Collective Phenomena in Photonic Crystals

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Abstract

Photonic crystals are solid materials that exhibit a selective behavior towards different modes of electromagnetic radiation. Due to a high contrast in periodic dielectric constant they can forbid the existence of light within a certain range of frequencies, called a photonic bandgap, within their core. In this paper we present some of the collective phenomena associated with photonic bandgaps, as well as possible applications and experimental realization of optical devices that employ these principles.

1 Introduction

Photonic crystals are an exquisite exemplification of fundamental physics at work towards cutting-edge technological advancement. Over the past two decades photonic crystals received a lot of attention from physicists around the globe. They offer a rich field of exploration, both theoretically and experimentally. While models flourish and provide insight into new venues of exploitation, fabrication and actual commercialization of derived devices have posed a considerable challenge but, nevertheless, major steps have been taken in overcoming it.

Schematically a photonic crystal is a periodic structure whose two main constituents have different indices of refraction. This contrast represents the basis for the selective behavior of the crystal towards specific modes of light. The photonic crystal forbids the existence of electromagnetic radiation whose energy lies within a range - called photonic bandgap - that depends on the given configuration, (i.e. the photonic crystal in question and the set of parameters that define its state - we shall see later that there exists the possibility of tuning the bandgap without physically changing the sample).

Photonic crystals affect the properties of photons in a similar way that semiconductors affect the properties of electrons. The photonic bandgap is the optical analogue of the electronic bandgap in semiconductors. In a semiconductor an electron is confined from propagating in any direction in the lattice if its energy lies within the forbidden range. The factors determining the energy bandgap are the periodic potential of the atoms and the geometry of the lattice. In much the same way, in a photonic crystal the periodicity (or the lattice) is that of the dielectric constants of the constituent media instead of atoms.

This opens up the possibility of using light instead of the traditional electronic circuits as the information carrier. Ideally, the emerging technology would lead to all-optical integrated circuits. A few notable advantages of photons over electrons are their greater speed when travelling through a dielectric medium, the amount of information they can carry, and the reduced energy losses due to their weakly-interacting nature. Photonic crystals are the ideal candidates for achieving the above goal. While they can be designed to forbid the existence of light with frequencies within the photonic bandgap in the interior of the crystal, defects in the lattice can provide a path that forbidden light can take. The reluctant nature of the photonic crystal will insure that the photons do not escape this path. Hence defects can lead

to the localization of light within the gap in the crystal. The properties of localization depend on the nature and shape of the defects. One can imagine a microcavity (point defect) that would act like a trap for photons in the bandgap, a line defect as a waveguide, or a planar defect as a perfect mirror for the forbidden frequencies.

New phenomena stem from the physics of defects in photonic crystals. Waveguides provide a unique ability for guiding light along narrow channels (in air or different media) and around tight bends (a superior feature that traditional manipulation of photons, based on total internal reflection at the interface between a high-dielectric propagation medium and a low-dielectric material, does not posses). Microcavities on the other hand allow for complete tunability in both defect frequency and symmetry, the latter of which leads to the concept of orbital angular momentum of photons. These types of defects represent basic ingredients for using photonic crystals to control the spontaneous emission of atoms in materials. This leads to atom-photon bound states that exhibit spontaneous symmetry breaking, a quantum optical spin-glass state of impurity two-level atoms, and a Bose-glass state of photons. We shall take a closer look at these phenomena in the following sections of this paper.

From a technical point of view, photonic crystals are artificially fabricated. The higher the contrast in the indices of refraction of the constituent materials, the better is the performance of the crystal. A classical combination is silica spheres and air (opals). However, it is possible to infiltrate the crystals with other media and thus produce a variety of materials with different bandgaps. Ideally, one would like to have a material with a tunable bandgap. It has been theoretically predicted (1999) that a liquid crystal photonic-bandgap material would have such properties. Not surprisingly, last year a group from Denmark achieved the fabrication of such a system. This is a fine example of how emergent states of matter blend together to produce state-of-the-art devices with greatly enhanced performance compared to the more traditional technologies.

