Vortex lattice pinning in high-temperature superconductors.

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Abstract.

Vortex matter in high temperature superconductors has many peculiar properties such as melting of the vortex lattice, creation of new vortex-liquid phases etc. These effects are not seen in conventional superconductors. This is mainly due to the fact that HTc compounds are strongly type two superconductors with Ginzburg-Landau ratio up to ~110 that makes thermal and quantum fluctuations more profound. Another class of unusual vortex matter properties in HTc materials is related to their structural features. Most important are strong layering and structural defects such as dislocations and grain boundaries. Moreover structural defects can be introduced artificially irradiating samples with high-energy ions. In this term paper I am going to discuss effect of structural defects on vortex lattice behavior in particular on vortex lattice pinning.

Introduction.

High-temperature superconductors (HTS) belong to one of the most intensively studied area of contemporary condensed matter physics. Not only this is because of still unraveled microscopic mechanism of superconductivity but also because of the variety of phenomenological properties of these layered and strongly type II superconductors and their possible technological applications. Most important of them are related to the behavior of superconductors in the presence of the magnetic field and their ability to carry dissipation-free currents.

As it is well known the magnitude of dissipation-free current in a superconductor cannot exceed some critical value j_c which is called critical current density. Currents with density j greater than j_c break up Cooper pairs and destroy superconductivity hence j_c is naturally called depairing critical current. Simple estimate for maximal Cooper pair velocity v_c can be obtained if we notice that the pair is broken if an electron from it acquires additional energy of the order of the energy gap Δ that leads to the electron's velocity being changed by $\sim \Delta / p_F$ and hence $v_c \sim \Delta / p_F$. More accurate value can be obtained from Ginzburg-Landau theory and is given (up to a numerical constant) by

$$j_c = \frac{c\phi_o}{\lambda^2 \xi} \qquad (1)$$

where ϕ_o is the flux quantum, λ and ξ - penetration depth and coherence length respectively. Plugging in typical values of the parameters for YBCO compound which is a common representative of HTS materials ($T_c \approx 90K$; at T=77K $\lambda \approx 2600\,\mathring{A}$, $\xi \approx 35\,\mathring{A}$) we get

$$j_c \sim 5 \cdot 10^7\, A/cm^2 \text{ at T=77K} \qquad (2)$$

The truth of life is that typical experimental values of j_c are ranging from $1 \cdot 10^4\, A/cm^2$ in bulk YBCO samples to $5 \cdot 10^5\, A/cm^2$ in thin films which is at least two orders of magnitude less than predicted value (2). This means that superconductivity is destroyed not by depairing but by another mechanism that "turns on" much earlier before actual break down of the pairs. The key to this mechanism may be the fact that in the growth of samples of HTS materials a large scale grain structure can arise with rather large angles of mutual misorientation of the grains both in the plane of the layer (*ab*) and with respect to the direction of the *c* axis. Here Josephson weak links can be formed at the boundaries between the grains, substantially suppressing the

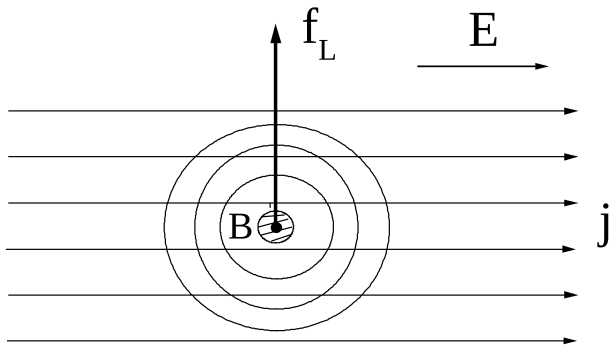

fig.1

superconducting current j_c. This model is supported by electron and scanning tunneling microscopy observations of YBCO samples.

