at different times and distances from the perturbation. This is also sourced from [1].

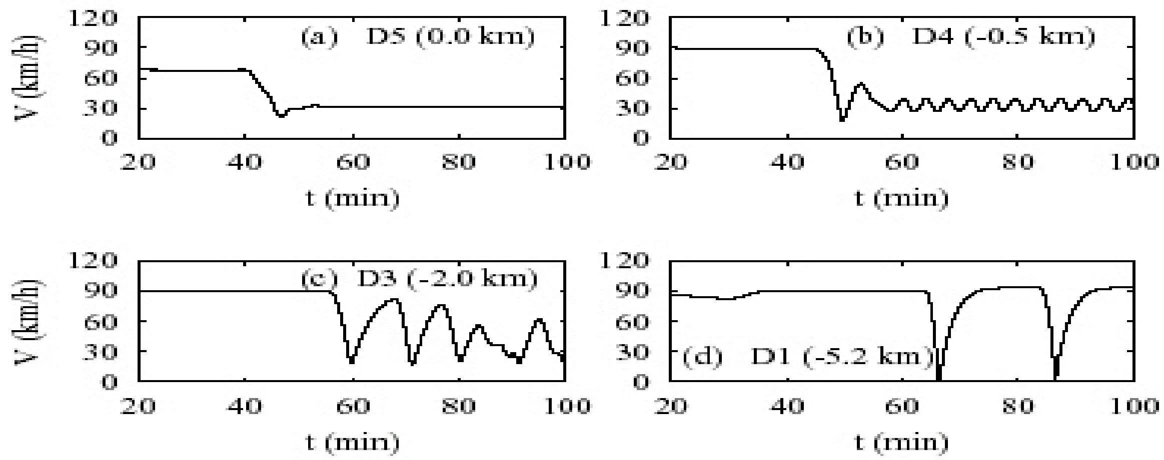

FIG. 2. Temporal evolution of the average velocity determined from the individual velocities of the vehicles that pass cross sections of the freeway during one-minute intervals at the four "detector" positions D1, D3, D4 (upstream of the inhomogeneity), and D5 (at the inhomogeneity). The naming of the detectors and their distances with respect to the inhomogeneity are the same as in Ref. [13]. The inhomogeneity is realized by a drop of the desired velocity (see the main text). (a) Breakdown to homogeneous synchronized traffic around $t = 45$ min. (b) Oscillating synchronized traffic ("pinch region"), (c) developing stop-and-go waves, and (d) resulting traffic jams.

Model by E. Levine, G. Ziv, L. Gray, and D. Mukamel

The last model that shall be discussed is the model of E. Levine, G. Ziv, L. Gray, and D Mukamel. In their paper, they focus on the phase transition that occurs between the jammed and free flowing states rather than the emergent jams that result behind the original perturbation to the system. Namely, they are looking for whether a phase transition exists at all, or if the transition from one regime to another is simply smooth.

The problem has been studied before and proposed mechanisms for phase transitions include the zero range process [3], two species driven models [3], and the chipping model. [3] The authors propose that an asymmetric chipping process quite accurately describes many traffic models. When modeling traffic the choice of the chipping model is an excellent one because it incorporates dynamical processes, which are very similar to the ones we're trying to model in traffic systems. [3] This would hopefully lead to accurate descriptions of real traffic systems. They use a cellular automata approach, examining a correspondence with the chipping model.

Probalistic Cellular Automata have been used already to analyze traffic flow in a number of models [3]. These models treat time and space as discrete quantities. The physical state of the system is updated according to some update scheme decided upon by the modeler. Each of the previous models seems to have problems with it though. [3] The primary problem is that the phase transition that occurs only does so in some limiting situation, where dynamical processes become deterministic. [3] These leaves open the question of the existence of a phase transtition for jamming when the dynamical processes are non-deterministic. The authors suspected that the correspondence between cellular automata based traffic models and the chipping model for non-deterministic dynamics would lead to no phase transition being observed, in other words, a smooth crossover between states, when the chipping process is symmetric.

The chipping model is similar to the other models used in that it considers a periodic lattice. Each site can contain any number of particles. "The dynamics is defined through the rates by which two nearest neighbor sites containing k and m particles, respectively, exchange particles". [3] This is important, because it's been shown that if the chipping part of the process is symmetric, condensation will occur at some critical density, and we'll have a macroscopically occupied site. However, if the chipping is asymmetric, then no phase transition can occur at any density. [3] Other similarities include the labeling of three distinct regions, similar to those of Helbing and Treiber. "…a free-flow regime at low densities; a regime of wide moving jams at high densities; and a synchronized flow regime, where jams and free-flow coexist, at intermediate densities." [3] What they characterize as a low density region has been known as a gap, and the high density regions as jams.