2 Collective phenomena

2.1 Emergence of photonic bandgaps

The solutions of the macroscopic Maxwell's equations give a complete description of the electromagnetic properties of photonic crystals. In the absence of external currents and sources the governing equation for the magnetic field is

$$\vec{\nabla} \times \left(\frac{1}{\epsilon(\vec{r})} \vec{\nabla} \times \vec{H}_\omega(\vec{r}) \right) = \left(\frac{\omega}{c} \right)^2 \vec{H}_\omega(\vec{r}) \tag{1}$$

where $\vec{H}_\omega(\vec{r})$ is the magnetic field of the photon with frequency ω, $\epsilon(\vec{r})$ is the spatially periodic dielectric function, and c is the speed of light. In addition, $\vec{H}_\omega(\vec{r})$ satisfies the transversality condition $\vec{\nabla} \cdot \vec{H}_\omega(\vec{r}) = 0$. The periodicity of the dielectric function requires the solution of the Maxwell's equation to satisfy the Bloch-Floquet theorem:

$$\vec{H}_\omega(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} \vec{H}_{\omega,\vec{k}}(\vec{r})$$

where $\vec{k}$ is the Bloch vector (crystal momentum) and is restricted to lie in the first Brillouin zone. $\vec{H}_{\omega,\vec{k}}(\vec{r})$ denotes the lattice periodic part of the Bloch function: $\vec{H}_{\omega,\vec{k}}(\vec{r}+\vec{R}) = \vec{H}_{\omega,\vec{k}}(\vec{r})$ for all lattice vectors $\vec{R}$. Restricting the Bloch vector to the first Brillouin zone corresponds to back-folding the dispersion relation for the infinitely-extended $\vec{k}$-space therein by translations through reciprocal lattice vectors. Consequently a set of discrete solutions (called *band structure*) emerge; they are indexed by a natural number n and have a particular wave vector $\vec{k}$: $\{[\omega_n(\vec{k}), \vec{H}^n_{\omega,\vec{k}}], \vec{k} \in$ first Brillouin zone, $n \in N\}$. To explicitly solve Eq.(1) the Bloch component of the magnetic field is expanded in a set of transverse plane waves, leading to a standard hermitian eigenvalue problem. We illustrate in Fig. 1 two photonic band structures calculated accordingly. Note the excellent agreement of experimental results with the theoretical predictions.

Particular dielectric geometries of the photonic crystal lead to a complete bandgap in the band structure, namely a region of frequencies with no allowed photon modes for any value of the wave vector $\vec{k}$ inside the first Brillouin zone.

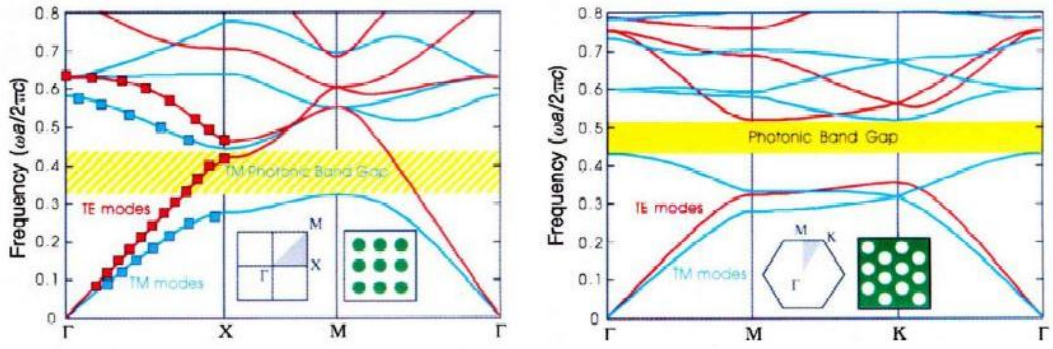

Figure 1: Left, photonic band structure for a square lattice of dielectric ($\epsilon = 8.9$) rods in air. Transverse magnetic (TM) modes are shown in red and transverse electric (TE) modes in red. The solid lines are from theory and the squares represent experimental measurements. Right, photonic band structure for a triangular lattice of air cylinders in dielectric ($\epsilon = 13$). Note the presence of a complete photonic bandgap for both TE and TM polarizations. In both cases green indicated high-dielectric material.

