The Quark-Gluon Plasma

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Abstract

The quark-gluon plasma is an exotic state of matter that occurs at high density and temperature. I describe the basic theory of the quark-gluon plasma and its formation in terms of simple ideal models. I describe current experiments that are attempting to produce and observe the quark-gluon plamsa and discuss some of their results.

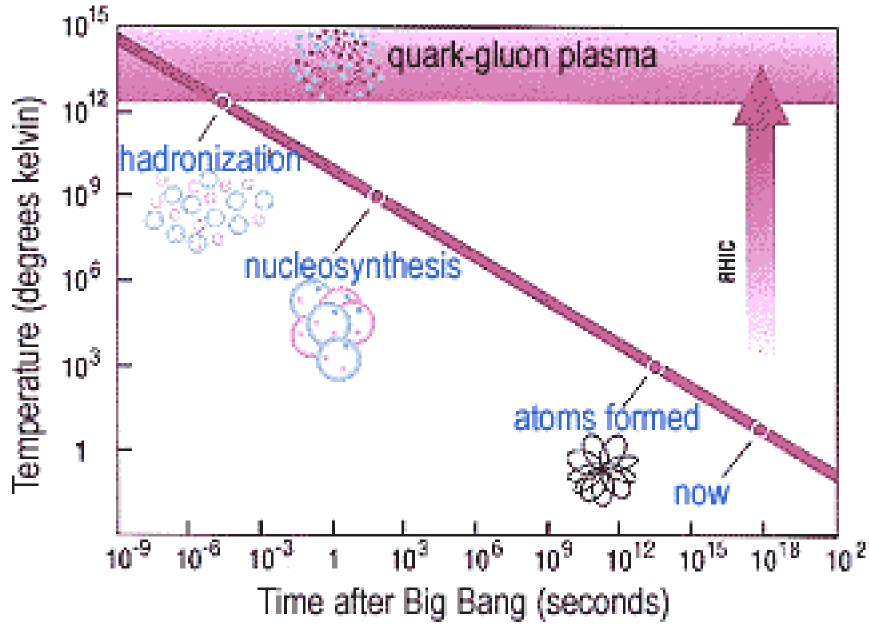

Figure 1: The temperature of the universe as a function for time from the Big Bang to the present. (Image from RHIC website [2])

1 Introduction

Within a few microseconds after the Big Bang, the universe consisted of a hot, dense gas of quarks and leptons moving about as free particles.[1] As this gas expanded, the quarks formed bound states of two or three quarks, hadrons, which now make up the *nuclear* matter of the present universe (see Figure 1). The leptons interact with the quarks only via the weak and electromagnetic forces (gravity is negligible). The quarks, however, interact via the weak, electromagnetic, and strong forces, and the strong force has a dominant effect over the other two. The gauge boson of the strong force is the gluon. As a result, we call this state a quark-gluon plasma.

"Traditional" exotic states of matter (e.g. superfluids, superconductors, etc.) exist at low temperature. We can understand these phenomena as arising when the interparticle spacing becomes comparable to the de Broglie wavelength of the particles. As the temperature decreases, the particle velocities decrease, and their de Broglie wavelengths increase. When the wavelengths of the particles are on the order of the interparticle spacing, we can no longer ignore quantum effects. However, the density of the particles can also increase, decreasing the interparticle spacing. Thus, at high temperature and sufficient densities, we also can not ignore quantum effects. If the quark-gluon plasma has a temperature of about 10^{13}K, then we will observe quantum effects at densities of about 10^{45}m^{-3}, which is the normal density of nuclear matter. However, the quark-gluon plasma is much different than regular nuclear matter, and the key distinction envolves quantum chromodynamics (QCD).

2 Quantum Chromodynamics

In both the classical and quantum descriptions, electrodynamics is an Abelian gauge theory (i.e. all gauge transformations commute with each other). However, a gauge theory need not be Abelian. Very strong evidence implies that both the strong and weak forces are non-Abelian gauge theories.