As it has been already mentioned there is another class of phenomenological properties of type II superconductors which is related to their behavior in the presence of magnetic field. If the magnitude of the field is less than some critical value* $H_{c1} \sim \phi_o / \lambda^2$ then the field is completely expelled from the bulk of the superconductor (Meissner phase). Upon increase of the field it penetrates into the superconductor in the form of flux lines of the field, surrounded by circular superconducting currents decaying on the length $\sim \lambda$. Such object is called vortex. Each vortex carries one flux quantum ϕ_o of the field and has energy ε

$$\varepsilon = \varepsilon_o \ln(\lambda / \xi); \qquad \varepsilon_o = (\phi_o / 4\pi\lambda)^2 \qquad (3)$$

per unit length. Superconductivity is suppressed only in the core of the vortex that has characteristic size ξ ($\xi << \lambda$ for HTS materials). Vortices interact with each other and form a regular lattice with the period

$$a_o \sim \sqrt{\phi_o / B} \qquad (4)$$

(exact value of a_o depends on the type of the lattice). Under further increase of the field a_o is decreasing and when it becomes $\sim \xi$, i.e. vortex cores starts overlapping, superconductivity disappears. This happens when the field reaches value $H_{c2} \sim \phi_o / \xi^2$.

If an external current density j is applied to the vortex system the flux lines start to experience the Lorentz force f_L (see fig.1):

$$f_L = \phi_o j / c \quad \text{for single vortex per unit length} \qquad (5)$$

As a result the vortex system is moving with velocity $\vec{V}$ and finite electric field $\vec{E} = \vec{B} \times \vec{V} / c$ is generated. Since both $\vec{j}$ and $\vec{E}$ run parallel the finite power $P = \vec{j}\vec{E}$ is dissipated in the system. Thus, it seems that under application of arbitrary small* magnetic field the technological advantage of a superconductor i.e. its ability to sustain dissipation-free current flow is lost. In

* H_{c1} strongly depends on the geometry of the sample. It is maximal for the cylindrically shaped sample with the magnetic field applied along the cylinder's axis (for YBCO H_{c1}~100G). For a film with perpendicular magnetic field H_{c1}~ 0.

order to recover the desired property the flux lines have to be pinned such that $\vec{V} = 0$ even though $f_L \neq 0$. In this case the driving Lorentz force is counteracted by the pinning force F_{pin}. Fortunately, any static disorder such as grain structure described above will contribute to a finite pinning force F_{pin} and thus reestablish technological usefulness of type II superconductors. However, maximal dissipation-free current is determined now by the pinning force F_{pin} and is given by

$$j_c = cF_{pin} / B \tag{6}$$

This depinning critical current is usually at least an order of magnitude less than the depairing critical current defined by eq. (1).
The effects described above take place in any, either conventional or HTS type II superconductor. However, due to a special range of parameters, behavior of the vortex system in HTS materials is much more richer and allows experimental testing of new theoretical ideas. One can name several reasons for this:

--small coherence length ξ, which is only a few times larger than the crystal lattice spacing. This makes vortices sensible to the details of the microscopic structure of the crystal lattice and introduces the notion of quenched (static) disorder. In HTS materials quenched disorder can be created by oxygen vacancies in CuO planes (uncorrelated disorder) and by structural defects of the crystal lattice such as dislocations, grain boundaries etc. (correlated disorder)

--importance of thermal fluctuations which is characterized by the Ginzburg number *Gi*. The Ginzburg number is defined as a square of a ratio of the characteristic thermal energy T_c to the condensation energy $H_c^2 \xi^3$ of the volume of size ξ. Here H_c is thermodynamic critical field $H_c \sim \phi_o / \lambda\xi$. In conventional superconductors $Gi \sim 10^{-8}$ while in HTS materials $Gi \sim 10^{-2}$. This allows experimental observation of non- mean-field behavior of the vortex system.

Vortex matter is a very rich and broad subject of modern condensed matter physics* [1]. In this term paper I am going to focus only on the simplest, essentially “one-particle” properties of the vortex matter, and discuss their relation to the vortex pinning and to the problem of determination of critical currents in HTS materials. We start with the discussion of the phenomenological phase diagram of the vortex matter.

Phenomenological phase diagram of the vortex matter.

