Defect Induced Structure Formation in the Early Universe

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Abstract When describing the early universe as an FRW metric and being filled with perfect fluids, there is no first principle understanding of structure formation. Instead, at high temperatures one must describe the matter as a quantum field theory. As the universe expands, this field theory will undergo phase transitions, and the possibility for topological defects arise. This paper will discuss how topological defects form, and their role in structure formation in the early universe, focusing on cosmic strings.

1 Introduction

There are many goals of modern cosmology, including, understanding mechanisms leading to inflation, understanding the origins of dark matter and energy, and understaning the origin of structure formation. In this paper we will review attempts to account for the last of these through topological defects.

The early universe was a very energetic place. At such high energies one expects particle physics to be important. The standard model of particle physics is described by a non-Abelian gauge theory, with gauge group $SU(3)\times SU(2)\times U(1)$. This theory is assumed to undergo a phase transition near at the electro-weak scale resulting in the QED dominated "real world" we see at visible energy scales. This phase transition is thought to proceed via the usual Higgs mechanism.

While the interactions are classically scale independent, the coupling constants receive logarithmic quantum corrections. At energies on the order of $10^{16}GeV$ these couplings are approximately equal. This is suggestive that new physics occurs at this scale. One usually studies larger grand unified theories (GUT's) which may be broken to the standard model via a second Higgs mechanism at the GUT scale, assumed to be $10^{16}GeV$. Beyond this scale the physics is assumed to be governed by a unified gauge theory, which may or may not be the low energy effective description of another field or string theory. Typical GUT theories are based upon $SU(5)$ or $SO(10)$ gauge theory.

Associated with phase transitions is the possibility of topological defect formation. The Kibble mechanism tells us that if a defect is allowed to form, it will develop in the early universe. These defects possess energy and will interact gravitationally at the very least. The objects may then provide the early seeds for structure formation in the early universe.

This paper will begin with a lightning review of the standard model of cosmology and inflation. In section three defects allowed in the simplest extensions of the standard model of particle physics are discussed. In section four the implication of these defects on structure formation is discussed.

2 The Standard Model of Cosmology and Inflation

2.1 Standard Model of Cosmology

From astrophysical observations we note that on scales beyond 100 Mpc there does not seem to be any additional structure formation. Also, the cosmic microwave background (CMB) is consistent with a spatially flat universe, possessing density perturbations $\frac{\delta\rho}{\rho}\sim 10^{-5}$. [1] The solution of Einstein's equations consistent with a homogeneous, isotropic universe is the Friedmann-Robertson-Walker metric,

$$ds^2 = dt^2 - a^2(t)\left(\frac{dr^2}{(1-kr^2)^{\frac{1}{2}}} + r^2 d\Omega^2\right). \tag{1}$$

$$\begin{aligned} k &= -1 \rightarrow \text{Closed} \\ k &= 0 \rightarrow \quad \text{Flat} \\ k &= 1 \rightarrow \quad \text{Open} \end{aligned}$$

We may characterize the light observed from distant objects by their redshift.

$$z + 1 \equiv \frac{\lambda_o}{\lambda_e} \tag{2}$$

Here λ_o and λ_e are the observed and emitted wavelengths of light. The FRW metric then implies,

$$z \simeq H(t_o)d_l. \tag{3}$$

The Hubble parameter, $H(t)$ is defined to equal $\frac{\dot{a}}{a}$. d_l is the spatial distance between two points, determined by the metric.[2]

The physics in this universe is presumed to be governed by the Einstein-Hilbert action augmented by a matter action. The Euler lagrange equation for the metric

$$\begin{aligned} \frac{2}{\sqrt{g}}\frac{\delta S_{tot}}{\delta g_{\mu\nu}} &= 0 \\ \rightarrow R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} &= 8\pi G T^M_{\mu\nu} \end{aligned} \tag{4}$$

and energy momentum conservation,

$$\nabla^\mu (T^{GR}_{\mu\nu} + T^M_{\mu\nu}) = 0. \tag{5}$$

If we define the matter density, $\rho = T^M_{00}$ and $p = \frac{1}{3}\sum_i T^M_{ii}$ we find,

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{6}$$

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3}(\rho + 3p) \tag{7}$$

Depending on the equation of state, different forms of energy scale differently with time. For radiation we have $p = \frac{1}{3}\rho$ and

$$\rho_{rad} \propto a(t)^{-4}. \tag{8}$$

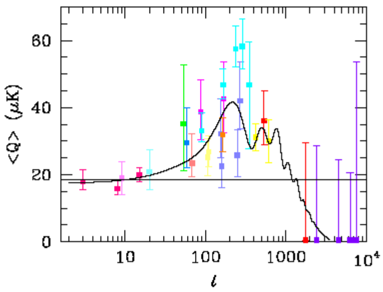

Figure 1: Angular decomposition of the CMB temperature correlations into moments l.[3]

