The emergence of money

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Abstract

Money, defined as the media of exchange, is demonstrated to be an emergent property of interacting-agent systems. An agent-based model of markets is discussed. In the model, agents are able to trade among a set of commodities, none of which has any special feature, i.e. they are symmetric to each other. The symmetry is then spontaneously broken due to fluctuation and money is emerged. Comparison to other models is discussed and drawback of these models is addressed.

I. INTRODUCTION

If there was only one individual in this world, he could only produce, *e.g.,* hunting and consume, *e.g.,* eating. And there would be nothing interesting about the system. But if we add more individuals to the systems. They could start doing different things, some of them might be hunting, some of them might be fishing, the others might be finding places to settle down. This is the division of labor, which enables everyone to specialize and focus on one or a few jobs, doing jobs better, advancing technologies, *etc.* All of these are not possible for just a single individual.

If we continue adding more and more individuals to the systems, different new phenomena might arise. People might organize themselves into villages. There might be languages, cultures, laws. Different disciplines emerge. Different social structures, such as markets, schools, shops, are being formed. The point is none of these structures is prescribed nor assumed in the very first place. They just come out autonomously, due to the complicated interaction between the individuals. These are emergent properties of this particular system. And that is why I am writing about economics in a course of emergent state of matter.

Economic systems are systems with many degrees of freedom (buyers, sellers, governments, companies, countries, *etc.*) interacting (producing, trading and consuming) with each others through some mechanisms (markets, barter and monetary exchange) over different spatial and temporal scales. These are inherently complex systems. Depending on the levels of description, the basic entities of these systems change (individuals, institutes, countries). The obvious basic entity is individual human being, with a set of things that they are able to produce, trade and consume. The goal of the game is to get whatever one wants to get. It can be food, clothes, premises, luxuries, knowledge, *etc.* As we discussed before, there are tons of examples of emergent phenomenon in this system. But we are going to focus only on one of them, namely the emergence of money.

People studying economics discovered that money can autonomously emerge from exchanges of goods among individuals without any supervision. The idea is that when people becomes aware of their economic interest, they would give their commodities in exchange for other more salable commodities, even though they might not need them for immediate consumption purpose. By such an invisible hand in the market, we can everywhere observe the phenomenon of certain number of goods becoming acceptable to everyone in the trade, that is, becoming money. We will come back to this later.

People are trying to convert this intuition into models. And since the system is so complex, many of them are trying to tackle the problem using the so called agent-based computational (ACE) models. ACE modeling is a general technique used by people building models in economics and finance. Its basic idea is to model a complicated system by creating a set of entities, called agents, which are assigned with a set of attributes, and being able to interact with each other by following a set of rules. The specifics of those rules and attributes depends upon the system that is to be modeled. A computer simulation is then carried out to study this ACE model.

Different ACE models are built to study the emergence of money in a system of trading agents.[1][2][3] Virtually they all support the statement that money is a emergent property. With the hope that this article is not too technical and boring, we are going to study in detail the simplest one of these models[1], which should enable us to gain some insight into the subject. In later sections, we will compare different ACE models studied. Note that the focus of this paper is not any of these particular ACE models, but the emergence of money in a general interacting-agent system. We spend time detailing a particular ACE model because it would let us have a clearer idea of what ACE modeling is, which in turn let us see clearly the drawbacks of it.

We will see that in that particular model, the emergence of money resembles the theory of phase transition. One of the commodities spontaneously breaks the symmetry between the commodities and be promoted to money. The coupling constant in this case is the threshold of exchange

This article is organized as follows. Sec. II describes precisely the model we are going to study, and why we expect the emergence of money. Sec. III shows the simulation results. Sec. IV discusses and interprets the significance of the results, with comparison with other models is made. Drawbacks of ACE models are also discussed. Finally, Sec. V is the conclusion.

II. THE SYSTEM

In this section, we first give an intuitive picture of the system, and explain why we expect emergence of money. Then we discuss the details of our ACE model.

