Magnetic Monopoles from 1931 to 2002

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Abstract

The physics of magnetic monopoles is described as introduced by Dirac and later 't Hooft and Polyakov. Experimental searches for monopoles are described. Monopoles are introduces as possible "confiners" in the dual superconductor model and the phenomena of abelian dominance and monopole dominance are explained. Confinement in examined in $U(1)$ Lattice Gauge Theory (LGT). The techniques are generalized to $SU(2)$ LGT. Complications arising in extension to $SU(3)$ LGT are discussed along with a prospects for future research.

1. Introduction

In 1931, P.A.M. Dirac proposed that particles with magnetic charge could exist and that the existence of these charges would in turn imply quantization of electric charge. The problem of how to handle the mathematics of the resulting Dirac string was left largely unresolved. Generalization to non-abelian gauge theories (NAGT's) was difficult [1,2].

Interest resurfaced in 1974 when 't Hooft and Polyakov independently realized that the introduction of a Higgs field to NAGT could produce monopoles when the theory was broken to $U(1)$. These monopoles did not need to have Dirac strings. Predictions of monopole mass could be made by examining how the NAGT broke into $U(1)$ [2,3].

Searches for these "classical" and 't Hooft monopoles were carried out using several methods. Initial searches for classical monopoles relied on light monopole masses to result in large velocities and hence large energy losses in traditional detectors. Later searches used techniques such as superconducting induction to locate the heavier monopoles that would result from the breaking of Grand Unified Theories (GUT's) [4,5].

Current interest in monopoles involves "colored" monopoles. These monopoles are agents of confinement in the dual superconductor model, in a situation similar to the way the magnetic monopoles would be confined in a type-II superconductor [6]. In $SU(N)$ NAGT, the principle of abelian dominance states that the long-range degrees of freedom can be squeezed into an abelian sector of the gauge field. The resulting colored monopoles may then provide mechanism for confinement in NAGT [7,8].

Testing of dual superconductivity has been done extensively in $U(1)$ LGT. Monopoles are located by finding their Dirac strings on the lattice. An evaluation of the field strength tensor from magnetic current gives Wilson loop values and thus an indication of the contribution of the monopoles to the string tension [7,9].

For an $SU(2)$ field on the lattice, gauge fixing is generally necessary before projecting onto the $U(1)$ part of the field. The maximum abelian gauge (MAG) has been the gauge of choice for much of the research done in this direction. Once there is only a $U(1)$ gauge field left, monopoles can be defined as before and the idea of monopole dominance may be tested [7].

It is desirable to extend the work done on $SU(2)$ into $SU(3)$. Complications arise with the gauge conditions and the projection process. Results are not as promising as in the case of $SU(2)$ LGT. It is possible the addition a Higgs field during the gauge-fixing process will correct this [10].

2. The Dirac Monopole

Maxwell's equations suggest a dual symmetry between the electric field $\vec{E}$ and the magnetic field $\vec{B}$. The duality is not complete, however, due to the absence of magnetic charge. If we allow magnetic charge we now get two new equations

$$\vec{\nabla} \cdot \vec{B} = \rho_m, \ \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = j_m \tag{1}$$

where ρ_m and j_m are the magnetic charge and current densities respectively.

The Dirac monopole with magnetic charge q_m can be described by magnetic and gauge fields

$$\vec{B} = \frac{q_m}{r}\hat{r}, \ \ \vec{A} = \frac{q_m}{r}\frac{1-\cos\theta}{\sin\theta}\hat{\phi}. \tag{2}$$

One thing to note is that the vector potential is not defined for $\theta = \pi$. This is the Dirac string, carrying magnetic flux into the monopole. It is non-physical with a location that depends on the choice of gauge. In order to get consistent results for the phase change of an electrically charged particle moving on a path, the charge of the monopole must obey $eq_m = n/2$. This condition is known as Dirac quantization. An additional condition, the Dirac veto, states that particles interacting with the monopole must have vanishing wavefunction on the string.

As it stands, there are the usual problems with infinite self-energy of the monopole and string. We would much rather not deal with strings and the Dirac veto makes calculations problematic. While the string will exist for any $U(1)$ gauge theory, it is possible to avoid the Dirac veto condition and even (with some difficulty) develop a quantum field theory of electric and magnetic charge [1].

