Propagation Modes in Heisenberg Spin Glasses

Shizhong Zhang

Department of Physics, University of Illinois at Urbana-Champaign, IL 61801

Spin glasses is a fascinating field, from which a variety of theoretical models and concepts were invented. As an emergent state from the random magnetic system, it defers from other statistical systems in several ways. In particular, the quenched variables prevent the system to be ergodic, and the competitive interactions gives rise to non-trivial degeneracies of the ground states as well as the excitations. In this essay, I will briefly discuss these generic features of spin glasses by presenting some of the characteristic experimental results followed by theoretic analysis. The broken symmetries are discussed with compare to other magnetic systems. Hydrodynamic theory, as well as numerical investigations are discussed aiming to identify the low lying excitations.

Contents

I. INTRODUCTION

Magnetic systems always provide us with a lot of interesting phenomena to study. Such as ferromagnetic(FM) and anti-ferromagnetism(AFM) etc., both systems are associated with spontaneously broken symmetries. In ferromagnetic, one breaks the global $SO(3)$ symmetry, and the order parameter(OP) which signatures the phase transition of the system is the average magnetism, defined by $\mathbf{M}_F = 1/N \sum_i \langle \mathbf{S}_i \rangle$. In anti-ferromagnetism, the global $SO(3)$ symmetry is also broken, and we have the order parameter defined as "staggered magnetism", $\mathbf{M}_{AF} = 1/N \sum_i e^{-i\mathbf{K}\cdot\mathbf{r_i}} \langle \mathbf{S}_i \rangle$. Those systems are well behaved in a sense that all spins in a symmetry broken state are align collinearly along a particular axis in space, i.e, the system breaks its symmetry from $SO(3)$ to $SO(2)$. However, it is noticed later, when people was trying to understand the low-temperature behavior of certain kinds of well-known alloys like AuFe or CuMn, under a critical temperature T_f, those systems fell into a state which is not like that of a paramagnetism, ferromagnetism, ferrimagnetism or anti-ferromagnetism. It is a state with a frozen disorder, not a uniform pattern we expect in conventional magnets. Those systems are generally referred as spin glasses in literature. Some believed that there is a phase transition happened near the transition temperature T_f, and some other evidence showed that the systems is not in equilibrium state, because the very long relaxation time of the system. Here some interesting questions emerges: 1), Is there any phase transitions near the critical temperature, if there is, then 2) what is broken symmetry and 3) what is the order parameter to describe those system. Also, if the relaxation time of the system is very long, then nonergodicity becomes a problem, 4) how can we deal those system with a lot quenched parameters.

The theoretic work dated back to the 1960s, when people use Ising model with RKKY interactions to describe spin glasses. It was not until when Edwards and Anderson(1) published their paper on the theory of spin glass, that this field became a very excited area. In their paper, Edwards and Anderson introduced the random bond model and the spin glass order parameter

$$q_{EA} = \lim_{t\to\infty} \lim_{N\to\infty} [S_i(t_0) S_i(t_0 + t)]_{av} \tag{1}$$

in which we use $[\,]_{av}$ to denote the average over disorder. We will discuss this model in detail afterwards.

The arrangement of this essay will be as following. In first section, I will describe the experimental observations of the system, and present what is the necessary ingredients for a system to be treated as a spin glass. In second

section, I will describe some basic theoretical concepts in spin glass, including broken symmetry, order parameter and low energy excitations in spin glass, with compare to other magnetic order, particularly, paramagnetic, ferromagnetic and anti-ferromagnetic. In the third section, I am going to describe the hydrodynamic theory of spin glass, and another possible way to find the spin waves in those disorder systems, using some approximate decouple scheme in the equation of motion, and show some numerical results obtained so far.

II. EXPERIMENTAL STUDIES IN SPIN GLASSES

In this section, I will present some of the characteristic experiments in spin glasses systems, and compare them to that in conventional magnetic systems. I will concentrate on two major findings in spin glass, one is the time-dependent susceptibility, and another is the specific heat of spin glasses.