An important property of QCD, and non-Abelian gauge theories in general, is asymptotic freedom. The electric field of a point charge in a dielectric medium decreases with increasing distance from the charge. Due to quantum fluctuations that produce charged particle-antiparticle pairs, the vacuum behaves like a dielectric medium. Thus, the effective charge of a free electron decreases with distance. However, in QCD, the quantum fluctuation can produce both quark-antiquark pairs *plus* gluon pairs and triplets. The quark-antiquark pairs screen the color of a quark, but the gluon pairs and triplets have an opposite effect. As a result, the strength of the chromodynamic interaction increases with distance from the quark. Similarly, the strength decreases with increasing momentum.

The coupling of QCD for large momentum, k, is

$$\alpha_s(k) = \alpha_s(\Lambda)\left[1 + \frac{\alpha_s(\Lambda)}{4\pi}\left(11 - \frac{2}{3}n_f\right)\ln\left(k^2/\Lambda^2\right)\right]^{-1} \tag{1}$$

where n_f is the number of flavors of massless quarks, and Λ is a dimensionful parameter, which is about 200MeV/c.[4] From experimental measurements, $\alpha_s(m_Z) \approx 0.12$, where $m_Z \approx 91.19$GeV, the mass of the Z-boson. So, $\alpha_s(\lambda) \sim O(1)$.

Asymptotic freedom gives rise to the notion of confinement; nobody has observed an individual free quark. The typical seperation of two quarks in nuclear matter is 1fm = 5.07GeV^{-1}. Thus, for nuclear matter, $\alpha_s(0.5\text{GeV}) \sim O(1)$. If the quarks tried to move further away from each other, they would feel a stronger attraction. So they are *confined* inside the (colorless) hadrons. But if the quarks move closer together, they interact less. For a collection of quarks, in the limit that the average quark seperation goes to zero, the quarks behave more like free particles. And thus, the quarks are deconfined from their hadronic forms and start behaving *like* an ideal gas. Of course, there are still strong interactions, so we must consider both the quarks and gluons.

3 The MIT Bag Model and Thermodynamics

Consider a collection of relativistic, massless, noninteracting quark confined within a sphere, $\mathbb{S}$, of radius, R (i.e. a bag).[6, 7, 8, 9] Then the quarks are described by the free Dirac equation,

$$i\gamma^\mu \partial_\mu \psi = 0, \tag{2}$$

with the boundary condition,

$$n^\mu J_\mu = n^\mu (\bar{\psi}\gamma_\mu \psi)\Big|_{\partial\mathbb{S}} = 0. \tag{3}$$

γ^μ are the Dirac matrices, and n^μ, is the normal (four) vector to the bag. (I let $\hbar = c = 1$ throughout this paper.) For a static bag, the time component of the normal vector is zero. The above boundary condition maintains current conservation: quarks do not enter or leave the bag. In addition, the bag must obey energy-momentum conservation. At the surface of the bag, we impose the additional boundary condition,

$$\frac{1}{2}\sum_{\rm i} n^\mu \partial_\mu (\bar{\psi}_{\rm i}\psi_{\rm i})\Big|_{\partial\mathbb{S}} + B = 0. \tag{4}$$

The sum is over the quarks labeled by i. To satisfy Lorentz invariance, B is a scalar quantity; we call B the bag constant, which has units of energy density. This boundary condition is a statement of pressure balance at the surface of the bag. The first term describes the pressure due to the impact of the quarks on the surface of the bag, and the bag constant is a negative internal pressure (effectively due to the attraction of the quarks, which we have neglected). The pressure outside the bag is zero.