3. The ‘t Hooft-Polyakov Monopole

The next question we would like to answer is how does this situation occur if our field theory has a non-abelian gauge group. The answer is that it is now possible to create solutions without those pesky Dirac strings. In fact, if the Electromagnetic (EM) $U(1)$ gauge symmetry is a result of the breaking of some compact symmetry, monopoles are required to exist [2].

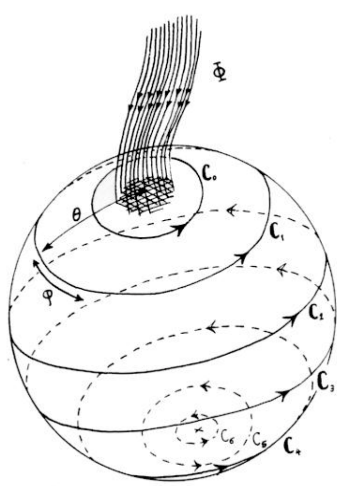


Fig. 1 Deforming the Contour C over a sphere surrounding a monopole [3].

We may visualize a Dirac string with flux F entering a sphere surrounding a monopole (Fig. 1). The potential $\vec{A}$ on the contour C obeys $\oint \vec{A} \cdot d\vec{x} = \Phi$. But we can lower the contour over the sphere (C_0-C_6) until reaching the bottom and at the same time continuously deform certain Φ's (multiples of 4π for $SO(3)$) to zero.

The mass of such objects can be computed and end up 137 times the mass of a vector boson. For this to work, there must be a compact covering group for the model, eliminating $SU(2)\times U(1)$ electroweak symmetry breaking as a mechanism.[3]

4. Monopole Searches

Since 1931 there have been many experiments looking for monopoles. Early on, experimenters were looking for "classical monopoles." While Dirac's formulation did not set the monopole mass, assuming properties of the electron (other than charge) gave these monopoles an energy of about 2.4 GeV. Later, when it was realized that monopoles could be the result of the breaking of some GUT, the mass estimates shot up to anywhere from 10^{10} GeV to 10^{19} GeV. While looking for classical monopoles could typically be done at a particle accelerator, current monopole energies could only come from Cosmic rays.

Classical monopoles (whether cosmic or mad-made) would be characterized by relativistic velocities and thus have very large energy losses in matter. These particles would be especially easy to spot with scintillation counters or gaseous detectors. Monopoles from a GUT would have somewhat lower velocities, perhaps with energy losses too low for ionization. A better way to detect such monopoles would be with a superconducting induction device, in which a monopole passing through a coil created an EMF in that coil (Fig. 2,3).

Of course, no magnetic monopoles have been found as of yet (to put it optimistically). Measurements that experimenters have been able to make have included lower limits on monopole mass (actually, upper limits on cross sections for low mass monopoles) and upper limits on the possible flux of monopoles from space. The latter puts constraints on the formation of the universe. It favors inflationary models, in which the universe would grow so fast that monopoles would not be given a chance to form when the symmetry of the unified gauge field was broken [4]. More recently it has been

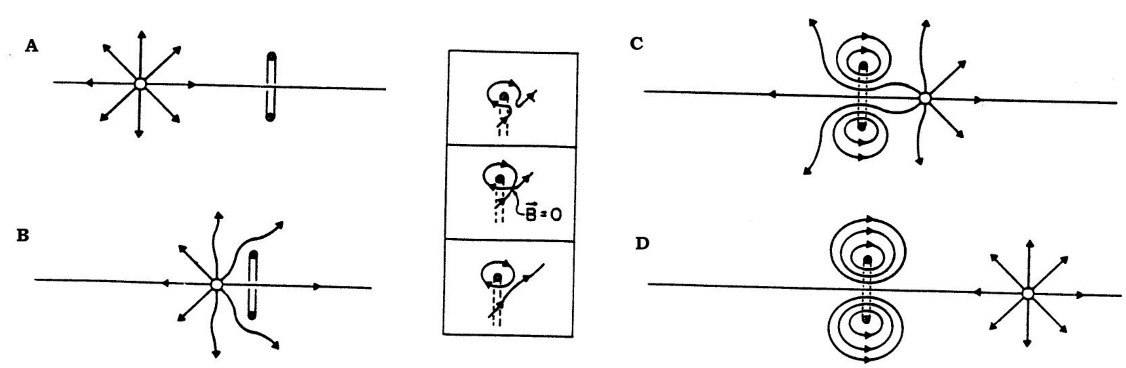
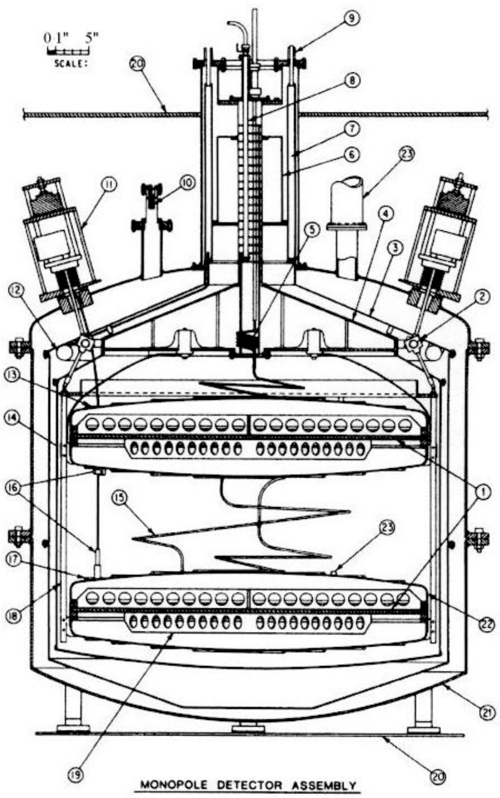