We know that in high-temperature, the susceptibility of the magnetic system obeys the so called Curie-Weiss law:$\chi = C/(T-\theta)$, where C is the Curie constant. Surprisingly, in spin glass system, one found that the deviation from the Curie-Weiss law even well above the spin glass transition temperature, around $T = 5T_f$. This means strong correlation, at least locally, has developed in the system. One typical data for the zero-filed frequency dependent susceptibility is shown in Fig.1(2).

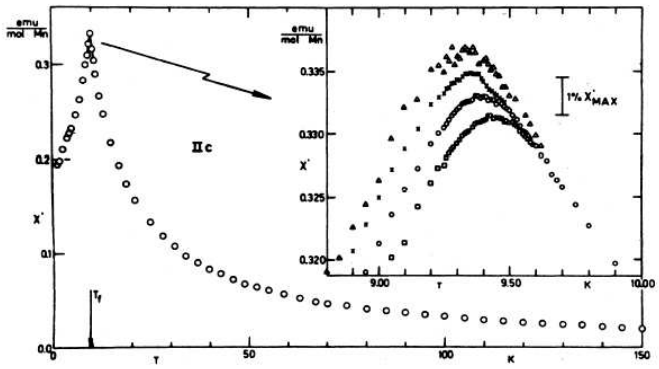

FIG. 1 Real part of susceptibility $\chi(\omega)$ as a function of temperature for sample IIc. Inset reveals frequency dependence and frequency dependence of the cusp by using of strongly expanded coordinate scale. After Mulder et al.(2).

In the above figure, we can see that there is a frequency dependent cusp in the low temperature susceptibility, and it turns out that this is a fairly universal behavior in spin glass system.

Another striking feature of the spin glass system is the strong preparation-dependent d.c susceptibility. The susceptibility obtained by cooling the system in the measurement field (FC: field cooling) yielded a higher value than that obtained from first cool the system and then applying the magnetic field(ZFC: zero-filed cooling). In Fig.2(3), we show a demonstration of this phenomena. Those preparation-dependence are also seen in the remanent magnetization,i.e, result is higher if we cool the system in the external field and them remove the field than that in the case we cool in a zero external field and then measure the remanent magnetization. All these phenomena suggests that there are certain degrees of freedom of the system in choosing its state in low temperature, i.e, a set of states with free energy very close to each other but has a high energy barrier between them. If in a external field, system will try to relax itself to a state in which it has the lowest free energy,i.e, largest magnetization along the external field; if it is not in a external magnetic field, then due to the high free energy barrier, the system will reside in one of those free energy valleys (depending on the initial conditions, which also illustrate the preparation-dependent of the susceptibility), and have a relatively small susceptibility.

Caution should be taken to interpret those phenomena. Is the system in true equilibrium state? In 1984, Wegner and Mydosh(4) observed that the fc susceptibility of the cobalt aluminosilicate spin glass depends on the cooling rate; and also fc susceptibility will slowly vary with time as observed by Lundgren et al(5). The zero-field cooling susceptibility depends strongly on the time the sample is kept in the constant temperature after cooling prior to the field application. All these suggest that maybe the true equilibrium state hasn't yet been achieved. Lundgren et al. estimated the relaxation time of a CuMn spin glass, which yield a relaxation time as long as 10^{20} sec. at $T = 23K$,(while $T_f = 27K$ for Cu with 4 at. % Mn)(5).

Now, let's now check the specific heat. There is a rather broad peak at temperature exceeding the freezing temperature at about 20%; at $T < T_f$, it varies approximately linearly with T. In the magnetic field, the specific heat

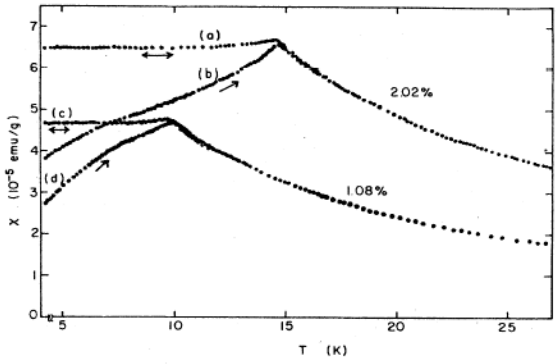

FIG. 2 Static susceptibility of CuMn vs temperature for different concentrations of Mn. After zero-field cooling ($H < 0.05Oe$),initial susceptibilities (b) and (d) were taken for increasing temperature in $H = 5.9Oe$. The susceptibilities (a) and (c) are obtained in the field $H = 5.9Oe$, which is applied above the freezing temperature T_f before cooling the sample, After Nagata et al..(3)

