Critical Dynamics of The Superconductor Transition

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This paper aims to be a brief survey to recent development on critical dynamics on high temperature superconductor made by cuprate oxide. Instead of looking at the normal-superconducting phase transition, focus is on the antiferromagnetic-superconducting phase. The AF-SC phase leads to the formulation of the SO(5) theory, which seems to be quite popular amount many research institute overseas.

1 Introduction

For nearly twenty after cuprate superconductors have been discovered, many interesting, but yet out of reach puzzles remains. The behavior of classical low T_c superconductors are well studied by the famous BCS theory by Bardeen, Cooper and Schrieffer, as well as the Landau and Ginzburg theory near the transition point. Unfortunately, the BCS theory cannot account for the existence of high T_c superconductor.

The BCS theory predicts that, there is a theoretical maximum to the critical temperature of transition from normal material to superconductor. It is about 30 40 K. Above this thermal energy would cause electron-phonon interactions, results high energy states that will destroy Cooper pairs.

In 1986, a Ba-La-Cu-O material was found to be in superconducting state at 35 K. Soon, materials were found that would transform to superconducting state at 77K – the melting point of liquid nitrogen. And in 1994, the record for T_c was 164 K for cuprate with structure $HgBa_2Ca_2Cu_3O_{8+x}$, under 30 GPa of pressure. These discoveries apparently contradict the prediction from the BCS theory. Therefore, there should be some mechanism prevails at high temperature that BSC theory was not accounted for.

One of the mechanism believed to be the "holes" varies with the doping (x). These holes are formed by the partially ionized Cu atoms (Cu2+, Cu3+ ions). They correspond to pinning vortices in the superconducting material, just like the impurities in a typical type II superconductor. As the concentration of this kind of holes varies, the critical temperature varies accordingly. However, too little or too much of the "holes" presents in the material.

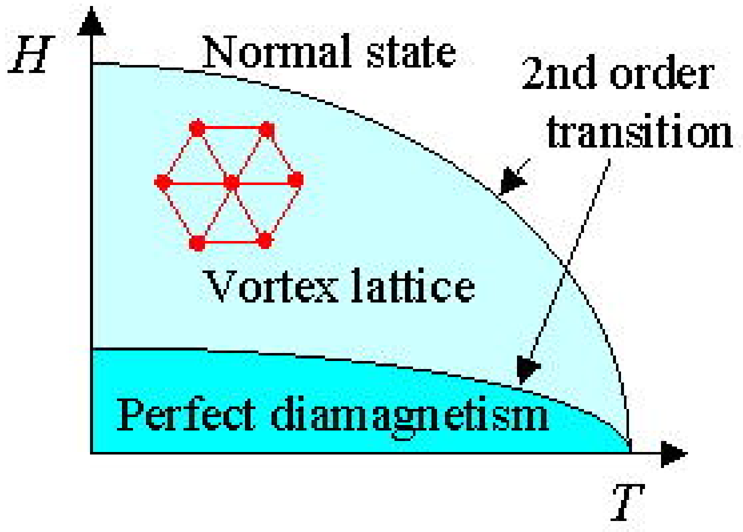

Figure 1: Phase diagram for type II superconductor

From the experimentally result of the high T_c superconductor phase transition, it shows a competition between magnetism and superconductivity. The system is struggling between the antiferromagnetic state or the superconducting state.

In this paper, we will look at the difficulties for formulating a quantitative theory for high T_c superconductor, as well as the phenomenological theory formulated by scientist in Stanford University known as the SO(5) theory. The theory itself is a Landau-Ginzburg like theory with five order parameter, three describes the antiferromagnetic phase degree of freedom and the other two describe the superconducting phase (i.e. X-Y model). At the end of the paper, we shall look at some interesting result of the phase diagram when melting transition presents.