A. Conceptual Discussion

Let us consider a system of N agents. For simplicity, let us assume that they do not produce nor consume.[1] All they have is a set of goods that they can trade with each other. And they also have demands that they want a certain kind of goods which they may or may not possess. The goal of the game is to get whatever they demand. The only way they can do this is to trade with other agents. But two agents can only trade if they both have what the other wants. An example can help illustrate this point. In Fig. 1, a hypothetical demand network is shown. The arrows represent demands. For example,

$$B \rightarrow A$$

means agent B demands the possession of agent A. It is clear that agents N and P can swap their possession so that both of them can get what they want. This is called the 'double coincidence of wants', meaning that two agents want each other's possession. They can just swap their possession. This is an ideal case but it is very rare if the number of commodities is large. Now, look at the other agents in Fig. 1. None of them have a 'double coincidence of wants'. So they cannot trade and cannot get what they demand. As a result, their living standard cannot be improved. At this instance, if the agents are clever enough, they might want to change their demands in the sense that they may want to get what the others want, in addition to what he wants.[2] For example, agents C and G may want to have the possession of agent A, so that they can use those possession to trade with B. By this logic, more and more agents want the possession of agent A since it has a higher chance to be accepted by the other agents. This positive feedback mechanism keeps going on and finally, all the agents accept the possession of agent A. And all the arrows in the demand network point to A. That means this possession is promoted to be the media of exchange, which is called money. And we say that money is emerged in this system.[3]

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[1] We will see that production and consumption are necessary for money to survive after its emergence.

[2] Note that the emergence of money requires agents to have a view of what the other agents demand.

[3] Notice that this happens only when the agents adjust their demands, that means it is necessary for the agents to have a view of what the other agents demand.

But since there is no production nor consumption. Those agents who have already got their demands demand no more goods. And they no longer need money. They would do no more trading. The rest of the agents keep on trading, until they get what they want. Finally, they would not want money anymore and money would then die away. So production and consumption is necessary to keep money survive, by not letting the system to go into a static equilibrium.

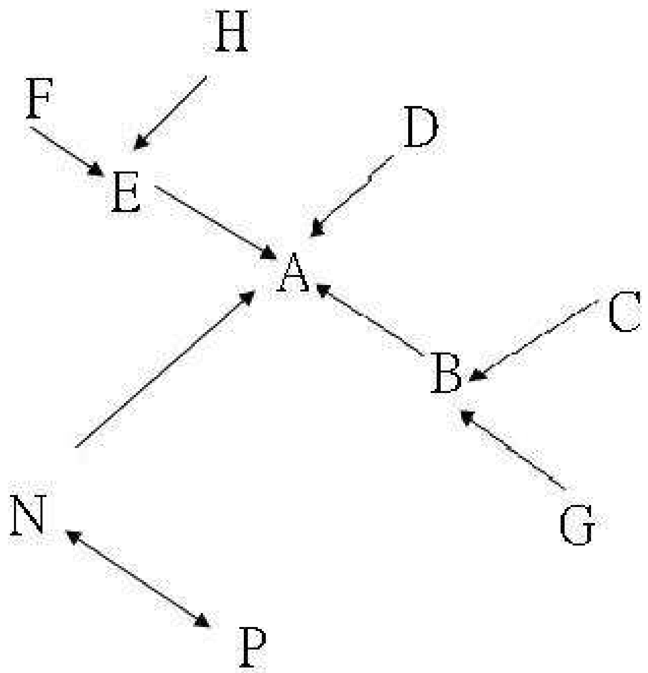

FIG. 1: A hypothetical demand network

B. The Model

In the last section, we see why we expect the emergence of money in a system of trading agents. The primary function of money is to serve as a media of exchange so that agents can get their demands much more easily, without requiring the 'double coincidence of wants'. We also see that the system must be dynamic, namely keep on producing and consuming goods, to ensure the survival of money. In this section, we are going to realize all these concepts by building up an ACE model, so as to justify our expectation.