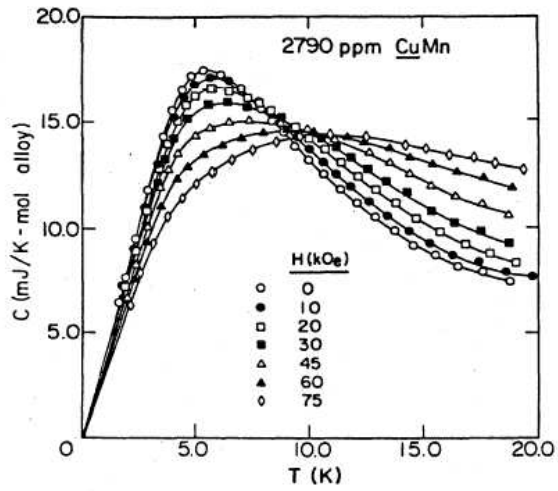

FIG. 3 Magnetic contribution of the specific heat of CuMn spin glasses with 2.79% Mn plotted vs temperature in various magnetic fileds. After Brodale et al..(6)

profile will get rounded. We show a typical behavior below in Fig.3.(6). We can see that in higher fields goes round and lose the peak in the specific heat, which is in contradict to the prediction of mean field theory, while mean field theory predict quite pronounced singularity in the low temperature susceptibility, although this does not necessarily mean that there is no phase transition at T_f. A close inspection gives the behavior of specific heat as

$$C_M \approx c_1 T + c_2 T^2 \tag{2}$$

Some other experiments shows that in amorphous $Gd - Al$ alloy, which in some respects like the spin glass system, have a $T^{3/2}$ dependence of the specific heat, as shown by J. M. D. Coey(8).

One can check the question of thermal equilibrium here by see the validity of the thermal dynamic relation. Fogle et al.(9) studied the Maxwell relation

$$\left[\frac{\partial^2 M}{\partial T^2}\right] = \frac{1}{T}\left[\frac{\partial C_M}{\partial H}\right]_T \tag{3}$$

surprisingly, no measurable deviation from (3) existed.

We have shown the most characteristic experimental phenomena for the spin glass state. 1),Low-field, low-frequency a.c. susceptibility $\chi(T)$ exhibits a cusp at transition temperature T_f; 2), no sharp anomaly appears in the specific heat; 3),history dependent of the susceptibility; also there are some other characteristic phenomena exists in spin

glass such as 4)lucking of Bragg peaks below T_f, which means no long range order exist in the system. We will not discuss all those phenomena in the following, but, we will focus, in particular, the specific heat observed in spin glass systems, comparing them to other conventional magnetic ordering states, and try to illustrate a way which could, possibly, explains the behavior of specific heat at low temperature.

III. BASIC CONCEPTS IN SPIN GLASSES

In this section, I will present the basic concepts needed to understand the spin glass system. First I will introduce two models to describe spin glass system, which capture the basic ingredients of spin glass. Then I will describe what is the broken symmetry and order parameters, and finally the spin waves in spin glass.

As mentioned before in the introduction, there is certain kind of disorder in the system, as magnetic impurities occupy the lattice site randomly. But this is not sufficient to be a spin glass. In addition, we have shown in spin glass system, there is a set of states with similar free energy, but with high free energy barrier between them. Above T_f, the system can make its way from one state to any other states, and achieve the thermal equilibrium, technically, the system is ergodic, it can wonder around the whole phase space. But as the temperature approaching the freezing temperature, the free energy barrier develops, and the system may reside in one of those states which is a local free energy minima. Because of the high free energy barrier between those states, the system will sit in one of those state for a long time compared to the observation time, so the system breaks ergodicty, it can not wonder around a whole phase space, but just part of it. Those meta-stable states in spin glass are due to some competition interactions, which can not be satisfied simultaneously. which is called **frustration**.