2 Impurities, Holes and Cuprate Oxide

In a typical type II superconductor, it has three phases (see figure 1). The normal state, where the material is not a superconductor. The Meissner phase, which the magnetic fields are totally expelled from the superconductor. Also, the vortex state (Abrikosov) state, which the magnetic field can stay inside the superconductor in the form of vortex, or fluxion as what it is call in text book.

For the vortex state to exist, there has to be impurities inside the material to pin down the vortices. Otherwise, a slight thermal fluctuation will drive the vortices into motion, and the moving vortices will destroy the superconducting state.

In high T_c superconductor, the thermal fluctuation slightly below the transition point is very large. It is believe that a similar mechanism pins down the vortex, so that the vortex superconducting state can be form even in high temperature. However, instead of impurities in the solid, it is believed to be the Copper ions (Cu2+ and Cu3+) form holes to pin down the vortices.

Except for pinning vortex, there is yet another explanation for these ions to be a catalyst for superconductivity. High temperature superconductor are usually compounds such as $YBa_2Cu_3O_{7-x}$ or $La_{2-x}Sr_xCuO_4$. Often, the copper atom will be oxidized to from ions. However, it is impossible to completely ionized (oxidized) the entire piece of compound. Therefore, there are some fraction of Cu2+ and Cu3+ ions (depending on x and temperature) flowing around in the material. These Cu2+ and Cu3+ ions are essentially holes (positive charge carriers) within the lattice.

Cu3+ will tend to get an electron from somewhere to reduce its energy. They cannot gain an electron from other holes like Cu2+, since this doesn't really change the situation. The holes are usually stabilized by the oxygen (in the form of O2-, etc). However, if there is an electron in neighborhood lattice plane passes through a hole, due to coulomb repulsion it pushes the orbiting electron in the ion to one side and distort the lattice. This effectively causes Cu3+ to get an electron from the neighboring Cu2+. Then the hole is moving in the lattice due to the motion of electron. The current generated by hole moving backward is equivalent to the supercurrent.

In fact, amount of holes present in the system depends on doping of the material. As what we would expect from impurities from a typical type II superconductor, too many or too few holes will destroys the superconducting state.

This is not the end of the story yet. On one hand, the copper atom seems to be responsible for the formation of high temperature conductor, but on the other hand, cooper-cooper interaction leads to antiferromagnetism. Therefore, the presents of cuprates can put the system in either antiferromagnetic state and superconducting state.

3 AF-SC Phase and Neutron Diffraction Experiment

Except the combine effect of strong thermal fluctuation of vortex and random potential due to defects, the dual role played by the copper atom is another challenge posed by the high T_c