This model is studied in great detail by Yasutomi[1][4]. The model is as follows. Assume there are N agents and N kinds of goods. Each agent is capable of producing one kind of good which he does not want. Additionally, we assume he wants only one kind of good at a time. In this

[4] The content of this subsection follows closely to Yasutomi[1], we still put this here to make this article self-contained.

situation, every agent is suffered from the difficulty of 'double coincidence of wants'. In order words, it is difficult for them to trade when there is no media of exchange. Let us assume that one unit of every good is equivalent in value to one unit of any other goods. An agent, say i, has four vectors, U_i, P_i, D_i and V_i. The utility vector, U_i is defined as:

$$U_i = (u_{i1}, u_{i1}, ..., u_{iN}) \tag{2.1}$$

with $u_{ij} = 1$ if agent i wants good j and $u_{ij} = 1$ otherwise. Since each agent only wants one kind of good at a time, we have $\sum_{j=1}^{N} u_{ij} = 1$ for all i.

$$P_i = (p_{i1}, p_{i1}, ..., p_{iN}) \tag{2.2}$$

with p_{ij} being non-negative integers is the possession vector. Agent i has p_{ij} units of good j. Initially, $p_{ij} = \delta_{ij}$.

$$D_i = (d_{i1}, d_{i1}, ..., d_{iN}) \tag{2.3}$$

with d_{ij} being non-negative integers is the demand vector. Agent i demands d_{ij} units of goods j. As we have discussed before, agents must have a sense of what the other agents want in order for the money to emerge. So we set up a world view vector, V_i, defined as

$$V_i = (v_{ij}, v_{ij}, ..., v_{ij}) \tag{2.4}$$

Agent i thinks that there are v_{ij} agents demands good j. Naturally, we have

$$0 \leq \sum_{j=1}^{N} v_{ij} \leq N \tag{2.5}$$

since there are only N agents.

The rules of the games are stated below.

Step 1. One agent, say agent k is randomly selected, who brings his possessions to a "market".

Step 2. Agent k nominates one agent, say agent l, who has the greatest number of good j which agent k wants. Agent l brings his possessions to the "market".[5]

Step 3. Agent k and l show their goods to each other.

Step 4. Agent k and l exchange their views. If agent k demanded good j but could not acquire good j in the latest trade in which he took part in, he increases v_{kj} by one unit. Agent l does the same. Then they exchange their view by a simple averaging, namely

$$v_{kj}, v_{lj} \rightarrow \frac{v_{kj} + v_{lj}}{2} \tag{2.6}$$

Then they normalize v_{ij}'s by

$$v_{ij} \rightarrow \frac{N}{\sum_{j=1}^{N} v_{ij}} \quad v_{ij} \tag{2.7}$$

to reinforce the constraint $0 \leq \sum_{j=1}^{N} v_{ij} \leq N$. This normalization reflects that new information is more important than the old ones.

[5] The labeling of the goods and the agents are matched in the sense that agent k is capable of producing good k.

Step 5. For each good j, if agent l has good j that either agent k wants it or he has a view that at least x agents demand it, agent k demands all the good j which agent l has. Otherwise agent k does not demand any. That is to say,

$$d_{kj} = \begin{cases} p_{lj} & \text{if} \quad p_{lj} > 0 \quad \text{and} \quad (u_{kj} = 1 \quad \text{or} \quad v_{kj} \geq x) \\ 0 & \text{otherwise} \end{cases} \tag{2.8}$$

We call x the threshold of exchange, where $0 < x < N + 1$. When x is large, the agent does not want to acquire what the other wants. When x is small, the agent does want to do so, in addition to what he himself wants.[6] We will see that x acts as a coupling constant in this system. The change of x will result in phase transition, that is the emergence of money.

Step 6. If the units of goods which agent k demands, say m_k, is equal to that of agent l, that is m_l, then they swap those goods. If $m_k > m_l$, then agent k must select m_l units from those goods which he demands. We assume that he takes one unit of good j whose d_{kj} is smallest and not 0, and decrease d_{kj} by one unit. He repeats this procedure m_l time. Then agent l takes all units of goods he demands. p_{ij} changes accordingly. If $m_l > m_k$, then they do the reverse.