The first model which captures essentially those two ingredients is the so called random site model. In which the randomness comes from the fact that the impurity atoms can occupy randomly the lattice sites, and the interaction takes the form of RKKY interaction[1]. The system in this class are generally called RKKY glasses. Another model introduced by Edwards and Anderson(1), in which they choose the spins to be a regular lattice(translationally invariant), but with random interactions between spins. The hamiltonian of the system is

$$H = -\frac{1}{2}\sum_{i,j} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \tag{4}$$

where i, j are lattice sites, and the interactions J_{ij} are taken to be random with a distribution which dependent on the distance $\mathbf{R}_i - \mathbf{R}_j$. In practice, it is convenient to choose the symmetric Gaussian distribution

$$P(J_{ij}) = \frac{1}{(2\pi\Delta_{ij})^{1/2}} \exp\left[-\frac{J_{ij}^2}{2\Delta_{ij}}\right] \tag{5}$$

There is further simplification of the model, which take the exchange interaction to be short range, for instance, nearest neighbor interaction. In the following, we will concentrate on such model.

As mentioned before, the spin glass system breaks ergodicity. This put a serious problem if we want to apply equilibrium statistics mechanics. One way to get rid of that is by using the appropriate boundary conditions to restrict the phase space to be in a certain part of the whole phase space. Like in two dimensional Ising spin, you can put all the spin on the boundary spin up, and this will restrict the whole system to be in a state with $M > 0$ to be the equilibrium state, rather than $M < 0$.Another way of doing this is by using of replica theory, by replicating N systems. Using the fact that

$$lnZ = \lim_{n\mapsto 0} \frac{Z^n - 1}{n} \tag{6}$$

to evaluate the quantity lnZ, which is self-averaging. I will not discuss this here. One can consult the paper written by David Sherrington (15).

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[1] The RKKY interaction has an oscillatory behavior, comes from the fact the impurity atoms polarize the conduction electrons of the host metal, and the spin susceptibility has the form

$$\chi_0(r) \propto \frac{cos2k_F r}{r^3}$$

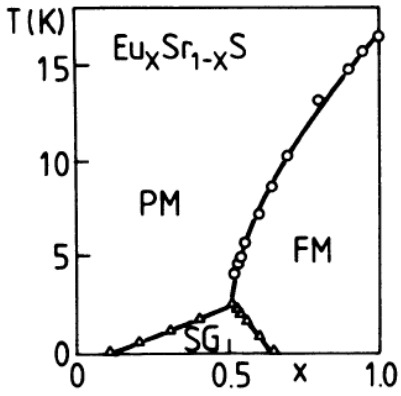

FIG. 4 Phase diagram of the spin glass system $Eu_xSr_{1-x}S$. After P. Monod et al..(10)

We now turn to the problem of broken symmetry in spin glass and introduce the order parameter. To do this, I will first give a phase diagram of a specific spin glass system, $Eu_xSr_{1-x}S$. There are three phases in the phase diagram. In paramagnetic phase(PM), the system has the highest SO(3) symmetry; in ferromagnetic and anti-ferromagnetic, one breaks the SO(3) to SO(2).In ferromagnetic, we have the order parameter as pointed out in the introduction.

$$\mathbf{M}_F = 1/N \sum_i \langle \mathbf{S}_i \rangle \tag{7}$$

Similarly, we have the order parameter for anti-ferromagnetic

$$\mathbf{M}_{AF} = 1/N \sum_i e^{-i\mathbf{K}\cdot\mathbf{r_i}} \langle \mathbf{S}_i \rangle \tag{8}$$

Those order parameter goes to zero in paramagnetic state and takes a finite value in the ferromagnetic (anti-ferromagnetic) state.What is the order parameter for spin glass then, from the above analysis, it seems that the spin glass state is more like a paramagnetic, in a sense that it has a disorder in it, as the random orientation of spins in paramagnetic. It turns out that the spin glass system also breaks the SO(3) symmetry, but with a zero total magnetization $\mathbf{M}_F = 1/N \sum_i \langle \mathbf{S}_i \rangle$=0. How can we tell the difference of a spin glass with a paramagnetic state? Here Edwards-Anderson come to rescue. They define the order parameter as in (1). Let us consider the static situation, which is then

$$q_{EAij} = [S_i \cdot S_j]_{av} = q_{EA}\delta_{ij} \tag{9}$$

We will simply taken the q_{EA} as the spin glass order parameter, which is believed to be zero in paramagnetic phase, and take a non-zero value in spin glass state. Here we list varies quantities that are important in magnetic systems.