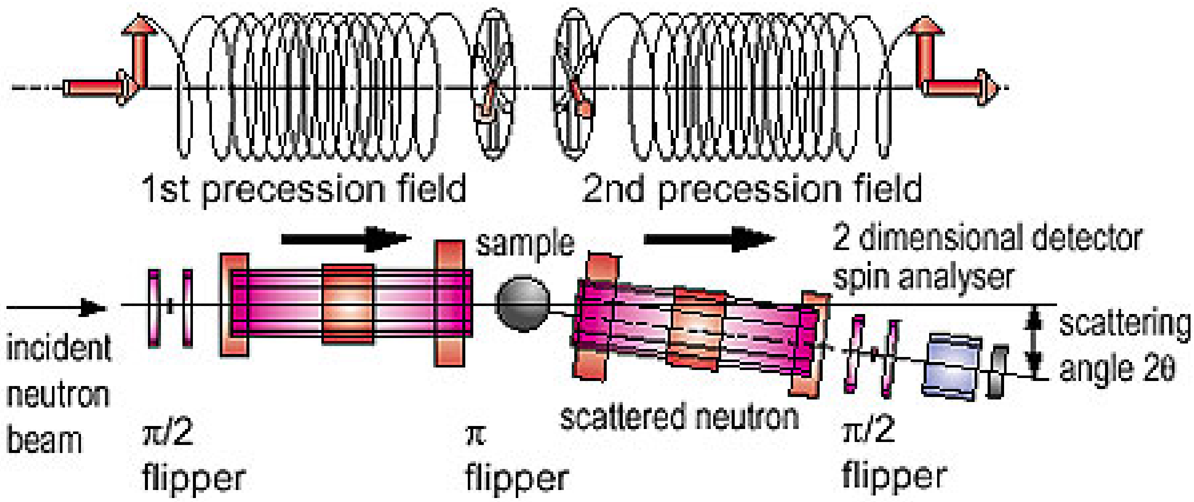

Figure 2: Set up for a typical neutron scattering experiment

superconductor.

More often, we think of magnetism is a mutually exclusive process of superconductivity. Simply enough, if we keep on increasing the external field of the system, the superconductivity will be reduced and then destroyed. It is in fact surprising to find that copper is responsible for both processes.

Evidence of the dual role of copper was found experimentally by using neutron scattering (see figure 2 for experimental set up). A sample of cuprate with the doping of our interest is put in the neutron scattering spectrometer. Neutron small-angle scattering detects the momentum transfer of the scattered neutrons. If we consider neutron beam as matter wave, the detector at the end of the spectrometer will detect scattering peak at the Bragg angles. The detector at the end of the spectrometer will measure the spin spin correlation function in fourier space (reciprocal, momentum space). To get the data in position space, we need to inverse fourier transform the raw data. This experiment allows us to get a snap shot of the structure of the sample of cuprate with varies level of doping.

The magnetic properties of solid is a consequence of the spin orientation of the atoms in it. Using neutron scattering to explore electronic structure has many advantage, neutron is not affected by electric charge of the scatterer, but it interacts with magnetic moments of electrons. So neutron scattering can gather information of the magnetic structure of the system.

It turns out neutron scattering (or diffraction) is a very powerful technique and used not only in exploring crystal structure, but also the energy spectrum. For example, the excitation spectrum (or the dispersion relation) of He4 [3] can be measured using this method. The draw back of neutron diffraction is that the instrument is expensive and rather difficult to build.

Using neutron scattering, Mook [5] et el. is able to detect for compound like lanthanum cuprate, the copper-copper interaction leads to antiferromagnetism. However, when this material is doped with 2 percents of holes per copper atom – we can do this by simply putting oxygen, or things like divalent strontium into interstitial site of the crystal – then the antiferromagnetism is destroyed. However, as the concentration of holes increases, below T_c, the material becomes superconductor. The dopant atoms are not replacing magnetic ones, so the copper is responsible for both superconductivity and magnetism.

For $La_{2-x}Sr_xCuO_4$, data from the Brookhaven National Laboratory [6] shows that the antiferromagnetic order disappear with the doping is at about x = 0.02, and the superconductivity appears when the doping is about x = 0.055. Magnetic correlation was found in the superconducting state but it is said to be incommensurate The fluctuation of magnetic moments on copper is trying to achieve the lowest energy configuration for the given lattice crystal structure.

4 SO(5) Theory

Knowing that we have two competing phase – antiferromagnetism and superconductivity, it's time to ask if there exists a theory describes the critical behavior between the transition from one to another.

It is so tempting to try to write down the Hamiltonian of the real system, then reduce it to some simpler form by the mean of perturbation or other approximation and solve for its energy spectrum, critical temperature and other related quantities. However, this strategy does not go so well. In the limit of strong thermal fluctuation and random potential due to defects, it is very hard to begin with a "real" Hamiltonian.

The research group in Stanford university leads by S. C. Zhang developed the SO(5) theory attempts to describe the antiferromagnetic (AF) phase and the (d-wave) superconductivity (SC) phase. According to Zhang [1], this idea is originated from the problem of Landau fermi liquid (i.e. Helium3).