Fig. 2 A monopole passing through a superconducting coil "leaves behind" field lines [4]



The detector assembly: (1) gradiometer plane, (2) support ring, (3) 78 K radiation shield, (4) helium reservoir, (5) heat exchanger, (6) heat pick-up shroud, (7) nitrogen reservoir, (8) helium fill line, (9) nitrogen vent, (10) vacuum gauge, (11) vibration isolator, (12) nitrogen reservoir, (13) detector cooling coils, (14) 4 K radiation and magnetic field shield, (15) helium vent, (16) SQUID, (17) detector cooling coils, (18) copper support rods, (19) *G*-10 cross braces, (20) steel pipe, (21) vacuum vessel, (22) lead lined copper shield, (23) pseudopole feedthrough block, (24) vacuum port.

Fig. 3 The superconducting induction monopole detector used by Incandela and Frisch circa 1984 [5].

proposed that upper limits on monopole mass may be inferred from the effect virtual monopoles would have on the anomalous magnetic moment of the electron and Z-boson decay [11].

There is, however, another side to monopoles. They are candidates for agents of confinement in QCD and other NAGT's. Here the monopoles would not exist as topological configurations stabilized by conservation of charge. Instead, they would represent unstable classical field configurations. The testing of related hypothesis must now take other methods. Specifically, we place the gauge fields on a lattice.

5. Dual Superconductivity and Non-Abelian Gauge Theories

If we did have magnetic charges, they would interact with each other as electric charges with Coulomb potential. However, if these charges were placed in a type-II superconductor, the flux could be confined into vortices by circulating electric currents. With the field lines unable to be spread out, the potential between two monopoles would now go as R instead of $1/R$. We can imagine the flux between the two charges to form a confining string (not to be confused with the previously introduced Dirac string). If $V(R)=\sigma R$ as $R\to\infty$, we refer to the quantity σ as the string tension.

Dual superconductivity refers to a situation in which the electric and magnetic fields are interchanged from the previous case: $\vec{B}\to\vec{E},\ \vec{E}\to-\vec{B}.$ We now have a linear potential between quarks caused by magnetic currents. The next step is to extend this model to the case of NAGT. Here we utilize the principle of abelian dominance. This principle states that the long-range degrees of freedom of the field $SU(N)$ are all contained in the $U(1)^{N-1}$ abelian sector of the field. Together the two yield monopole dominance: that the string tension can be accounted for by just the monopole degrees of freedom. We test this hypothesis by moving our fields to the lattice. For a recent treatment of the abelian dominance in the continuum see Kondo [12].

6 Working on the Lattice: U(1)

At this point, the easiest way to perform tests is to move to the lattice. The gauge field now takes values on links between lattice sites. In a full simulation, other fields would take their values on the lattice sites themselves. Monopoles now consist of 3-D cubes with magnetic currents being represented as links on the dual lattice. Dirac strings are a series of plaquettes (1x1 squares). At the end of the day, we can look at what happens to our measured quantities as we take the lattice spacing back to zero.