We assume that the bag is static, so $n^\mu = (0, \hat{r})$, where $\hat{r}$ is the radial unit vector. We solve equation (2) with boundary condition (3) and obtain the allowed energy states. When we include the bag constant, the total energy of the bag is,

$$E(R) = \frac{1}{R}\sum_{\rm i} \omega_{\rm i} R + \frac{4\pi}{3} B R^3, \tag{5}$$

where $\omega_{\rm i}$ is the lowest energy state for quark i (note: $\omega_{\rm i} R$ is a number independent of R). For a more detailed description, I refer the reader to [7, 9]. Nevertheless, according to boundary condition (4), the bag pressure must be in equilibrium. We enforce this by minimizing $E(R)$ with respect to R. Thus, we find

$$R_0{}^4 = \frac{\sum_{\rm i} \omega_{\rm i} R}{4\pi B}. \tag{6}$$

which gives,

$$E_{\rm min} = \frac{16\pi}{3} B R_0{}^3. \tag{7}$$

The energy of the bag can be refined by including contributions due to quantum fluctuations (e.g. vacuum fluctuations and QCD interations). The above description of the bag model is basically a mean field theory.

We can use the bag model to describe hadrons. The low energy states correspond to the masses of the hadrons. If we fit the energy spectrum to the hadron masses, we obtain $B^{1/4} \approx 200$MeV, or 60MeV/fm^3.[8, 9]

When the quarks deconfine, the quarks behave as free particles amongst medium of gluons. We model the system as an ideal, relativistic gas of free quarks and gluons. Gluons are bosons which have two possible spin states ($s_z = \pm 1$) and 8 possible color states. The average energy density of the gluons is,

$$\mathcal{E}_{\rm gluon} = 2 \times 8 \times \int \frac{p}{e^{\beta p} - 1} \frac{{\rm d}^3 p}{(2\pi)^3} = \frac{8\pi^2}{15} T^4. \qquad (8)$$

where p is the gluon momentum, and T is the temperature ($\beta = 1/T$). The number of gluons is not conserved, so the chemical potential is zero. Equation (8) is the Stefan-Boltzmann law for gluons.

Quarks are fermions which have two possible spin states, three possible color states, and two possible flavors (only up and down are considered, the other four are more energetic). The average energy density of the free quarks is,

$$\mathcal{E}_{\rm quark} = 2 \times 3 \times 2 \times \int \frac{p}{e^{\beta(p-\mu)} + 1} \frac{{\rm d}^3 p}{(2\pi)^3} = \frac{6T^4}{\pi^2} \int_{-\beta\mu}^{\infty} \frac{(x + \beta\mu)^3}{e^x + 1} {\rm d}x, \qquad (9)$$

where μ is the chemical potential and $x = \beta(p + \mu)$. A similar equation gives the energy of an antiquark but with a modification to the chemical potential. Assuming the gas contains a constant difference of more quarks than antiquarks (a fixed positive baryon number), then the addition of a quark to the gas requires energy, μ. However, the addition of an antiquark can annihilate a quark in the gas, thus releasing energy; we must remove energy. Hence, the chemical potential of the antiquark components is $-\mu$, and the average energy density is

$$\mathcal{E}_{\rm antiquark} = 2 \times 3 \times 2 \times \int \frac{p}{e^{\beta(p+\mu)} + 1} \frac{{\rm d}^3 p}{(2\pi)^3} = \frac{6T^4}{\pi^2} \int_{\beta\mu}^{\infty} \frac{(x' - \beta\mu)^3}{e^{x'} + 1} {\rm d}x', \qquad (10)$$

where $x' = \beta(p - \mu)$.