Table 1 :Characteristics of the magnetic systems[2]

Systems	Broken Symmetry	Order Parameter	Transition Temperature
Paramagnetic	NO	$M = 0$,$M_{AF} = 0$, $q_{EA} = 0$	in high temperature
Ferromagnetic	$SO(3) \rightarrow SO(2)$	$M \neq 0$,$M_{AF} = 0$, $q_{EA} \neq 0$	$T < T_c$ (Curie temperature)
Anti-ferromagnetic	$SO(3) \rightarrow SO(2)$	$M = 0$,$M_{AF} \neq 0$, $q_{EA} \neq 0$	$T < T_n$ (Néel temperature)
Spin Glass	$\mathbf{M} \neq 0$, SO(3); $\mathbf{M} = 0$, NO	$M = 0$,$M_{AF} = 0$, $q_{EA} \neq 0$	$T < T_f$ (freezing temperature)

[2] Here, we only considering the symmetry associated with the global rotation, since this is responsible for the spontaneous magnetization of the system. And also, I left diamagnetism and ferrimagnetism, since it won't give any new insights into the symmetry consideration discussed here. A note about the symmetries in spin glasses. It is sure that the O(3) symmetry in SG transition is totally broken, if we assume the spin lives in three dimensions.

Knowing order parameters and broken symmetry, we can then tell the differences between each phase, and more importantly, we can construct the free energy of the system, based on the symmetry requirement. We will show this later.

We now turn to discuss the spin waves in magnetism system and give the dispersion relation of these spin wave modes. We will give here very simplified consideration based on the symmetry principles. For more technical derivation, see the review article by J. Van Kranendonk and J. H. van Vleck in Reviews of Modern Physics(11). For simplicity, we consider the one dimensional spin system, which we show schematically in the following graph. We now analyze

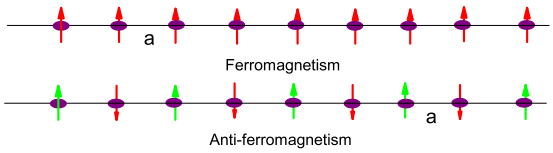

FIG. 5 Schematic graph of the Ising spin with ferromagnetic and anti-ferromagnetic alignment. The lattice constant is a.

the symmetries of both cases. In ferromagnetic case, one preserve the translational symmetry, denoting by $U(a)$, meaning that moving the system by one lattice site in the positive direction, and $U(-a)$ in the negative direction.In the mean while, the system breaks the time reversal symmetry, denoting by T, meaning that we transfer $t \to -t$. So the corresponding equation of motion for the spin waves should be invariant under the action of $U(a)$, but changes under the action of T. In this situation, we should have the spin wave dispersion relation goes like $\omega \propto k^2$. Similarly, for anti-ferromagnetic case, one breaks the translational and also time reversal symmetry, but we can combine these two to give a invariant action $TU(a)$, which will take the system back to the original state. In this case, the dispersion relation takes the linear form as $\omega \propto k$. The situation is not so clear in spin glass however. There have been several unsuccessful attempts to detect the spin waves using neutron scattering. There are several possible reasons for the failures, which will be discussed in the next section.

For the moment, let's make a connection between spin waves and the linear specific heat observed in experiments. There should be at least two sources connected with magnetism which are responsible for the specific heat[3]. One is associated with the field energy proportional to χH^2, and another comes from the excitations independent of external field. We can do experiments in zero external field, so we will only discuss the second contribution. Let us assume that there are certain moods with energy ϵ_k, k is the wave vector, and density of states $\rho(\epsilon)$. Then the energy of the system is

$$E_M = \int_0^\infty d\epsilon f(\epsilon)\epsilon\rho(\epsilon) \tag{10}$$

where $f(\epsilon) = [\exp(\beta\epsilon) - 1]^{-1}$ is the Bose-Einstein distribution. Those excitation are actually called magnon. If we assume a linear density of states $\rho(\epsilon) \propto \epsilon$, then we can get the specific heat linear with T. This implies that you have a quasi-particles dispersion relation like $\epsilon(k) \propto k^{3/2}$, if we have $\epsilon(k) \propto k^2$, then we have $C \propto T^{3/2}$. Compared to the low temperature specific heat observed experimentally before, we see that it could possibly be interpreted as the contribution from the spin wave excitations. But what about the inter-valley transitions, as showed by P.W. Anderson(7), by using the two-level tunnelling model, which gives a linear specific heat at low temperature? At present, we think that the inter-valley transition plays a minor role in the low temperature behavior, because of the high free energy barrier between the local minimums.