We consider at first a system of vortices in a homogeneous type II superconductor with no driving currents. As has been already mentioned it exists in the region of magnetic fields bounded by H_{c1} and H_{c2} from below and above respectively. Values of these critical fields depend on temperature that gives two first-order phase transition lines on the corresponding (T,H) phase diagram (fig.2a). The main new result in the description of the vortex system in HTS is the appearance of a vortex liquid phase occupying a substantial portion of the phase diagram below $H_{c2}(T)$ and above $H_{c1}(T)$. Melting of the vortex lattice in the region of the phase diagram close to the upper critical field is caused by increasing thermal fluctuations of the vortex position $\vec{u}$. To determine the position and the shape of the vortex lattice melting line one uses simple Lindemann criterion $< \Delta\vec{u}^2 >= c_L^2 a_o^2$, where a_o is given by eq. (4) and $c_L \sim 0.1 - 0.4$ is

* For example, review [1] has more than two hundred pages and more than six hundreds refrences.

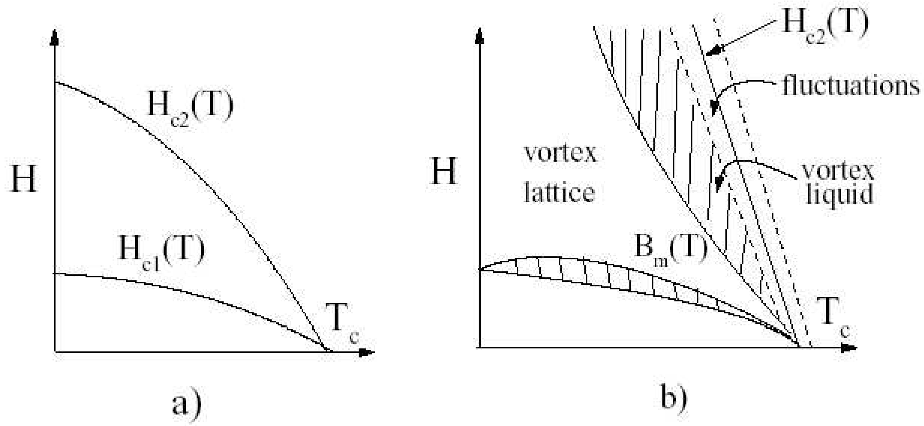

fig2. (adapted from ref. [1])

the Lindemann number. Considering vortices as non-interacting elastic strings with tension ε_o (eq. (3)) in a thermal bath one gets the following equation for the melting line:

$$B_m(T) \approx (5.6\, c_L^4 / Gi)\, H_{c2}(T)(1 - T/T_c) \tag{7}$$

The melting of the vortex lattice close to the lower critical field is caused by the weakening of the vortex-vortex interaction. With decreasing magnetic field, the distance between the vortices increases and eventually grows beyond the penetration depth λ. In this region the vortex-vortex interaction is exponentially small and the vortex lattice state is destroyed. As a result, the melting line develops the reentrant behavior shown on fig.2b. In homogeneous superconductors the width of the vortex-liquid phase close to H_{c1} is extremely narrow, of the order of 1G. However introducing disorder in the system allows substantial expansion of this phase (see below).

Another interesting region of the vortex matter phase diagram is the regime of critical fluctuations (see fig.2b). The width ΔT of this region is determined by the Ginzburg number $\Delta T \sim T_c\, Gi$. In spite of the largeness of Gi the width ΔT around the mean field transition line $H_{c2}(T)$, where fluctuations of the amplitude of the order parameter become relevant, is still small and is of the order 1K for YBCO compound. Outside of this region all the fluctuation degrees of freedom involve only the phase of the order parameter. Note that $H_{c2}(T)$ line now is just a crossover line and no longer describes a thermodynamic phase transition.