We obtain an exact expression for the total energy density of the quarks and antiquarks [9]:

$$\begin{aligned}
\mathcal{E}_{quarks} + \mathcal{E}_{antiquarks} &= \frac{6T^4}{\pi^2} \left[\int_{-\beta\mu}^{\infty} \frac{(x+\beta\mu)^3}{e^x + 1} {\rm d}x + \int_0^{\infty} \frac{(x'-\beta\mu)^3}{e^{x'} + 1} {\rm d}x' - \int_0^{\beta\mu} \frac{(x'-\beta\mu)^3}{e^{x'} + 1} {\rm d}x' \right] \\
&= \frac{6T^4}{\pi^2} \left[\int_0^{\infty} \left[\frac{(x+\beta\mu)^3}{e^x + 1} + \frac{(x-\beta\mu)^3}{e^x + 1} \right] {\rm d}x + \int_{-\beta\mu}^{0} \frac{(x+\beta\mu)^3}{e^x + 1} {\rm d}x \right. \\
&\quad \left. - \int_{-\beta\mu}^{0} (x+\beta\mu)^3 \left[1 - (e^x + 1)^{-1}\right] {\rm d}x \right] \\
&= \frac{6T^4}{\pi^2} \left[\int_0^{\infty} \left[\frac{(x+\beta\mu)^3 + (x-\beta\mu)^3}{e^x + 1} \right] {\rm d}x + \int_{-\beta\mu}^{0} (x+\beta\mu)^3) {\rm d}x \right]
\end{aligned}$$

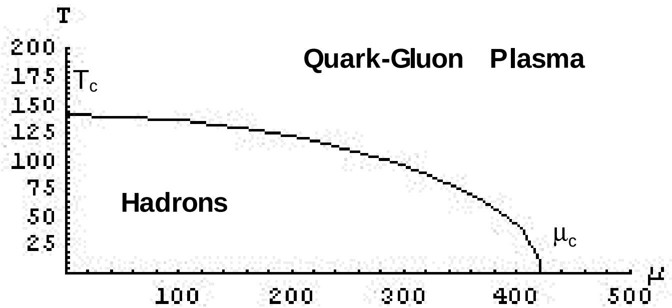

Figure 2: Sketch of the phase boundary of temperature versus chemical potential determined by equation (14) with $B^{\frac{1}{4}} = 200$ MeV. All values are in units of MeV.

where in the last integral of the second line, we have set $x' = -x$. Solving the integrals and adding the gluon energy density, the total energy density is

$$\mathcal{E}_{qgp}(\mu, T) = \frac{37\pi^2}{30}T^4 + 3\mu^2 T^2 + \frac{3}{2\pi^2}\mu^4. \tag{11}$$

Next, we can find the baryon density (one-third of the difference between the quark and antiquark densities):

$$n_{baryon} = \frac{1}{3} \times 12 \times \int \left[\frac{1}{e^{\beta(p-\mu)}+1} - \frac{1}{e^{\beta(p+\mu)}+1}\right]\frac{\mathrm{d}^3 p}{(2\pi)^3} = \frac{2}{3}\mu T^2 + \frac{2}{3\pi^2}\mu^3 \tag{12}$$

So for low temperatures, the baryon density is directly related to the chemical potential. Note, 1MeV is considered a small temperature for these systems but corresponds to 10^7K.

The pressure of the ideal quark-gluon gas is,

$$P_{qgp} = \frac{1}{3}\mathcal{E}_{qgp} = \frac{37\pi^3}{90}T^4 + \mu^2 T^2 + \frac{1}{2\pi^2}\mu^4. \tag{13}$$

The quark-gluon plasma is stable if $P \geq B$.

Thus, the phase boundary is defined for,

$$B = \frac{1}{3}\mathcal{E}_{qgp} = \frac{37\pi^3}{90}T_c{}^4 + \mu_c{}^2 T_c{}^2 + \frac{1}{2\pi^2}\mu_c{}^4 \tag{14}$$

where T_c and μ_c are the critical temperature and chemical potential, respectively. If we let the chemical potential of a quark equal one third the mass of a nucleon, so $\mu \approx 300$MeV, then the transition temperature is about 100MeV, or one billion Kelvin.

Equation (14) determines the phase boundary between the hadronic phase and the quark-gluon phase as a function of temperature and chemical potential. In this simple model, the number of degrees of freedom changes between the two phases, implying a first order phase transition. However, whether this phase transition is actually first order or continous is still debated.