IV. SPIN WAVES IN SPIN GLASSES: HYDRODYNAMIC THEORY AND NUMERICAL SIMULATIONS

We have shown in the previous section that the linear specific heat could be explained by assuming that the spin wave dispersion takes the form as $\epsilon(k) \propto k^{3/2}$. In this section, we will examine whether this is true or not. We will approach this problem in two ways. Firstly, I will discuss the hydrodynamic theory of spin glass, and secondly, I will discuss some results obtained by computer simulations.

[3] There are sure other contributions from phonon and conduction electron etc.. In practice, we can substrate them off and get the specific heat only due to magnetic freedom.

In hydrodynamic theory(12), one should considering all those modes which is slow than the slowest the microscopic mode. Let us denote the characteristic relaxation time in spin glass to be τ, then it follows that $\omega\tau \ll 1$, where ω is the frequency of the spin wave. The hydrodynamic mode are determined by the conservation laws and broken symmetries in the system. In the latter case, we could expect that there might be Goldstone mode present in the system, as the spin waves in ferromagnetism and anti-ferromagnetism, since the system has a lower symmetry in the ground state than that of the hamiltonian. A subtle point is that, as we showed before, there are very long relaxation time in spin glass, so it is hard to make clearly what is the scale of the frequency within which the hydrodynamics are valid. This is surely a big problem for experiment to identify the spin wave mode using neutron scattering measurement. Another fatal threat to hydrodynamic theory is the spin anisotropic interactions in real system, which always destroy the prediction of hydrodynamic theory in sufficient long wave lengths.

Since hydrodynamic theory only concerns about those modes with very long wave lengths, so we can neglect lattice details and define the "coarse-grained" quantities(12). Like the spin density $\mathbf{S}(\mathbf{r})$

$$\mathbf{S}(\mathbf{r}) = \frac{1}{N_R}\sum_{j\in R}\mathbf{S}_j \tag{11}$$

where R denote a region which contains N_R spins labelled as j. From the spin commutation relation $[S_i^r, S_j^s] = i\epsilon^{rst}S_i^t\delta_{ij}$, we have the commutation relation for the "coarse-grained" spin operator

$$[S^r(\mathbf{r}), S^s(\mathbf{r}')] = i\epsilon^{rst}\delta(\mathbf{r}-\mathbf{r}') \tag{12}$$

To find the order parameter, let us first pick up one particular ground state, say g. we then define the quantity

$$t^{rs}(\mathbf{r}) = \frac{1}{N_R}\sum_{j\in R}\langle S_i^r\rangle_g S_j^s \tag{13}$$

It can be seen that the Edwards-Anderson order parameter in the coarse-grained form is just

$$\langle t^{rs}(\mathbf{r})\rangle_g = q_{EA}(\delta^{rs} + \epsilon^{rst}\Theta^t(\mathbf{r})) \tag{14}$$

Now let's consider a small rotation about the ground state defined by the rotation operator

$$U = \exp[i\sum_j \mathbf{\Theta}_j\cdot\mathbf{S}_j] \tag{15}$$

we get a new state, say, g', then we have

$$\begin{aligned}\langle S_j^r\rangle_{g'} &= Z^{-1}Tr(U^{-1}\rho_g U S_j^r) &(16)\\ &= \langle S_j^r\rangle_g + \epsilon^{rst}\Theta^t(\mathbf{r}) &(17)\end{aligned}$$

where ρ_g is the density matrix in ground state g. From (16), we can solve for $\Theta^t(\mathbf{r})$, it has the form

$$\Theta^r(\mathbf{r}) = \frac{1}{2q_{EA}}\epsilon^{rst}\langle t^{rs}(\mathbf{r})\rangle_g \tag{18}$$