Step 7. Now the agents start to consume the goods. For every good j, if agent k finds himself to have good j which he wants, he consumes them. That means,

$$p_{kj} \rightarrow \begin{cases} 0 & \text{if} \quad p_{kj} > 0 \quad \text{and} \quad u_{kj} = 1 \\ p_{kj} & \text{otherwise} \end{cases} \tag{2.9}$$

And if he does not have good k, then he produces one unit of good k.

When he succeeds to consume what he wants, his wants for that good will be extinguished, and he will come to want another good. So we assume that when agent k consumes some goods, U_k is reshuffled. One good distinct from good k, say $p \neq k$, is randomly selected, and then $u_{kj} = \delta_{jp}$.

Agent l does the same.

We regard such a process as one trade and N trades as a turn. This is our ACE model.

Clearly, the emergence of money depends upon the value of x, the threshold of exchange. If $x = N+1$, then no agents would acquire goods that are not demanded nor produced. And they would suffer from the difficulties of trading due to the rarity of 'double coincidence of wants'. This is the barter exchange. If x is very small, then it is easier for the agents to accept goods that they think others would accept. This helps the emergence of money to happen. After money is emerged, the system is said to be using monetary exchange.

III. SIMULATION RESULTS

Before we discuss the simulation results. We have to introduce some parameters to describe the system. The first one acts as an 'order parameter', which tells us if money is emerged in the system. We define **money supply** as the number of goods that are not consumed nor produced by the agents. Clearly, in the case of barter exchange ($x = N + 1$), no agents would acquire goods that are not demanded nor produced. So **money supply** would be very small. If monetary exchange takes place, then agents are willing to possess goods that are not demanded

[6] The value of x is set fixed in this model. There are models that let x to evolve, resulting in both the emergence and collapse of money, we will discuss it in later sections.

nor produced, but might be demanded by other agents. Then **money supply** would increase. It is in this sense that **money supply** serves as the order parameter.

The second quantity is called **cons**, which is the total number of goods that is consumed in one turn, as the name suggested. If agents can get what they want and consume them, then **cons** would be high. It would be low otherwise. In this sense, **cons** represents the living standard of the agents. From the discussion of last section, it is natural to think that the increase in **money supply** would increases **cons**, which is the same as saying the emergence of money smoothen the trading processes, which in turn improves the living standard. We can now test this statement.

First, a simulation of 50 agents ($N = 50$) and $x = N + 1$ is run. This simulates the barter exchange. The **cons** is shown in Fig. 2. From the graph, we can see that there is not much consumption by agents. That means agents cannot trade effectively in this model, which can only due to the rarity of 'double coincidence of wants'.

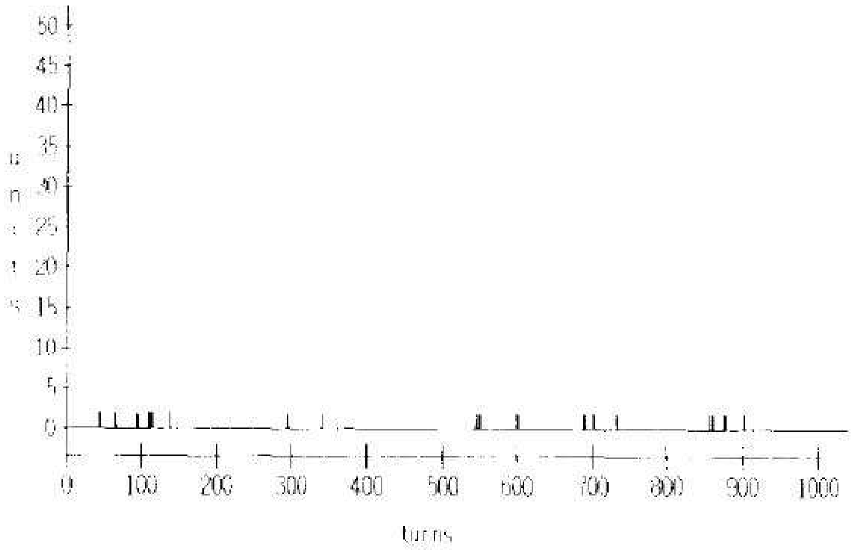

FIG. 2: The time variation of **cons** for a system with $N = 50$ and $x = N + 1$. (barter exchange) One see that there is almost no consumption in this system. The quality of the graphs are limited by the quality of the graphs in the original electronic copy.