2.2 Symmetry breaking in photonic crystals

Spontaneous emission is the natural tendency of an excited atom to fall into a lower energy state by releasing radiation. Due to the discrete nature of energy levels in the atom, there are well-determined frequencies that the emitted wave can have. The rate at which the atom decays depends on the coupling between the atom and the photon, as well as on the density of states available for the emitted photon. In a photonic crystal both of these factors can be tuned by changing the properties of the defects introduced in the bulk of the lattice.

The existence of a photonic bandgap around an atom or a set of atoms that normally radiate forbidden modes can confine or even suppress spontaneous emission and thus lead to the formation of strongly-localized states of light and photon-atom bound states. The spontaneous emission in this case has been shown to display an oscillatory behavior due to the strong interaction between the atom and its own localized radiation. This is quite different from a normal exponential decay occurring in vacuum, and leads to naming this phenomenon *superradiance*.

Consider a system of N identical two-level atoms coupled to their radiation field in a three-dimensional periodic lattice, confined to a region smaller than the wavelength of light, which lies in the vicinity of the bandgap of the crystal. The Hamiltonian for the interaction is

$$H = \sum_{\lambda} \hbar \Delta_{\lambda} a_{\lambda}^{\dagger} + i\hbar \sum_{\lambda} g_{\lambda} (a_{\lambda}^{\dagger} J_{12} - J_{21} a_{\lambda}),$$

where $J_{ij} = \sum_{k=1}^{N} |i\rangle_{kk}\langle j|(i,j = 1,2)$ are the collective atomic operators, a_{λ} and $a_{\lambda}^{\dagger}$ are the radiation field annihilation and creation operators, $\Delta_{\lambda} = \omega_{\lambda} - \omega_{21}$ is a detuning of the radiation mode frequency ω_{λ} from the atomic resonant frequency ω_{21}, and g_{λ} is the atomic field coupling constant.

S. John proposed two models for the superradiance of this system, an isotropic one that makes use of the Markovian approximation, and an anisotropic one that employs numerical methods and gives a more accurate description. However, qualitatively both models render similar results: localized superradiance and spontaneous symmetry breaking when the atomic resonance frequency lies at the edge of photonic bandgap.

To see how spontaneous symmetry occurs in superradiance, a small external perturbation is introduced, to the effect of polarizing the atomic system by an infinitesimal amount. It turns out that this initial polarization of the atomic dipoles gives rise to a macroscopic polarization in the steady-state limit $t \to \infty$. This is distinct from the free-space superradiance where the atomic polarization in this limit is zero. This spontaneous symmetry breaking in the atomic polarization field is analogous to lasing without a cavity mode and hints at the possibility of observing macroscopic quantum coherent superpositions of states. The steady-state atomic polarization also has a rotating phase whose frequency is proportional to the magnitude of vacuum Rabi splitting. This spontaneous polarization is analogous to the emergence of a "superfluid" order parameter for photons.

In an anisotropic three-dimensional photonic bandgap material the collective decay rate of superradiance scales as N^2, and consequently the peak superradiance intensity is proportional to N^6. This shows that the superradiant emission occurs much faster and outputs a greater power than conventional spontaneous emission. This suggests that lasers operating near a photonic band edge will exhibit ultrafast modulation and swithching speeds that would make them an ideal medium for data transfer and computing.