To study the response of the vortex matter driven by an external current we need to introduce disorder. In this way the vortices become pinned and under certain conditions the dissipation-free current can flow in the system. However, even being pinned, the vortices, subject to thermal fluctuations, can still 'jump' from one pinning site to another. This phenomenon is called creep and it is made possible by the fact that pinned vortex lattice driven by a current is a metastable state of the vortex system. Creep of the vortex lines is equivalent to their small but finite directed motion and thus leads to the dissipation that again raises the doubts about existence of 'truly superconducting' vortex state. The crucial question here is about existence of creep down to the limit of zero driving currents. There are two possibilities:

1) if the dissipation due to creep and hence the resistivity vanish in the limit $j \to 0$ then in the thermodynamic sense the system is able to sustain the superconducting current for exponentially long time;

2) if, however, the dissipation remains finite then the superconducting currents are subject to a fast decay.

In the first case the system is said to be in the vortex-glass state and is characterized by non-analytic response to the vanishing current $E \sim \exp(-j_c / j)^{\mu}$, where E is electric field developed by the system and $\mu > 0$. On the phase diagram of the vortex matter in the presence of driving currents and pinning forces the vortex-glass state takes the place of the vortex lattice phase on the diagram from fig.2b.
In the second case the system shows Ohmic behavior and is in general in the liquid phase (fig.2b). In the high field regime, close to the superconducting transition line, the vortex liquid is essentially free to move (unpinned) and the corresponding regime is called flux flow. In the low field regime just above $H_{c1}(T)$ the vortex liquid is in the pinned state and the dissipation in the system is performed by thermally assisted flux flow.

Thus, the introduction of disorder (pinning forces) in the vortex system has important consequences on the vortex matter phase diagram. In the next chapter we are going to review nature of the pinning forces in HTS materials.

Pinning of the flux lines.

The elementary pinning forces of individual vortices can be due to various causes: interaction of the normal core of the vortex with microscopic cavities in the superconductor, magnetic interaction of the vortex currents with their mirror images near the surface of the superconductor and with small ferromagnetic particles, due to nonuniformities of the electron's mean free path etc. In most cases pinning energy of the vortex is of the order of the condensation energy $H_c^2 R^3$, where R is the characteristic size of the defect.

There is another relevant source of pinning forces in HTS materials. Due to the small coherence length ξ the flux lines become sensitive to the details of the microscopic structure of the crystal lattice such as oxygen vacancies (defects) in CuO planes. Each individual defect cannot pin a vortex line but collective action of several defects can substantially affect the vortex dynamics. This introduces the idea of the collective pinning. When the individual pinning forces acting on the vortex line are summed up, the contributions of various defects will add up only randomly, i.e. only fluctuations in the density and force of the defects can pin the flux line in a definite position. Assuming a density n_i of defects acting with an individual force f_{pin} on the vortex line, the total force for a vortex segment of the length L is

$$F_{pin} \approx f_{pin} \sqrt{n_i \, \xi \, L} \tag{8}$$

that shows only square-root behavior with the segment length L. On the other hand the Lorentz force (5) grows linearly which would imply that, driven by a current, the vortex remains unpinned. On the other hand, as the vortex can accommodate itself to the pinning potential by elastic deformation, the flux line can bend in order to find most favorable energetic position. The bending energy will compete with the pinning energy over some distance L_c determined by the condition $\varepsilon_{elastic}(L_c) \approx \varepsilon_{pin}(L_c)$. The vortex then breaks into segments of the length L_c, each of the segments behaving separately under the action of the Lorentz force. This leads to a finite critical current density

$$j_c \approx j_{co} (\xi\gamma / \varepsilon_o^2)^{2/3} \tag{9}$$

where j_{co} is the depairing critical current given by eq. (1), ε_o is the elastic energy of the vortex (eq. (3)) and coefficient γ characterizes the strength of the disorder. The response of the vortex system driven by a current and subject to the collective pinning will be essentially the same as described at the end of the previous chapter. Weak pinning mechanism of the critical current limitation is most probably realized in bulk HTS samples and produces low values of the critical current density.