For non-relativistic matter we have $p = 0$ and

$$\rho_{mat} \propto a^{-3}. \tag{9}$$

For vacuum energies of a quantum field we have $p = -\rho$ and

$$\rho_{vac} = \text{const}. \tag{10}$$

At a temperature of $t_c = 1000K$, the photons and charged matter fall out of thermal equilibrium. This is characterized by the formation of hydrogen and the release of photons. These photons then have a blackbody distribution characterized by the combination temperature, t_c. If there are density perturbations in the universe the photons will experience the variations as potential wells. This will lead to variations in the thermal distribution. We have $\delta\rho \propto \delta E = k_B \delta T$ and

$$\frac{\delta T}{T} \propto \frac{\delta\rho}{\rho}. \tag{11}$$

Observationally it is found $\frac{\delta T}{T} \sim 10^{-5}$.[1]

2.2 Inflation

The cosmology described by the FRW metric several difficulties, two of which are the flatness and horizon problems. The universe is observed to be nearly flat. An analysis of the FRW equations of motion show that a flat metric is not a stable solution. In order to achieve the current nearly flat universe one must accept a high amount of fine tuning. The second problem is that the observed CMB spectrum is uniform across scales the FRW metric says were never in causal contact. This again causes an unreasonable degree of fine tuning in the initial conditions of the universe.

An accepted explanation of these observations is to postulate that the early universe underwent an inflationary phase characterized by exponential growth. This will account for the observed observations. The exponential growth pushes the universe towards flatness regardless of initial conditions. The horizon problem is solved because separated points were in causal contact in the pre-inflationary phase. During the inflation, the points fall out of causal contact. The evolution proceeds according to the FRW metric after the inflationary phase.

One mechanism to generate this inflationary phase is a phase transition [4]. One postulates that the physics of the early universe contained a scalar field, whose potential is invariant under a symmetry. At a critical temperature the theory is assumed to undergo a first or second order phase transition. In the case of a second order phase transition, a tachyonic mode develops driving the field away from the peak of the potential. as the field rolls away from the peak it radiates energy which drives inflationary growth. The inflationary phase is assumed to end when the field reaches its minimum. The expectation value of the field $\langle\phi\rangle$ is only expected to be coherent over some coherence length. This length typically varies inversely with the coupling constant of the field and the position of the potential minimum in the broken phase [4].

If the phase transition is a first order transition, on imagines an unbroken phase where the field has a zero expectation value. Below some critical temperature the minimum of the potential discontinuously shifts to some non-zero field value. There is a barrier which, classically, traps the field in a metastable state. Quantum mechanically, the field can tunnel through the barrier. The scalar field will settle into the minimum, releasing energy into the other degrees of freedom of the theory. Here the inflation is driven by the vacuum energy of the field resting in the metastable state. The inflation proceeds for a time set by the tunnelling decay rate. This is set by the size of the potential barrier.

3 Phase Transitions in Particle Physics

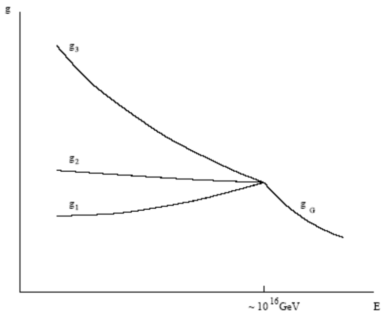

Figure 2: The unification of the MSSM coupling constants. [3]

In the standard model there are two phase transitions. At the electro-weak scale, the $SU(2) \times U(1)$ is broken to the observed $U(1)$ of electricity and magnetism. In addition, at energies near $.2GeV$ hadronization occurs. This is the transition where individual quarks are no longer seen, being bound into composite mesons and nucleons. As mentioned in the introduction, the unification of the coupling constants is suggestive that at energies near $10^{16}GeV$ another phase transition occurs. Further, if one assumes the minimal supersymmetric standard model the coupling constants meet exactly.

The usual Higgs mechanism leads to a first order phase transitions, proceeding via nucleation and bubble formation. [5] Depending upon the internal symmetry group, there is the possibility of defect formation. When bubble formation begins, regions of the "true vacuum" form in a sea of false vacua. These bubbles expand and the local field attempts to align with the true vacuum so as to minimize the energy of the system. In some cases only "large" fluctuations can lower the energy of the system, the system will then contain defects whose lifetime is determined by the probability of a "large" fluctuation.