We redefine $\Theta(\mathbf{r})$ as an operator

$$\Theta^r(\mathbf{r}) = \frac{1}{2q_{EA}}\epsilon^{rst}t^{rs}(\mathbf{r}) \tag{19}$$

from which we can compute the commutation for $\Theta(\mathbf{r})$, it is simply

$$[\Theta^r(\mathbf{r}), \Theta^s(\mathbf{r}')] = \frac{i}{4q_{EA}\rho^2}\epsilon^{rst}S^t(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') \tag{20}$$

$$[S^r(\mathbf{r}), \Theta^s(\mathbf{r}')] = -i(\delta^{rs} + \frac{1}{2}\epsilon^{rst}\Theta^t(\mathbf{r}))\delta(\mathbf{r}-\mathbf{r}') \tag{21}$$

If we assume that the coarse-grained magnetism is zero, i.e, $\mathbf{S}(\mathbf{r}) = 0$, then the commutation relation (12),(20) and (21) simply reduce to[4]

$$[S^r(\mathbf{r}), S^s(\mathbf{r}')] = [\Theta^r(\mathbf{r}), \Theta^s(\mathbf{r}')] = 0 \tag{22}$$

$$[S^r(\mathbf{r}), \Theta^s(\mathbf{r}')] \;=\; -i\delta^{rs}\delta(\mathbf{r}-\mathbf{r}') \tag{23}$$

Now, we consider the hamiltonian of the system, as said before, can be constructed from the symmetry considerations. We drop terms that are high in order of $\Theta^n, (n = 2, 4, 6\cdots)$, since the energy is independent of the rotation of all spins by the same angle Θ. In long range wavelength limit, the most important part has the form $(\nabla\Theta)^2 \equiv \sum_r (\nabla\Theta^r)^2$. So the effective hamiltonian takes the form as

$$H_{eff} = \frac{1}{2}\int d^3r(\chi_0|S^r(\mathbf{r})|^2 + \rho_s(\nabla\Theta)^2) \tag{24}$$

here χ_0 is the uniform susceptibility and ρ_s is the spin wave stiffness, both are phenomenological constants.

Neglecting the quantum fluctuations, we replace the commutation relation by Poisson brackets, and using the canonical equation, we have

$$\partial_t S^r = \{H_{eff}, S^r\} = -\frac{\delta h_{eff}}{\delta\Theta^r} = \rho_s\nabla^2\Theta^r \tag{25}$$

$$\partial_t \Theta^r = \{H_{eff}, \Theta^r\} = -\frac{\delta h_{eff}}{\delta S^r} = \chi_0^{-1}S^r \tag{26}$$

The solution to the above two equations gives the solution

$$\omega = \pm c_0|k|, \quad c_0 = \frac{\rho_s}{\chi_0} \tag{27}$$

Further investigation of the hydrodynamic theory will give a damping term proportional to k^2.

Several remarks here. Firstly, as mentioned before, the condition for the hydrodynamic to be valid, one should have $\omega\tau \ll 1$. It is not quite sure whether this is satisfied for spin waves to be detected practically. Secondly, we assume in the hamiltonian that the spin stiffness is finite, but this could be wrong. There are several computer simulations which showed that there is a large density of states in the low energy limit, which indicates that the spin stiffness maybe zero. Thirdly, there are faster damping modes presenting in the system to which the spin wave modes are strongly coupled, causing dramatic dissipations. The unsuccessful experiments so far could possibly be understood from the facts list above.

The situation of spin waves is still not so clear even now. Several simulations seems to be quite good for certain kinds of spin glass. As an exercise, we are trying to redo the calculation in a larger system using the method by W.Y.Ching et al.(13), called the equation of motion technique. By using an approximate decoupling procedure, one can calculate the one-magnon zero-temperature dynamic structure factor, which tells the dispersion relation of spin waves. Due to the limitation of space, I will just outline the principal steps below, more information can be found in reference (13).