As it has been already mentioned in the Introduction, the magnitude of the critical current strongly depends on the sample preparation and reaches its maximal value for the thin film samples. This fact is probably related to the presence of a large number of crystal lattice defects in epitaxially grown films of HTS materials, in particular in YBCO films, which contain large number of edge dislocations oriented both along the *c* axis and in the *ab* plane. It is energetically favorable for the edge dislocations to align into quasiperiodic chains to compensate elastic deformation energy of the crystal lattice. As a result of this, a film separates along its entire thickness into a system of single-crystal slightly misoriented grains (blocks). The size of the grains can vary from several hundred to several thousands angstroms. The boundaries of the grains, composed of edge dislocations, can serve as effective pinning centers for the flux lines. To calculate the pinning force acting on a flux line due to the presence of a dislocation we need to consider the dislocation structure. In the simplest case the dislocation can be modeled as a non-superconducting non-metallic core* with the radius r_o. The pinning energy of the vortex collinear with the dislocation is given then by the following expression:

$$\varepsilon_{pin}(\vec{u}) = -\int d^2 r U_{pin}(\vec{r})(1 - |\psi(\vec{r} - \vec{u})|^2) \tag{10}$$

where $\vec{u}$ is the distance between the vortex and the dislocation, U_{pin} is the pinning potential, ψ gives the distribution of the order parameter around the vortex. Corresponding pinning energy has a maximum at $u = 0$:

$$\varepsilon_{pin}(0) = -\varepsilon_o r_o^2 / 2\xi^2 \tag{11}$$

and yields critical current densities comparable with the depairing current [3].
When the dislocations are aligned in a grain boundary they create highly anisotropic pinning potential despite the fact that the contribution of each individual dislocation is isotropic. The pinning potential has a shape of a deep 'gully' (fig.3) with longitudinal pinning forces being much smaller than the transverse ones. Such anisotropic pinning forces can substantially affect

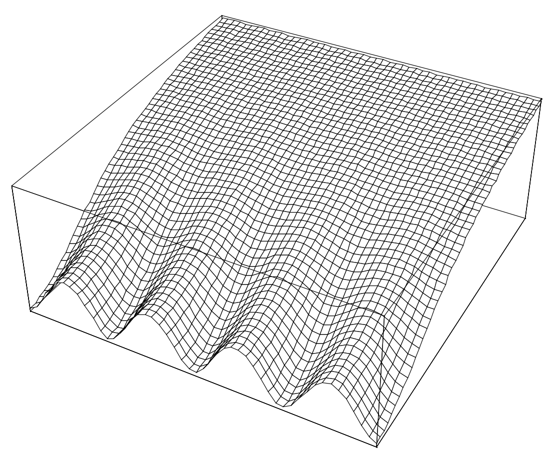

fig3.

* The physics of dislocations in HTS materials is actually much richer. In particular, it can be shown [2] that elastic strain fields around the dislocations can cause local enhancement of the critical temperature T_c

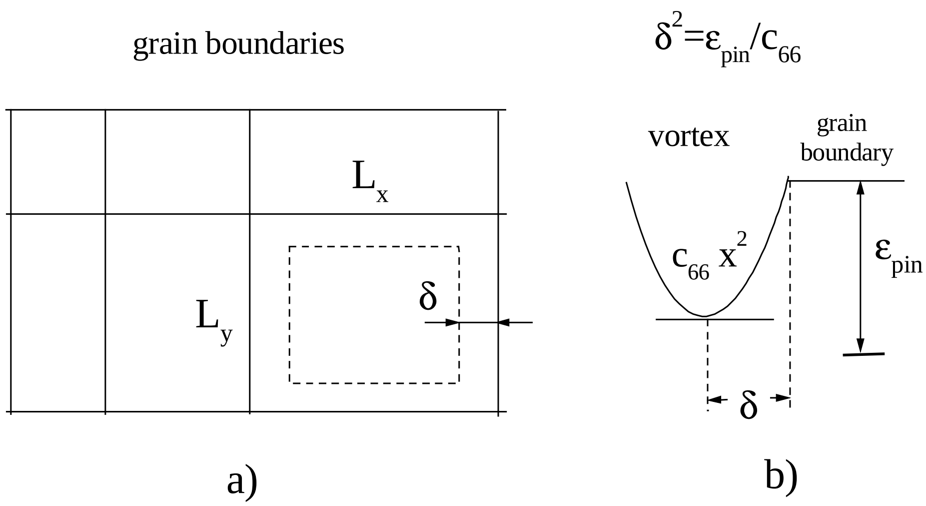

fig4.