There are many reasons to move to the lattice. The finite volume of the lattice takes care of any infrared divergences. Since confinement is regarded to be an infrared phenomenon this makes sense. Even more important is that we now have the typical lattice momentum cutoff. With the field having values at a finite number of locations, calculations may now be computed using path integral Monte-Carlo. The Wick rotation ($t \rightarrow i\tau$) that we use to solve path-integrals goes along with the lattice being Euclidean. Finally, if we're looking at a compact U(1) model, the energy of the Dirac strings on the lattice will be zero (instead of diverging). Chucking Lorentz invariance is a small price to pay.

Gauge transformations are generated with links assigned values of $U_\mu = \exp(iagA_\mu) = \exp(i\theta_\mu)$ in order to maintain gauge invariance on the lattice. For lattice spacing *a* and coupling *g*, gauge configurations can be generated with cosine action

$$S_E^{U(1)} = \frac{1}{2g^2}\sum_{x,\mu\nu} 1 - \cos(a^2 g F_{\mu\nu}(x)) = \frac{1}{2g^2}\sum_{x,\mu\nu} 1 - \cos(\Theta_{\mu\nu}(x)) \,. \qquad (3)$$

The quantity $\Theta_{\mu\nu}(x)$ is known as the plaquette angle. It can be found by adding up the angles on the sides of the $\mu\nu$ plaquette $\Theta_{\mu\nu} = \theta_\nu(x) + \theta_\mu(x+\hat{\nu}) - \theta_\nu(x+\hat{\mu}) - \theta_\mu(x)$. Measured quantities will be found by averaging over some large number of configurations.

The string tension associated with this setup can be determined by measuring Wilson loop values on the configuration. The Wilson loop $W(R,T)$ may be thought of as the action cost of creating a quark-antiquark pair, separating them at distance R lattice spacings, time evolving the pair for T lattice spacings, and the bringing them back together. It is a gauge invariant quantity and for $T \gg R$ goes as $\exp(-V(R)T)$. From Wilson loop measurements at various (R,T), we may determine V(R) and determine the string tension σ along with Coulomb term α is and perimeter term V_0 using the ansatz $V(R) = V_0 + \alpha R^{-1} + \sigma R$.

We may now look for the magnetic currents themselves. This is accomplished by looking for the accompanying Dirac strings. A plaquette is considered to be pierced by a string (two strings) if it has $\Theta_{\mu\nu}(x)$ outside the range of $[\pi,-\pi]$ $([3\pi,-3\pi])$. Sign matters. A cube that has a different number of strings entering and leaving contains magnetic current. While the locations of the Dirac strings depend on choice of gauge, the locations of the monopoles do not [9].

It is now possible to take the magnetic currents on the dual lattice and from them obtain the field-strength tensor. From the field strength tensor, Wilson loops may be calculated as before. For $U(1)$ LGT, monopoles recreate the string tension of the entire configuration to within error [13].

7 Non-Abelian Lattice Gauge Theory: SU(2)

We now move on to the case of an $SU(2)$ gauge field. Here, the link values are stored by parameterizing the $SU(2)$ group

$$U_\mu(x) = \exp(igA_\mu^i(x)\frac{\sigma_i}{2}a) \tag{4}$$

where σ_i are the 2x2 Pauli spin matrices. Gauge fields configurations are generated using the Wilson action which is for $SU(N)$ gauge theory

$$S_E^{SU(2)} = \beta\left[\sum_{x,\mu\nu} 1 - \frac{1}{N}\mathrm{Re}(\mathrm{Tr}[U_{\mu\nu}(x)])\right],\ \beta = \frac{2N}{g^2}. \tag{5}$$

The $SU(2)$ plaquettes variable $U_{\mu\nu}(x)$ is given as

$$U_{\mu\nu} = U_\nu(x)U_\mu(x+\hat{\nu})U_\nu^\dagger(x+\hat{\mu})U_\mu^\dagger(x). \tag{6}$$

Monte Carlo updates are done using standard heatbath techniques and Wilson loops are found in the usual way.

We then wish to project our $SU(2)$ group element valued links field onto a $U(1)$ subgroup. The projection process is straight forward, mapping

$$U_\mu(x) = \exp(igA_\mu^a(x)\frac{\sigma_a}{2}a) = a_\mu^0(x)I + i\vec{a}_\mu(x)\cdot\vec{\sigma} \rightarrow \theta(x) = \tan^{-1}\left(\frac{a_3}{a_0}\right). \tag{7}$$

However, projecting immediately without any gauge-fixing yields too much noise for effective string tension measurements. An additional step is needed.