We can define an order parameter called the Polyakov loop expectation value:

$$\langle \widehat{L}(\vec{x}) \rangle = e^{-\beta F(\vec{x})} = \begin{cases} 0 & T \leq T_c \\ \text{finite} & T > T_c \end{cases}$$

where $F(\vec{x})$ is the free energy of a quark at position, $\vec{x}$.[5, 11] In the confined phase, the free energy is infinite; in the deconfined phase, the free energy is finite. Furthermore, for two quarks,

$$\langle \widehat{L}(\vec{x}_1)\widehat{L}^\dagger(\vec{x}_2) \rangle = e^{-\beta F(\vec{x}_1, \vec{x}_2)} \tag{15}$$

Suppose the distance between the two quarks is taken to infinity. If the quarks are in the confined phase, then the energy between the two quarks becomes infinite. But if the quarks are in the deconfined phase, the two quarks are uncorrelated. Thus, as $|\vec{x}_1 - \vec{x}_2| \to \infty$ then

$$\langle \widehat{L}(\vec{x}_1)\widehat{L}^\dagger(\vec{x}_2) \rangle \longrightarrow \begin{cases} 0 & T \leq T_c \\ \langle \widehat{L}(\vec{x}_2) \rangle \langle \widehat{L}(\vec{x}_2) \rangle & T > T_c \end{cases}$$

Thus, the quark-gluon plasma exhibits long-range order.

Above, we ignored the myriad of interactions in the quark-gluon plasma, and there is a serious problem in ignoring them. We partially accounted for the interactions by including the gluons in the gas, but we did not include a coupling of the gluons to the quarks. More rigourous approaches use finite temperature field theory.[5, 10] There are also powerful computational methods, namely lattice QCD.[12] Lattice QCD calculations predict the formation of the quark-gluon plasma at a temperature of $150 - 170$MeV and energy densities of 4GeV/fm^3. [13]

A final important property of the quark-gluon plasma is chiral symmetry. Chiral symmetry is exhibited by free, massless fermions. The up and down quarks have mass, but it is small. So chiral symmetry is an approximate symmetry. In the hadronic phase, quarks are not chiral symmetric due to the confining interactions. However, in the deconfined state, the quarks apparently exhibit chiral symmetry. Thus, there is a chirial symmetry restoration in the formation of the quark gluon plasma. Whether, the chiral symmetry restoration is connected with the confinement-deconfinement transition is still debated, but lattice QCD calculation suggest a plausible connection.

4 Experimental Production of the Quark-Gluon Plasma

Recreating the first few microseconds after the Big Bang in the laboratory is not a simple task. As a result, experiments on the quark-gluon plasma are at a pre-natal stage. Physicists are currently running experiments to make or find the existence of a quark-gluon plamsa state. They are reluctant to confirm observing the formation of the quark-gluon plasma, but there is "compelling evidence" that is has been fomed.[14]

The current trend is colliding heavy nuclear ions together at very relativistic speeds (see figure 3); this is what physicists are doing at the new relativistic heavy ion collider

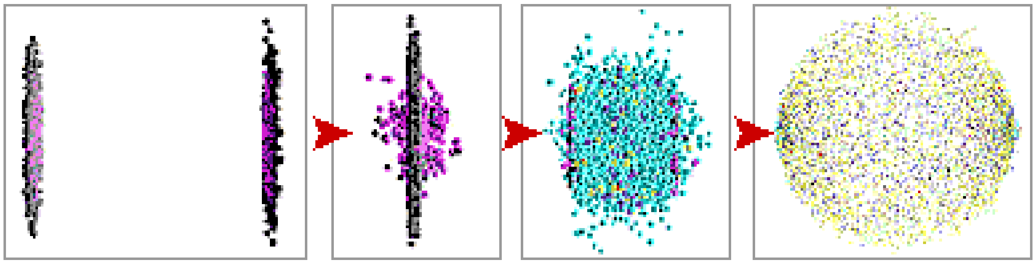

Figure 3: Stages in the collision of two heavy ions traveling at relativistic speeds. The two ions initially look like two pancakes due to Lorentz contraction. (Image from the RHIC website.[2])