We take the Heisenberg hamiltonian

$$H = -\frac{1}{2}\sum_{i,j} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \tag{28}$$

Treating it as a classic Heisenberg spin glass, we minimize the energy (28) subject to the constraint $\mathbf{S}_i \cdot \mathbf{S}_i = 1$. We can do this by successively rotating each spin to align with its local field[5]. After this, we get the equilibrium configurations of the system. Then using Holstein-Primakoff transformation to construct the local creation and annihilation operator

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[4] We will not discuss the situation where $\mathbf{M} \neq 0$. Hydrodynamic theory can be applied to non-zero magnetization essentially the same as that to the zero magnetization case. Also note that with $\mathbf{M} \neq 0$, we break the SO(3) rotation symmetry, as shown in Table 1.

[5] You can prove that by doing this, you are guaranteed to get to the ground state.

(in one-magnon approximation). By linearizing the hamiltonian in terms of magnon creation or annihilation operators, we can using the quantum-mechanical equation of motion to obtain the equation for magnon operators. Solving the equation, we can then compute the dynamic structure factor in a straight forward manner. I won't give the detail derivation here, but simply quote some of the result here[6]. Ching et al. used this method for Edwards-Anderson model in a $16 \times 16 \times 16$ cubic lattice with periodic boundary conditions. They found no long-wavelength propagating modes. Here is a typical curve they got from their simulation.

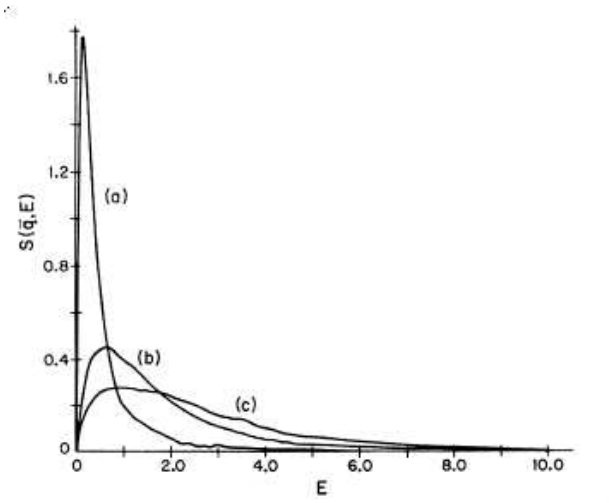

FIG. 6 $S(\mathbf{q}, E)$ vs E. (a), $\mathbf{q} = (\pi/8)(1,0,0)$; (b), $\mathbf{q} = (\pi/2)(1,0,0)$; (c), $\mathbf{q} = \pi(1,0,0)$. $16 \times 16 \times 16$ cubic lattice with periodic boundary conditions. All curves normalized to unite area.

It is evidence that as $|\mathbf{q}| \to 0$, the spectral weight becomes increasingly concentrated near $E = 0$. It is simply because of the spectral function at $\mathbf{q} = 0$ is a constant of motion, called kinematic slowing down. Even though there is a clear peak in the spectral function, but, whether it can be identified with long wavelength propagating mode is not so evident. Is is just an artifact of numerical analysis? There are much left to be done about this question. In a word,by now, there is as yet no satisfactory theory about the dynamical factor of Heisenberg spin glass, and so with the spin waves in those systems.

V. CONCLUSION

In conclusion, we have shown in this essay some aspects of the spin glass system, and outlined a unsolved problem which deserves further investigation. This essay is by no means complete and there are a lot interesting points which are missing here. Mostly due to the little acquaintance of spin glass of the author. Those who are interest in spin glass system should consult books by K.H. Fischer and J.A. Hertz entitled *Spin Glasses* and also there are excellent review papers on spin glass like K. Binder and A.P.Yang(14) appeared in Reviews of Modern Physics. In the end, these lines by David Sherrington gives a more comprehensive meaning of spin glass.

Spin glass behavior is not limited to alloy systems, we also encounter this kind of situation in biology, evolution, organization dynamics, hard-optimization, and environmental and social structures. In consequence, the expression spin glass has now taken on a wider interpretation to refer to complex glassy behavior arising from a combination of quenched disorder and competitive interactions or constraints, and to systems exhibiting such behavior.©*David Sherrington(15)*

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[6] There are still one problem with my code. I haven't got enough time to debug it. So for the moment, I will simply quote some of the result obtained in early 1980s.

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