The SO(5) theory starts with the phase diagram obtained experimentally of the cuprates. Then we ask, what is the Hamiltonian associates with this kind of system with superconducting state exist only in ground state in the clean (no impurity) limit, assuming if the system will be in antiferromagnic state if it is not in superconducting state. There exists a lot of model will gives the superconducting phase at ground state. In fact, most models with purely attractive interaction will gives this result. However, not many of them is capable to generate and antiferromagnic phase when the conduction bands are half filled. After that, we can find an Hamiltonian for a special model related to above but adiabatically connected to the system to interest. Finally, one can formulate a general theory that captures all the qualitative physics of the system. After some works, Arovas [1] et el. suggest that this problem can be formulate by considering five order parameters: Three of them corresponds to the antiferromagnetic state and the other two (i.e. X-Y model) corresponds to the superconducting state. These five components combine to form a vector in five dimension phase space called superspin. It turns out that this superspin transform under symmetry group SO(5). This is how the theory got its name.

We let the component of the superspin be labelled as n_1, n_2, n_3, n_4, n_5, where $\psi = n_1 + in_5$ and $\mathbf{m} = n_2\mathbf{x} + n_3\mathbf{y} + n_4\mathbf{z}$ are the SC(ψ) and AF($\mathbf{m}$) order parameters. These two order parameter is coupled due to the constraint $\psi^*\psi + m^2 = 1$. If we allow only quadratic symmetry breaking term, and isotropic system, the SO(5) theory gives the free energy density takes the form as equation 1

$$F = \frac{1}{2}|(\nabla + \frac{\mathbf{ie^*}}{\mathbf{\hbar c}}\mathbf{A})\phi|^{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{2}}\chi(\mathbf{2}\mu)^{\mathbf{2}}|\phi|^{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{2}}\rho|\nabla\mathbf{m}|^{\mathbf{2}} - \frac{\mathbf{1}}{\mathbf{2}}\mathbf{gm^2} + \frac{\mathbf{1}}{\mathbf{8}\pi}(\nabla\mathbf{A})^{\mathbf{2}} \tag{1}$$

There are many interesting result can be obtained using this free energy, which we are not going to quote all of them here. We are interested in the competition between the AF and the SC state, so let's ask if a vortex state with antiferromagnetic core observable. Let $\psi = ncos\theta \exp i\phi$ and $\mathbf{m} = \mathbf{nsin}\theta\hat{\mathbf{m}}$ with $\phi = tan^{-1}(y/x)$. Further assume that $\mathbf{m}$ is constant and ignore magnetic field. The free energy density is then (equation(2)).

$$F = \frac{1}{2}[(\nabla\mathbf{n})^{\mathbf{2}} + \mathbf{n^2}(\nabla\theta)^{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{r^2}}\mathbf{n^2\cos^2}\theta] + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{an^2} + \frac{\mathbf{1}}{\mathbf{4}}\mathbf{g}(\mathbf{cos2}\theta - \mathbf{1})\mathbf{n^2} + \frac{\mathbf{1}}{\mathbf{4}}\mathbf{bn^4} \tag{2}$$

where aT = a'(T - T_c). Now consider the normal vortex core $n(r) = n_o tanh(r/\xi)$, $\theta(r) = 0$ and the antiferromagnetic core $n(r) = n_o$, $\theta(r) = \frac{\pi}{2}exp(-rl)$. ξ and l are variational parameters, while $n_o = \sqrt{-a/b}$ is the superspin magnitude far from the vortex core. The energy different between a vortex with antiferromagnetic core and with normal is given by equation ferro.

$$F_{AF} - F_{normal} = \pi\rho n_o^2(\chi(\lambda) - \chi_c) \tag{3}$$

$\lambda = 1.852\sqrt{g/a}$, $\chi_c = 0.3214$, and

$$\chi(\lambda) = \int_0^\infty \frac{du}{u}[tanh^2 u - cos^2(\frac{\pi}{2}e^{-\lambda u})] \tag{4}$$

The AF core with lower energy (the AF core is energetically preferred) with $\lambda \leq 0.5683$. This corresponds to $4\chi(\mu^2 - mu_c^2) \leq 0.0941(T_c$ - T). Therefore, as T $\rightarrow$ 0, the antiferromagnetic phase should observable. However, as the temperature increases or doping (μ) increases, the AF state become unfavorable, and the vortex core will be normal core.