(RHIC) at Brookhaven National Laboratory. At RHIC, two heavy ions traveling at 99% the speed of light collide with each other. The nucleons in the heavy ions are compressed to a sufficiently high density that deconfines the quarks in the nucleons. Thus, the quark-gluon plasma is formed - in theory. But, the conglomerate of quarks and gluons (know as a "fireball") expands, the density decreases, and the quarks hadronize - much like at the beginning of the universe.

But we can not simply look and see if the quark-gluon plasma has been formed. Instead we must look for signatures of its formation, and there are a variety of things to look for.[14] Physicists basically look for two things: (1) creation or suppression of particles associated with the formation of the quark-gluon plasma, and (2) spatial distributions of dynamical qualities (energy, momentum, etc.) of particles.

An example of the first is the suppression of the J/Ψ meson (a charm-anticharm pair); this particle is strongly suppressed by the quark-gluon plasma. Two gluons interact to produce a charm-anticharm pair. Normally, the two quarks would pair to form the J/Ψ. But in the quark-gluon plasma, color screening prevents the two from pairing, and so they wander away from each other. When hadronization occurs, the two quarks will pair with other quarks, which is mostlikely not charm flavored.[15] We can easily single out the J/Ψ from the other particles eminating from heavy ion collisions because they are more massive than the observed other mesons.

Analysis of data collected from experiments in 1998 at the Super Proton Synchrotron (SPS) at CERN indicate a clear suppression of the J/Ψ at high energy densities.[16] Out of about 80 million events, about $40,000 J/\Psi$ particles were detected. Figure 4 displays the relative yields of the J/Ψ versus energy density from the data collected in 1998 and previous years. The energy densities are estimated using a Monte Carlo event generator - a common technique used in experimental high energy physics. The graph indicates suppression beginning at an energy density of about $2.5\mathrm{GeV/fm}^3$. In our simple calculations with the bag model, we predicted that the deconfinement transition occurs for energy densities at about $200\mathrm{MeV/fm}^3$, and about $2\mathrm{GeV/fm}^3$ from lattice QCD calculations.

Experiments at RHIC are not yet at the stage to observe J/Ψ suppression. Current

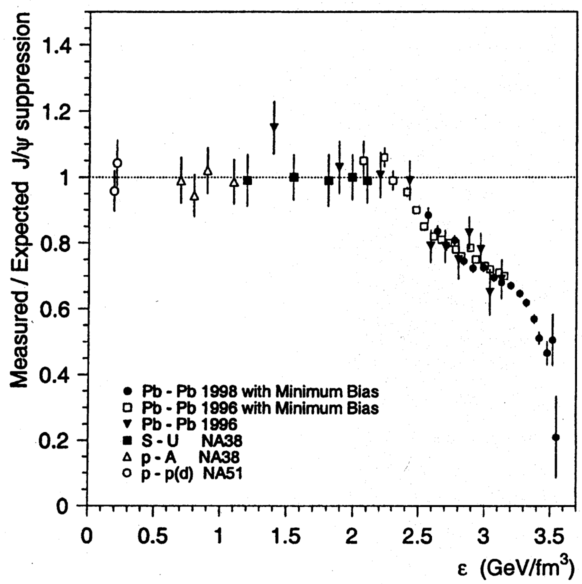

Figure 4: "Measured J/Ψ production yields, normalized to the yields expected assuming that the only source of suppression is the ordinarry absorption by the nuclear medium. The data is shown as a function of the energy density reached in the several collision systems."[16]