2.3 Optical spin-glass state. Photon Bose-glass state

Let us consider a system of N impurity two-level atoms imbedded in a photonic crystal as before, with two important differences: the resonant fre-

quency of the atoms lies well within the photonic bandgap, and the volume that superradiance is confined to (i.e. the volume of the defect) is larger than a cubic wavelength. Under these circumstances resonance dipole-dipole interactions (RDDI) between atoms prevail and can lead to a breaking of the permutation symmetry of the N-atom wave function. RDDI causes photons to propagate within the cavity (thus the atoms interchange energy), while the overall spontaneous emission from the system of N atoms is inhibited by the photonic crystal.

Employing multiple theoretical methods (the interaction picture, order parameters in analogy to spin-glass theory, and mean-field approximation) for the above system, S. John derives a remarkable description of the behavior of the ensemble in the steady-state limit, given certain nonequilibrium boundary conditions. Numerical simulations of the equations of motion have confirmed the qualitative picture. The parameters that offer insight into the evolution of the system are the global polarization density of the whole atomic system, $m(t)$, and the Edwards-Anderson order parameter that describes local, spontaneous atomic polarization, $q(t)$. What leads the author to foreseeing a glass-like state emerging from coherent but random RDDI is the fact that, although the overall atomic system acquires a steady-state polarization (quantified by m), its individual phase varies chaotically from atom to atom. Thus the resulting collective state resembles a quantum spin glass. On the other hand, the photons interacting with the impurity atoms tend to a steady state that is the optical analogue of a Bose gas. The frequencies they posses are not unique (they depend on the RDDI), but are nonetheless restricted by the chemistry of the atoms and the coupling to the defect in the crystal. Consequently the ensemble light state is intermediate between incoherent light given off by a thermal source and coherent light from a conventional laser.

The main result of this analysis is that $m(t)$ has an oscillatory behavior with a collective time scale factor of $N^{\frac{1}{2}}$ and tends to zero in the steady-state limit while $q(t)$ remains nonzero as $t \to \infty$, justifying the parallel to spin-glass systems. Moreover, similar behavior is deduced concomitantly for analogous parameters that describe the cavity mode (the field amplitude m_c, and the Bose-glass order parameter q_c). Thus, the cavity mode tends to a Bose-glass state in the steady-state limit. These results hold true as $t \to \infty$ for two different sets of initial states of the system, as illustrated in the graphs below.

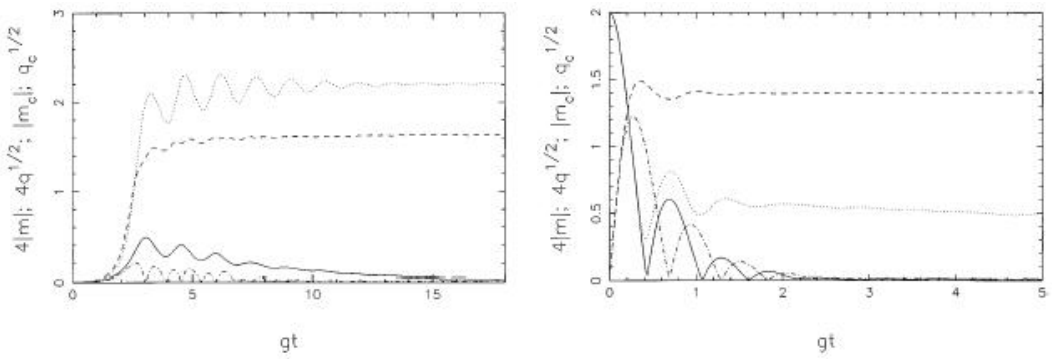

Figure 2: Macroscopic atomic polarization $|m|$ (dashed-dotted curve), spin glass order parameter $q^{1/2}$ (dashed curve), coherent state amplitude $|m_c|$ (solid curve), and Bose-glass order parameter $q_c^{1/}2$ (dotted curve), as a function of scale time gt for two distinct initial states: atoms excited, defect mode in vacuum (left); atoms in ground state, photons in defect mode in coherent state (right).