Not all the result can be solved analytically like what we did in last section. For non-isotropic system and system filled with defect, it is preferable to solve the problem using computer simulation. One of the very popular simulation is the large scale Monte Carlo simulation. the Monte Carlo simulations on a 3D SO(5) model is performed by Hu [2], [4]. B. Zheng[10] also performed the simulation using the X-Y model, for the short time scale.

The general goal of doing Monte Carlo simulation is tried to find the critical exponent. At the transition point, the spatial correlation length diverges. Therefore, the large scale behavior becomes more important in determining the behavior of the system then the microscopic details. A universal scaling between the correlation length (ξ) and the relaxation time (τ). The critical exponent,z, is defined to be $\tau = \xi^z$. Experimentally, the critical exponent is found to be around 1.5 to 2.

5 Near the Critical Point

There is actually a lot of interesting things going one as we get closer and closer to the boundary in phase diagram of the vortex state and the normal state. The high T_c superconductor has a large thermal fluctuation on vortex. Vortex lattice, therefore, undergoes a melting transition before it can goes from a superconductor to normal material. The melting transition is similar to melting to lattice. The vortex lattice will transform to vortex liquid (see the cartoon picture figure 3) as temperature increases. It is calculated by Hu et el. that the presence of point defects can separate a high T_c superconductor into four phases: Bragg glass, vortex glass, vortex liquid and vortex slush [8].

The melting phase transition does not only occurs in high T_c superconductor, but in the classical type II superconductor as well [9]. Ling et el uses a crystal of niobium (a low-temperature superconductor), they adjust the crystal until it reaches the narrow region of the peak effect. Using neutron diffraction (the same as the neutron scattering experiment mentioned earlier), they observed the melting of the vortex crystal. They also show that the vortex liquid turns into vortex crystal again after applying a oscillating external magnetic field.

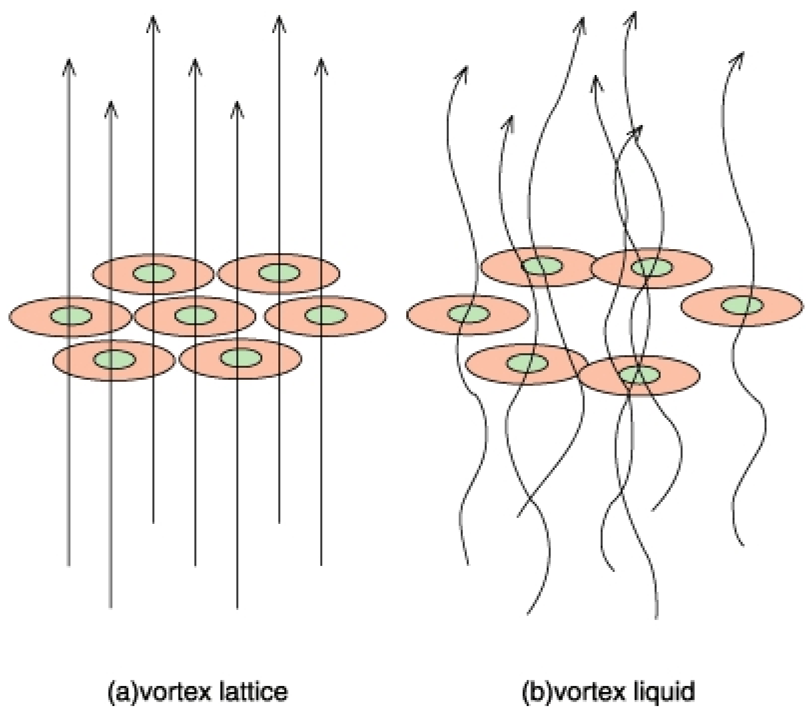

Figure 3: A cartoon picture showing the vortex crystal (appears in a hexagonal form) and a vortex liquid. The arrow lines are Josephson flux.