experiments involve measuring the distribution of transverse momentum.?? At energies below the formation of the quark-gluon plasma, heavy ion collisions result in particles scattering in all directions. The momentum component perpendicular to the collision axis is the transverse momentum. With the formation of the quark-gluon plasma, a phenomena called jet quenching occurs. When partons (a general name for constituents of hadrons - quarks and gluons) pass through the quark-gluon plasma, they lose energy by radiating gluons. Thus, we expect to see a suppression in the transverse momentum distribution of hadrons with deconfinement. Figure 5 presents how the nuclear modification factor, R_{AA}, changes with transverse momentum, p_T in the collisions of two gold nuclei. R_{AA} is a ratio of how the distibution of transverse momenta in nucleus-nucleus $(A+A)$ collisions compare to proton-proton scattering; $R_{AA}=1$ for hard scattering. The energy of a gold nuclei in these collisions is roughly 90GeV. In figure 5, R_{AA} increases towards unity for $p_T \lesssim$ 2GeV, and then plateaus for $p_T \gtrsim$ 2GeV. Thus, we observe suppression for large p_T. Since partons with large p_T have more energy to "give away" via gluon radiation, we expect a greater suppression in this regime. The analysis of the experiment does not confirm the formation of the quark-gluon plasma, but it does suggest the presence of a dense nuclear medium.[17]

There are several other methods of looking for evidence of the formation of the quark-gluon plasma. Since, heavy ion collisions result in a massive mess of particles, extracting important information out of the them is a challenge. The above experiments and analyses are strong indications of the presence of some nuclear phenomena. Whether this phenomena

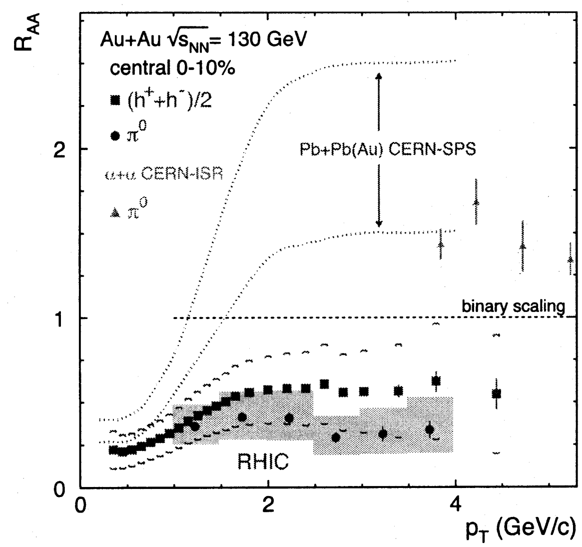

Figure 5: A graph of R_{AA} versus p_T in the collision of two gold nuclei. The data below $R_{AA} = 1$ were acquired at RHIC. The brackets and shaded bands are uncertainties. [17]

is the formation of the quark-gluon plasma is unclear. There is still much to be done.

5 Conclusion

Our description of quark matter is not complete, and there are a other phases at higher density regions of the phase diagram. One such phase is diquark matter, or the color superconductor.[20] This state is believed to exist in place of the quark-gluon plasma at high densities (high chemical potential). Two quarks of different flavor and color are attracted to each other and can form a Cooper pair. The quarks must have opposite spin and chirality. The color superconductor is very similar to traditional electronic superconductors. There is even an Anderson-Higgs mechanism where $SU(3)$ color symmetry is broken and five gluons aquire mass.

Recently, there is *possible* new evidence for the existence of quark stars.[21] In contrast to the quark-gluon plasma formed at RHIC, the quark-gluon plasma in quark stars is stable. The internal pressure due to gravitational keeps the inside of the star at high density and temperature. However, these recent observations are very questionable.

Experiments for studying the quark-gluon plasma state are important. They will help us understand the QCD and nuclear matter where the theory has encountered many snags. An understanding is also important for applications to particle physics, astrophysics, cosmology, and many-body physics. Moreover, we still do not know if QCD provides a *completely* valid description of the strong force.

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