the vortex matter phase diagram, considered in the previous chapter under the assumption for the disorder being uncorrelated and weak. To see how the presence of correlated disorder can be reflected on the phase diagram we consider a thin film of type II superconductor in the perpendicular magnetic field with the thickness of the film $d < \lambda$ that makes unfavorable flux line bending. Next, we introduce correlated disorder in the form of a random rectangular grid (fig.4a) which models grain boundaries of the actual superconducting samples. The sizes *L* of the grains are described by the distribution function *P(L)*. We also restrict discussion to the magnetic field region where the distance between vortices a_o is larger than the average size of the grain $< L >$. Pinning properties of the grain boundaries are specified by the pinning potential ε_{pin} which is assumed to be localized on the boundary and can be taken from eq. (11).

Now let's superimpose a regular vortex lattice with the pinning grid. Each vortex will try to adjust its position to minimize the total energy. Displacement *x* of a vortex is opposed by the vortex lattice and will require the energy $c_{66}x^2$, where c_{66} is the shear modulus of the vortex lattice (fig4b.). For this change to be compensated by the pinning energy the vortex should be in a stripe of the thickness δ near the grain boundary. Equating the energies we get

$$\delta^2 = \varepsilon_{pin} / c_{66} . \tag{12}$$

Upon decrease of the magnetic field the shear modulus c_{66} goes to zero and hence δ grows. Let's denote the maximal grain size by $L_{\max}$. We can see that when the condition $\delta > L_{\max}$ is satisfied all vortices in the system are pinned at random places and the vortex system is neither in the liquid nor in the solid states. If the field is increased some part of the vortices is released from the pinning centers and forms either the vortex liquid or solid according to the phase diagram of the previous chapter.

For an arbitrary value of the field the relative number $n_p(B)$ of the pinned vortices is given by the simple relation [4]:

$$n_p(B) = 1 - \left[\int_{2\delta}^{\infty} P(L)(1 - 2\delta / L)\, dL \right]^2 \tag{13}$$

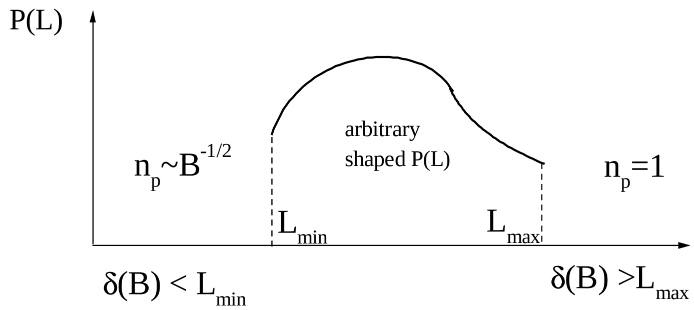

fig.5

We can extract more information about $n_p(B)$ making only general assumptions about the function *P(L)* and using an expression for the shear modulus in the intermediate range of the magnetic fields $c_{66} = \phi_o B/(8\pi\lambda)^2$. This information is summarized on fig.5

To recover physical significance of the eq.(13) we note that experimentally measured critical current density j_c is directly related to the function n_p:

$$j_c(B) = j_{c,B=0}\; n_p(B) \tag{14}$$

Experimental measurements are in the good agreement with eq. (13, 14) in the intermediate range of the magnetic fields.

Conclusions

In this short review we summarized some basic ideas of a relatively new and rapidly developing field of condensed matter physics - the vortex matter. Taking as an example YBCO high temperature superconductor we analyzed the vortex matter phase diagram and stressed the importance of the better understanding of the vortex pinning properties both for theoretical developments as well as for possible technological applications.

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