3 Liquid Crystal Photonic Babdgap Fibers

The localization of light and the controllable inhibition of spontaneous photon emission from defects in photonic crystals have opened up new venues for technological applications, particularly in the field of lasing and data transport. Versatile devices designed on the basis of the fundamental principles of photonic crystals (and defects) should offer some flexibility, i.e. the possibility to tune the photonic bandgap without physically changing the microstructure of the sample. This could be achieved by infiltrating an air-dielectric photonic crystal with a medium that has an externally-controllable index of refraction. Liquid crystals are ideal candidates for this, as their dielectric constant depends on the orientation of the nematic director $\hat{n}(\phi, \theta)$, which can be manipulated thermally.

S. John and K. Busch advanced the idea of a tunable liquid crystal photonic band gap material and developed a theoretical model that also offered considerable insight into the fabrication of such a material. The model shows how a complete photonic bandgap arises at a particular orientation of the director of the liquid crystal; the width of the bandgap changes subsequently, as $\hat{n}$ is rotated, because different high symmetry points in the Brillouin zone are affected unequally. We illustrate these effects in Fig. 3 for a silicon inverted opal infiltrated with the nematic liquid crystal (BEHA).

Combining this idea with existent photonic bandgap materials technol-

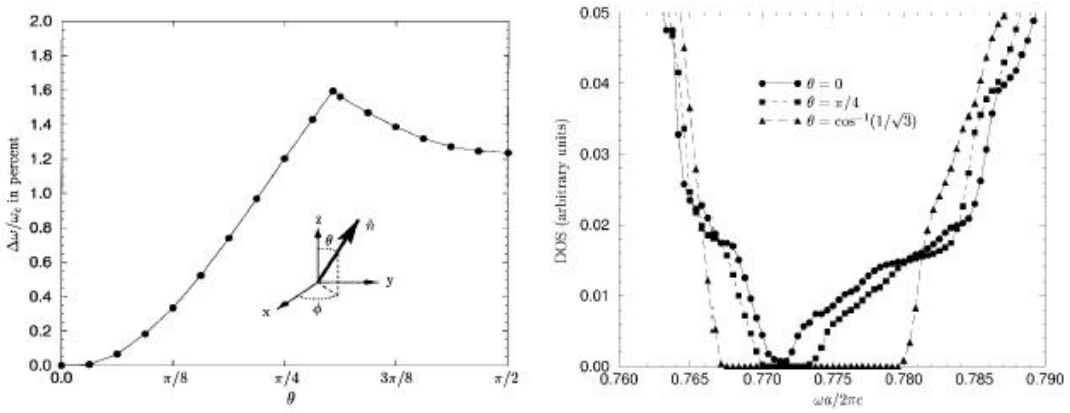

Figure 3: Photonic bandgap size (left) and total photon density of states (right) *vs.* orientation of the nematic director $\hat{n}(\phi, \theta)$ for a fixed angle $\phi = \pi/4$. Note that the photonic bandgap is closed for $\theta = 0$ and reaches a maximum value $\Delta\omega/\omega_c \simeq 1.6\%$ relative to its center frequency ω_c for $\hat{n} = (1, 1, 1)/\sqrt{3}$.

ogy, specifically photonic crystal fibers that act as optical waveguides, T.T. Larsen *et.at* designed and fabricated *liquid crystal photonic bandgap fibers*. A pictorial comparison between the "theoretical view" and the "experimental realization" is shown in Fig. 4.

Apart from behaving as predicted by theory (the sensitivity of the bandgap location was found to be 3nm/^{0}C for infrared light, and 1nm/^{0}C for visible wavelengths below the cholesteric to isotropic phase transition temperature $T_c = 94^0$C), the new devices brought to light a quite dramatic effect when operated at the phase-transition temperatures of the liquid crystal. The dominant bandgap evolved from green to yellow, then to an off state (due to the highly scattering behavior of the liquid crystal at the phase transition that causes the molecules to dis-align), and ultimately to blue, as shown in Fig. 5.