Gauge fixing is usually not needed in LGT as measured quantities are for the most part gauge-invariant. In this particular case, however, gauge fixing can dramatically cut noise such that *U*(1) string tension measurements go from nigh-impossible to far easier than the *SU*(2) case. The MAG relies on successive updates of links to maximize the lattice functional

$$G_{lat}^{SU(2)} = \sum_{x,\mu} \mathrm{Tr}[U_\mu(x)\sigma_3 U_\mu^\dagger(x)\sigma_3]. \tag{8}$$

This is akin to performing the gauge-fixing in the continuum so as to minimize the functional

$$G_{cont}^{SU(2)} = \int d^4x \left(\sum_\mu (A_\mu^{(1)})^2 + (A_\mu^{(2)})^2 \right). \tag{9}$$

The projected *U*(1) string tension has been found to be within error of the *SU*(2) value with monopoles coming in about 10% low. While extremely promising, this result comes with some caveats.

An alternative system of vortices carrying flux in the center of the group also does a good job of explaining confinement. Exactly how these abelian monopoles and center vortices tie in together is not fully understood. With many gauge-fixing scheme gribov copies arise. These are local maxima of the relevant functional. Taking into account such ambiguities complicates matters considerably. In addition results for monopole string tensions are not stable under small local cooling of the lattice. How this can be the case and monopoles still responsible for the long-range physics has yet to be satisfactorily resolved [7].

8 Generalization to include QCD: *SU*(3)

We would like to extend our results to the case of *SU*(3). The first change is that our link variables are now *SU*(3) elements. They can be represented as

$$U_\mu(x) = \exp(igA_\mu^a(x)\frac{\lambda_a}{2}a) \tag{10}$$

where λ_a are the eight Gell-Mann matrices. The action used to generate gauge field configurations simply given by Eqn. 5 with *N*=3. Updates are done with by individually updating *SU*(2) subgroups.

Abelian projection in from *SU*(3) to *U*(1)x*U*(1) raises some questions. First of all, there are two different kinds of monopoles. Also there are a variety of ways to carry out the actually projection whereas there was really only one for the projection of *SU*(2) onto *U*(1). Fortunately these different methods yield the same values for observed quantities.

There are also different forms of the MAG in *SU*(3). The most common form used is currently

$$G_{latt}^{SU(3)} = \sum_{x,\mu} \mathrm{Tr}[U_\mu(x)\lambda_3 U_\mu^\dagger(x)\lambda_3] + \mathrm{Tr}[U_\mu(x)\lambda_8 U_\mu^\dagger(x)\lambda_8]. \tag{11}$$

Results for this functional give a projected $U(1)\text{x}U(1)$ string tension that is low by 10% with the monopole string tension another 10% lower. Taking into account gribov effects brings the projected and monopole string tensions even lower. Changing lattice spacings in order to work toward the continuum results in an increase in this trend.

The problems with $SU(3)$ string tension may yet me resolved by introducing a new lattice functional for the MAG. This functional would involve a Higgs field which would be rotated to take values of the form

$$\phi(x) = \frac{1}{\sqrt{2}}(\lambda_3 \cos\theta(x) + \lambda_8 \sin\theta(x)) . \tag{12}$$

The total functional would be

$$G_{latt}^{new} = \sum_{x,\mu} \mathrm{Tr}[U_\mu(x)\phi(x+\hat{\mu})U(x)_\mu^\dagger(x)\phi(x)] . \tag{13}$$

It is hoped that this new form of the MAG will correct problem discussed above [9].

9 Conclusions

Research on monopoles has shown remarkable progress since 1931. This is especially impressive since no direct evidence of the existence of monopoles has been found. Even without such evidence, the realization that a compact gauge field in turn implies monopoles is important and legitimate physics. As quantities such as the anomalous magnetic moment of the electron get nailed down more concretely, new evidence for the existence of monopoles could be just on the horizon.

The origin of confinement in QCD remains an open question. Colored monopoles are promising candidates as confiners. There is still much to work out with $SU(2)$ and $SU(3)$ gauge theories, both on and off the lattice. This is good, however, because it gives me something to do.

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