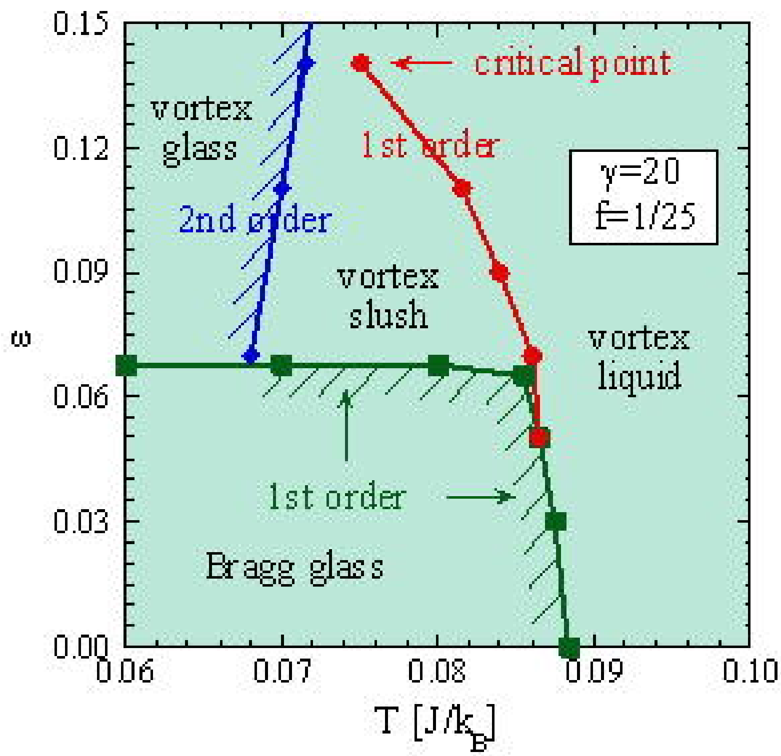

Figure 4: The varies vortex liquid and glass state near the normal-superconductor boundary in the phase diagram. Taken from Computational Materials Research Center.[8]

6 Conclusion

In this paper, we introduced high T_c superconductor which cannot be explained by the BCS theory. It is believed that the cuprate present in the high T_c superconductor responsible for pinning vortex even when the thermal fluctuation is large. From the neutron scattering experiment, competition between antiferromagnetic phase and superconducting phase was found, and this leads to the formulation of the SO(5) theory. When the antiferromagnetic degrees of freedom are turned off, the theory reduced to the X-Y model for describing normal-superconducting phase transition.

Due to the constraint on the pages of this term paper, a lot of interesting material, especially on the melting transition of superconductor is not presented. Also, most of the mathematically detail in the SO(5) theory is omitted on purpose. Description on the neutron scattering experiment to detect antiferromagnetic phase in cuprate is also shorten. It is arguable that the critical dynamic of superconductor should refer to the normal-superconducting state phase transition. However, search from the Los Alamos archive return more result than essay related to the normal-superconducting phase transition. Material presents here are chosen so that they are connected to one another.

To recapitulate and emphasize, although it is logical from the experimental result to formulate the SO(5) theory to describe the AF phase and SC phase. Unfortunately, there is still no sign of a microscopic theory the describes the behavior of high T_c superconductor, just like the BCS theory describes the classical superconductor. Undoubtedly, more work needed to be done to achieve more fundamental understanding of superconductivity.

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