Their process involves setting up the lattice or highway and then, "A domain of size k is then associated with a site of the CM occupied by k particles. One then proceeds by examining the evolution of the domains, and identifying their dynamical processes. As will be demonstrated, in many cases these processes are closely related to the diffusion and the chipping processes of the asymmetric CM." [3]

The density of the automobiles tells the story of the movement of the vehicles. First they consider what they call the "cruise control limit". This is where the probability of braking occurring is zero, in other words all cars are maintaining a constant velocity. There, as long as the density stays below some density $\rho < \rho_f = \dfrac{1}{v_{\max}+1}$ free flowing traffic persists. [3] Here all cars are moving deterministically, and one can express the current as $J(\rho)=v_{\max}\,\rho$. [3] As density increases, local jams form and current reduces. This leads to the conclusion that a phase transition, if one exists, must occur at some ρ_0 less than or equal to ρ_f.

However, as soon as we leave the cruise control state, the probability of braking, q, is no longer zero, and this phase transition disappears. ρ_0 and ρ_f are different values and when ρ is between them we have two phase coexistence of free flowing and jammed states in the thermodynamic limit. [3] Jammed states do not form, but if given an initial condition with a jammed state already in it, it is shown that this state slowly evolves back toward free flowing. The time necessary for this to occur, though, increases exponentially with system size. [3]

They go on to conclude that in the cruise control limit, there is no phase transition for densities greater than ρ_0. The impact this result has on the non CC limit is interesting though. The CC limit transition is expected to be smooth, and since there is no transition for $\rho > \rho_0$, it can be assumed that there is no transition even when q>0.

"Nevertheless, as long as the number of chipped particles r is bounded by a finite number, or the probability of chipping r particles u(r) decays sufficiently fast with r (say exponentially), the main results obtained from the CM are expected to be valid. Namely, condensation transitions should not take place as long as the chipping process is asymmetric." [3]

In summary, it has been shown that for traffic models which are non-deterministic, we do not see a phase transition occur between jammed and free flowing states. Rather a smooth crossover occurs as car density is increased. [3]

To help visualize the results seen, these diagrams have been borrowed from the paper. [3]

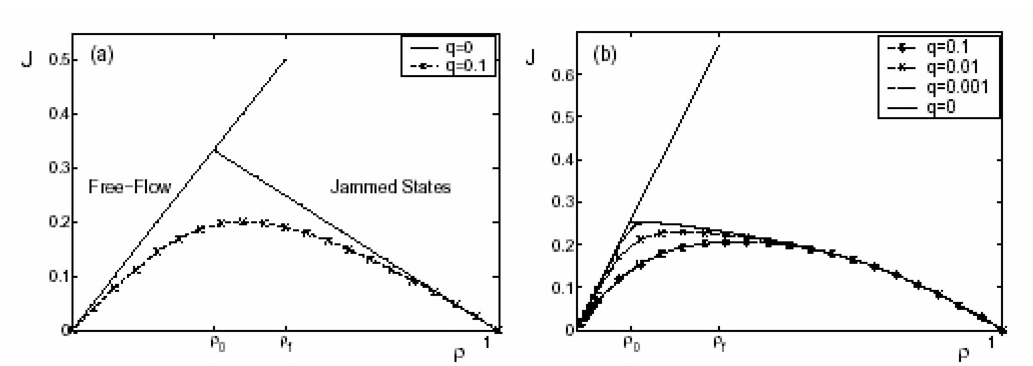

Fig. 1 – Fundamental diagram for the VDB model. Data for $q > 0$ was obtained from numerical simulations of systems of size 500. (a) $v_{max} = 1$ and $p = 1/2$. (b) $v_{max} = 2$ and $p = 0.6$; here the $q = 0$ free-flow branch is given by 2ρ, while the branch of jammed states in this case was extrapolated from numerical data.

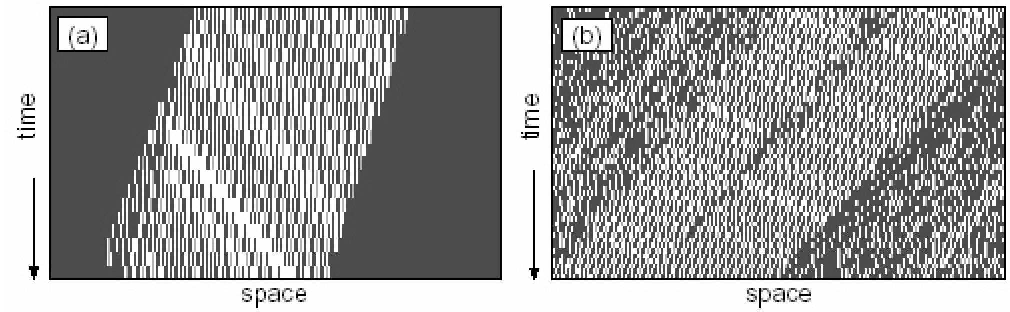

Fig. 2 – Space-time configurations of the VDB model. Stopped cars are in black, moving cars in gray, vacancies in white. (a) The evolution of a single domain is characterized by a biased diffusion and chipping of small domains. Every 5th sweep is presented. (b) Chipping and diffusion may lead to coalescence of neighboring domains. Configurations are presented every 30 sweeps.

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