It has thus been demonstrated both theoretically and experimentally that it is possible to obtain tuneable light localization in a photonic crystal by infiltrating it with a liquid crystals. The spectral characteristics of the fiber obtained this way depend on the original photonic crystal fiber and the alignment and optical properties of the liquid crystal. These results open up the possibility of designing advanced all-optical signal processing devices on a small scale.

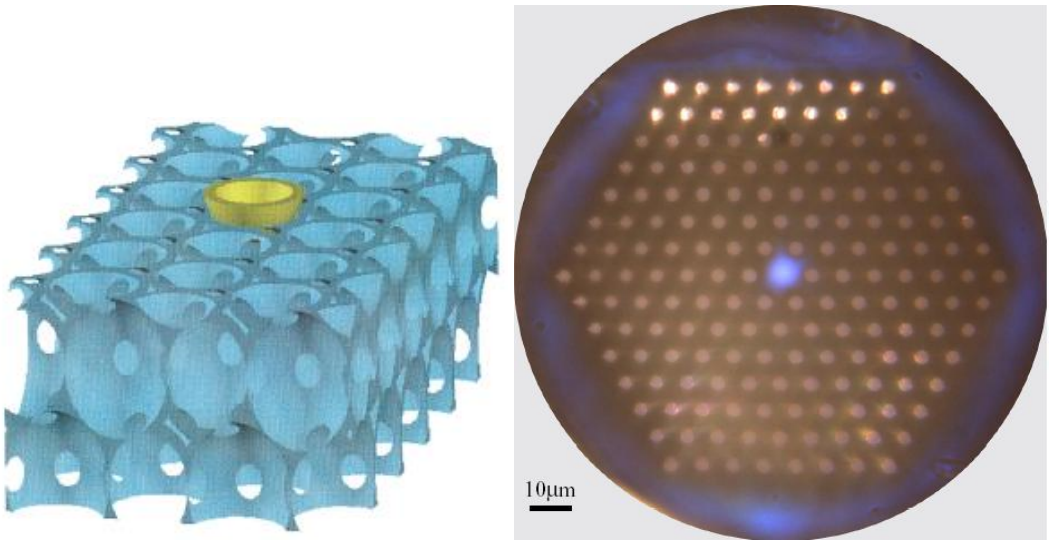

Figure 4: Left, opal structure - by adding patterns of liquid crystal, one can program transmission properties. Right, liquid crystal photonic crystal fiber supporting a blue mode.

4 Conclusion

We have exemplified how photonic crystals provide a rich field of study from two viewpoints. Firstly, new phenomena and states of matter arise from fundamental principles applied to photonic bandgap materials (controlled spontaneous emission, localization of light, atom-photon bound states, glass-like states of both atoms and photons). Secondly, as a consequence of these novel manifestations, (controllable) defects in photonic crystals allow the programming of these processes, thus providing a highly-efficient way of moulding the flow of light, and even tuning the forbidden modes.

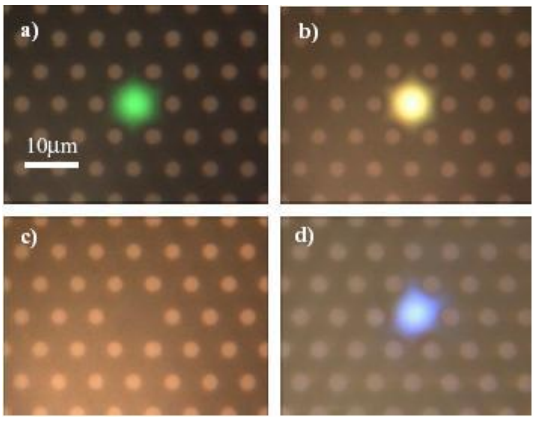

Figure 5: Micrographs of guided modes in a liquid crystal photonic bandgap fiber. (a) green at 77^0C, (b) yellow at 89^0C, (c) off state at 91^0C, (d) blue at $T_c = 94^0$C.

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