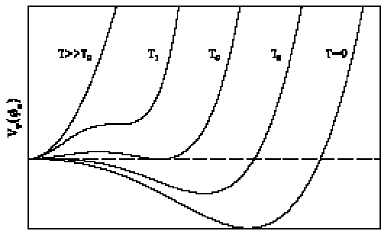

Figure 3: The first order Higgs phase transition.[6]

The Kibble mechanism is a concrete realization of spontaneous symmetry breaking in the early universe. One begins with a Lagrangian possessing a symmetry and a temperature effective potential. Above a critical temperature, T_c, the expectation value for the Higgs field vanishes. Below T_c, the minima degenerate. The field will "choose" one of the minima. At temperatures below the Ginsburg temperature, T_G, thermal fluctuation are insufficient to change the minima and the defects are effectively "frozen out." [7] This mechanism ensures that if the gauge theory allows defects to form, they will in the early universe.

3.1 Classifying Defects

A system's ability to form topological defects reduces to the mathematical study of homotopy. We may define the "vacuum" manifold $\mathcal{M}$. For a theory with symmetric phase symmetry group $\mathcal{G}$ and broken phase symmetry group $\mathcal{H}$, the vacuum manifold is $\mathcal{M} \equiv \mathcal{G}/\mathcal{H}$. The allowed defects correspond to the nontrivial homotopy classes of $\mathcal{M}$. Two maps ϕ_0 and ϕ_1 are homotopically equivalent, $\phi_0 \sim \phi_1$, if there is a continuous one parameter family of maps connecting them:

$$\begin{aligned} \phi(t) : S^n &\rightarrow \mathcal{M};\ t \,\epsilon\, [0,1] \\ \phi(0) = \phi_0 & \qquad \phi(1) = \phi_1. \end{aligned} \tag{12}$$

The n'th homotopy group is the set of n'th equivalence classes, $\pi_n(\mathcal{M})$. This is only a group for $n \neq 0$.[4][8]

For low n we give names for the π_n.

$$\begin{aligned} n &= 1 \rightarrow \text{domain wall} \\ n &= 2 \rightarrow \text{string} \\ n &= 3 \rightarrow \text{monopole} \end{aligned} \tag{13}$$

Defects arise in two varieties local or global. This depends on whether the broken symmetry is gauged or not. Gauge defects are characterized by the existence of a core.

The gauge group of the standard model, $SU(3) \times SU(2) \times U(1)$, does not allow stable defects. However, the popular GUT groups, $SU(5)$ and $SO(10)$, do allow for defects. For $SU(5)$,

$$\pi_2(\mathcal{M}) = \pi_1(U(1)) = \mathbb{Z}. \tag{14}$$

For $SO(10)$ one typically proceeds via two symmetry breakings,

$$SO(10) \rightarrow SU(5) \times \mathbb{Z} \rightarrow (SU(3) \times SU(2) \times U(1)) \times \mathbb{Z}. \tag{15}$$

This breaking will pickup an additional $\pi_1(\mathcal{M}) = \mathbb{Z}_2$. [4]

3.2 Examples of Defects

For domain walls, $n = 1$, the vacuum manifold is a discrete set of points, corresponding to different lowest energy vacuum field configurations. For global defects,the spatial sections of the space-time manifold possesses domains of differing field values. A Lagrangian possessing a domain wall solution is

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu - \frac{\lambda}{4}(\phi^2 - \eta^2)^2 \tag{16}$$

The boundaries between the domains of vacuum energy are called domain walls. Because the field does not take on a vacuum value along the wall, the domain wall solution costs energy. The typical size of a domain wall is $\sim (\lambda^{1/2}\eta)^{-1}$. The energy is $\sim \lambda\eta^4$. This leads to a surface energy $\sigma \sim \lambda^{1/2}\eta^3$.

An example of a string defect is given by a complex scalar field coupled to a $U(1)$ gauge field.

$$\begin{aligned}\mathcal{L} &= |D_\mu \phi|^2 - V(\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ V &= \frac{\lambda}{2}(|\phi|^2 - \eta^2/2)^2\end{aligned} \tag{17}$$

This is the famous "Mexican hat" potential. It is easy to see that $\mathcal{M} = S^1$. As we are looking for field configurations which are maps $S^2 \to S^1$, we may look for z independent solutions. Thinking of S^2 as $\mathbb{C} + \{\infty\}$, we may write down asymptotic solutions [8],

$$\phi = \frac{\eta}{\sqrt{2}} f(m_v r) e^{in\phi}, \ A^i = \frac{n}{er}\hat{\varphi}^i a(m_v r). \tag{18}$$