Then, we change the value of x to 7. The results are shown in Fig. 3. From the graph, one can see that the **money supply** is very low initially, but it increases dramatically around the $500th$ turn. This is the emergence of money. At this point, agents are willing to accept what the other accepts. Money is said to be emerged and monetary exchange is being used. Fig. 4 shows the time variation of **cons** for the same system. It can be seen that **cons** remains a low level before the $500th$ turn. After the emergence of money at that point, **cons** is dramatically increased, which implies a strong correlation in **money supply** and **cons**. This confirms our previous assertion.

These two special cases give us a taste of how the threshold x affects the emergence of money. We now have two extremes. For x very large, there is no money. For x very small, there is emergence of money. So, it is natural to ask what the critical value of x is, just like we have in the theory of phase transition. Fig. 5 shows the time for money to emerge for different values

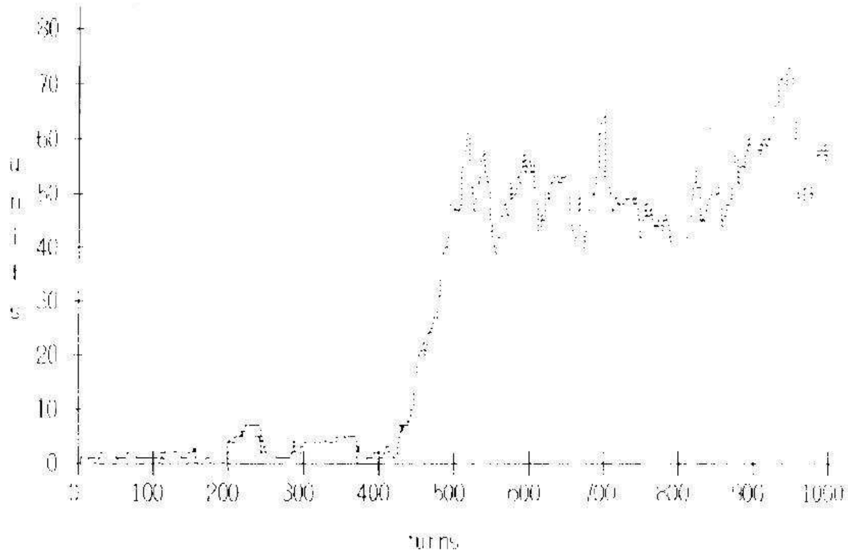

FIG. 3: The time variation of **money supply** for a system with $N = 50$ and $x = 7$. (emergence of money)

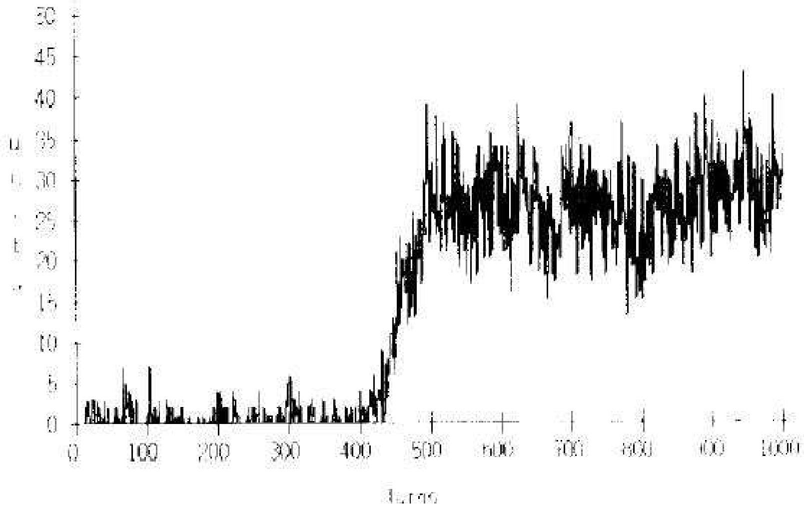

FIG. 4: The time variation of **cons** for a system with $N = 50$ and $x = 7$. (emergence of money)

of x. [7] From the graph, we see that the time needed for money to emerge increases with x, and eventually goes to infinity for $x > 13$ (not shown in the graph), that means there is no emergence of money for $x > 13$. Another observation is that money is not emerged for $x = 0$.

[7] Yasutormi[1] used a parameter called **opt** to define the emergence of money. It is similar to the **money supply** we introduced earlier.

This is easy to understand since when $x = 0$, agents would demands all the goods that the other agent possesses, so trading activities are maximized, and there is no need to have money in the system.

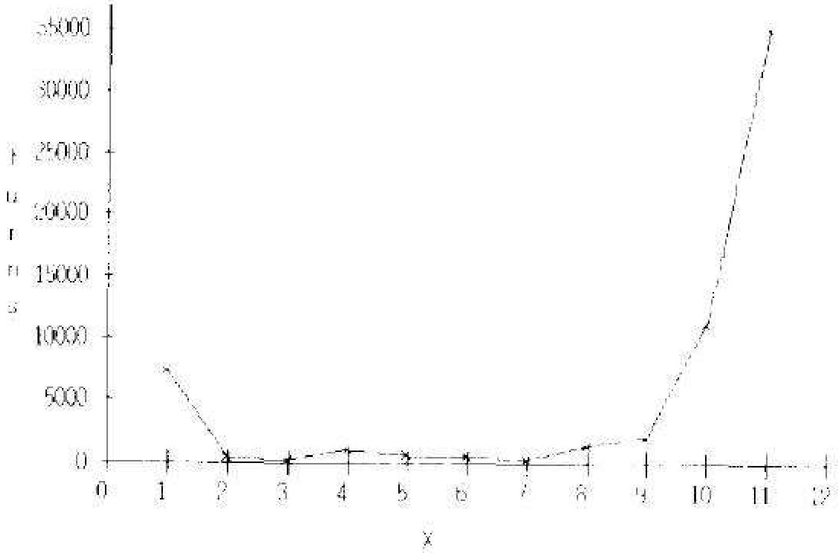

FIG. 5: The time needed for money to be emerged.

IV. DISCUSSION

The whole phenomenon of emergence of money resembles a phase transition. Initially all the commodities are symmetric in the sense that none of them is special nor preferred. As time goes on, the world view vectors, V_i, fluctuate. For $x > 13$, the symmetric phase is stable upon these fluctuations, and no money is emerged. But for $0 < x < 13$, there is a commodity singled out by fluctuation. The system is unstable upon fluctuation due to the positive feedback mechanism discussed in Sec. II. Eventually every agent accepts that commodity as a media of exchange and money is emerged. The symmetry among the commodities is spontaneously broken. This scenario is the same as the well-known ferromagnetic transition, where the threshold, x, acts as a discrete version of 'coupling constant' in the system. On the other hand, since the commodities are initially perfectly symmetrical. No one knows in advance which commodity will break the symmetry and be promoted to money. This is also similar to the ferromagnetic transition.

In what follows, we are going to discuss some other models people studied and compare them with this model. We would also like to address some of the drawbacks of ACE simulations.

ACE models are very specific models. Different models have different rules, even though they may model the same phenomenon. The one we discussed here is a relatively simple model. And we see the emergence of money in it. But how can we tell the conclusion is still valid in other similar ACE models, or even in real world? In the end, we are not trading with each other using such a simplified strategy. Before further elaborate on this question, let us study more ACE models that try to model the emergence of money.

Howitt[2] performed a ACE modeling to study the emergence of shops in the economy. In his model, shops are special agents that the other agents trade with. The result of their simulation supports the emergence of money in the economy. In addition, they found that shops are emerged from the system. They concluded by saying that one can 'grow' economical organizations from some simple rules, without a priori assigning such a structure to the system. This is the whole point of emergent properties.

Marimon[3] used a more detailed and complicated model to do the analysis. He introduced a classifier system. Classifier systems are systems that select a trading rule from a pool of such rules that is being used by the agents. As time goes by, the system would evolve and credit rules that outperformed the others. Agents then prefer rules with higher credits. So this is an evolutary system. The trading strategies are evolving with time. In this complicated dynamical system. Marimon[3] still found that money is emerged in the system.