Here f and a have the large r behavior

$$f \simeq 1 - f_1(m_v r)^{-1/2} e^{-\sqrt{\frac{\lambda}{e^2}}(m_v r)} \ a \simeq 1 - a_1 (m_v r)^{1/2} e^{-m_v r}. \tag{19}$$

In these expressions m_v is the low energy mass of the vector field, $e\eta$. The magnetic flux, through surfaces in the $x - y$ axis including the origin, is quantized in units of inverse electric charge. This solution demonstrates a general feature of gauge defects, we may generally expects a core of vanishing scalar field. The existence of a core indicates that the defect is stable, or topological. The width of the defect is $w \sim \eta^{-1}\lambda^{-\frac{1}{2}}$. The energy per unit length scales as $\mu \sim \eta^2$. This is not the whole story, though. The solution above is for a single string. Strings are allowed to interact both with themselves and other strings, creating new strings.

Monopoles are characterized by point-like cores and quantized winding numbers. The field monopole solutions vanish at spatial infinity. Since these have cores, they are topologically stable. The mass of a monopole is proportional to η. [7]

4 Defect Induced Structure formation

One our best tools for studying the early universe is the cosmic microwave background. The is the electromagnetic radiation released in the recombination phase of cosmic evolution. This is the era when protons and electrons combined to form hydrogen. This occurred at a temperature near 3000 K, or when the universe was about 100,000 years old. This era is the earliest we can hope to probe with electromagnetic radiation. Before recombination the universe was opaque to photons, being a soup of charged particles. It is observed that this background is very nearly blackbody. The temperature variations are $\frac{\delta T}{T} \leq 10^{-5}$. [1]

Does particle physics give possible sources for these perturbations? The existence of magnetic monopoles are generally predicted by grand unified theories. Once the defects are formed at $10^{16} GeV$, the number of monopoles is fixed, while the number of strings may change. Open strings are allowed to self-interact, producing both open and closed strings.

Without inflation, the expansion of the universe causes the density of monopoles to dilute as $a(t)^{-3}$. However, the density of radiation scales as $a(t)^{-4}$. This indicates that at some point in the universe's development the monopoles would dominate, causing the universe to collapse. This contradicts observation. Any viable theory must not allow defects to dominate the evolution of the universe.

As with other difficulties with the "standard" cosmology, inflation provides a resolution. The rapid inflation would serve to dilute the number of monopoles enough to make cosmology consistent with grand unification.

Similar considerations rule out models possessing domain walls. the energy of a domain wall solutions scales as t to the number of co-dimensions, t^2. Because the surface energy density is based upon the difference in vacuum energies, it does not scale with time. Other forms of matter scales like $\rho\ t^3$. The density scales like $\rho \sim \frac{1}{Gt^2}$. The ratio of domain wall energy to matter density would grow linearly in time, causing the universe to collapse.[3] Any viable GUT must not predict domain walls.

Cosmic strings are not as dangerous for the universe. Because their number is not fixed after formation, their scaling behavior is much more complicated. One must rely upon simulations to study string network dynamics. Without shape changing string density would scale as a^{-2}, more slowly than matter and radiation. However, closed strings scales as a^{-3}. The result is that string networks never have a vanishing effect, yet never dominate.[2] The fact that cosmic strings never dominate the development of the universe, yet are not inflated too much, makes them the only candidate defect to seed structure formation.

When a string passes between two objects, initially at rest, the objects move towards each other. If the one object is a source of radiation for the second, the observer will detect a jump in the frequency of radiation. This discontinuous jump is due to a Doppler shift. If the radiation is thermal, the Doppler shift give a deviation from the blackbody spectrum.

$$\frac{\delta T}{T} \propto G\mu \tag{20}$$

This is for a single string. In terms of the CMB, we are the observing object. To account for string networks in the early universe we must look at rms temperature fluctuations averaged over the entire sky. This yields

$$\left(\frac{\delta T}{T}\right)_{rms} \sim (5-20)G_N\mu. \tag{21}$$

The range of values depends on the redshift at which photons were last scattered. For $\eta \sim m_{GUT} \sim 10^{16}$ and $m_{Plank} \sim 10^{19}$, this gives fluctuations of the correct order of magnitude to seed structure formation. [7]

5 Summary

We have reviewed aspects of inflationary cosmology and particle physics. Generalizations of the standard model of particle physics generally predict the existence of topological defects. The existence of domain walls or monopoles could potentially cause the collapse of the universe. The grand unified theory of particle physics must not predict domain walls. The universe is rendered safe from monopoles by the inflationary phase. Cosmic string networks, however, never dominate the evolution of the universe. The Doppler shift from passing string networks is a reasonable candidate for the fluctuations in the cosmic microwave background.

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