Now we have a very interesting and important question. Is emergence of money generic in interacting-agent models? We see examples of models with very different trading rules, from the simple one we presented in this paper to the very complicated classifier system used by Marimon[3]. But no matter what the difference between the details of these models are, money, defined as a media of exchange, is somehow emerged.

This problem is a familiar one to physicists. We now have many microscopic (ACE) models give rise to the same phenomena (money). This is like what we have in fluid dynamics. There are many different kinds of fluids, e.g. water, oil, air, or even coffee. They have very different chemical compositions, different excitations and different inter-molecular interactions. But if we are only interested in the macroscopic, long-wavelength properties of the fluid, instead of building model for each of the fluid, one can coarse grain the system, ignore unwanted microscopic details, and end up with a unified theory, say, the Naiver-Stokes equation. Having this theory, we now only need to plug in a few parameters, such as viscosity, thermal diffusivity and so on, to model a particular fluid, without having to build up models for individual fluids. Now, in our case of emergence of money, what we have to do is to seek for a macroscopic, hydrodynamic description of the system. In this sense, we do not have to model details of the interaction between agents that is inherently complicated, if not impossible, to model. And we do not have to model each economy with tailer-made model. This would give us a unified theory of the emergence of money in interacting-agent systems. But how we can coarse-grain the system, and what parameters are critical to these systems, are yet open questions.

This line of thoughts can be used in many different situations in economics and finance to bulid up more robust and generic models, where very specific ACE models are widely used.

One last remark about this system is the collapse of money. In Yasutomi's paper[1], he modified their ACE models to describe an evolutary system. He introduced the concept of score. It works as follows. Every agent is assigned a score after each trading, with more successful agents having higher scores. From time to time, the few agents with lowest score change their thresholds randomly. As a result, the threshold may fluctuate to a very high value at which these agents do not accept money as a media of exchange. Agents trading with these agents may then be influenced and think that money is no longer accepted in the market. This thought propagates throughout the system and more and more agents think that money is not accepted. At this point, no agent would accept money and the money is collapsed. After the collapse of this money, the system is like in its initial state, then another money may accidentally be emerged due to fluctuation. This phenomena repeats itself. This is Yasutomi's theory on emergence and collapse of money.

We are conservative about this theory. There are two main reasons. First, as we have discussed in detail, ACE models are very specific models. What happens in a model does not necessarily happen in the other models. In the case of collapse of money, Yasutomi's model[1] is the only model that predict the collapse of money. We cannot find any other models predicting this. So, further studies are needed before we make any conclusion. Second, if we think of this system as a system with spontaneous symmetry breaking. The symmetry broken state should has a stiffness toward changing the direction of the order parameter. If the symmetry is continuous, we may have Goldstone modes. But if the symmetry is discrete, then there is ergodicity breaking. No one would expect an Ising magnet in the ferromagnetic phase, say with most spins pointing up, suddenly collapse due to thermal fluctuation and then restablise by having most spins pointing down. This is due to the stiffness of the spins as a whole. Flipping all the spins at the same time is very expansive in energy. The same concept applies to the case of money collapse. Once a money is emerged. We would expect that there is a large resistance towards changing the money. This is even more true if the number of agents is very large. This makes the collapse of money questionable.

V. CONCLUSION

We discussed a specific ACE model describing the emergence of money. The system resembles the theory of phase transition. Money is demostrated as an emergent property of the system. Different models give this same conclusion, which make us more confident in the validity of it. On contrast, the collapse of money lacks support from different models, and it is counter-intuitive to our knowledge of spontaneous symmetry breaking and emergent sniffness. The weakness of ACE models is discussed. The possible direction of further research is to seek for a macroscopic, hydrodynamic description of the system. The same line of thoughts may be used in the study of emergence of economical organizations, such as shops, banks and exchanges, to bulid up more robust and generic models.

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[2] Peter Howitt, Journal of Economic Behavior and Organization **41**, 55 (2000).
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