Thermally activated phase slips and quantum phase slips in one-dimensional superconductors

Mitrabhanu Sahu
Department of Physics
University of Illinois at Urbana-Champaign

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Abstract

We will briefly discuss the theories for thermally activated phase slips (TAPS) given by Langer, Ambegaokar, McCumber and Halperin(LAMH) and the theory of macroscopic quantum phase slips(QPS).Experimental evidence of thermally activated phase slips in superconducting nanowires (10-30 nm wide) and its agreement with LAMH theory will be discussed. We will then turn our attention to the question of quantum phase slips, by outlining various experimental data as well as different theoretical predictions surrounding the controversy whether QPS can be, or up to the present time, have been, observed experimentally

1.Introduction:

Superconductivity in two and one dimensions has been of great interest for condensed matter physicists for last few decades. According to Mermin-Wagner theorem superconducting long-range order is impossible in strictly one or two dimensional systems at any finite temperature. However, it is not clear how this theorem translates into experimental reality – how exactly is superconductivity extinguished in reduced dimensions?

Due to advancement of lithographic techniques and thin film deposition, researchers are now able to deposit continuous metallic films, which are few nm thick. Since the 1980s, superconductivity in two-dimensional thin films has been extensively studied and the mechanisms are relatively well understood. Generally speaking, a superconductor-insulator phase transition was observed, with the controlling parameter being Rsq = Rq, where Rsq = ρ/d is the sheet resistance of the thin film and Rq $=h/4e^2$ is the quantum resistance for superconductors. Such a phase transition arises from the enhanced Coulomb interaction among the electrons, and can be tuned by either disorder or an applied magnetic field (Hebard and Paalanen, 1990, Haviland *et al*., 1989, Fisher, 1990, Goldman and Markovic, 1998).

One of the outstanding problems in 1D superconductivity concerns thermally activated phase slips and quantum phase slips. Phase slips give rise to resistance in a thin superconducting wire below *Tc*. During a phase slip, the superconducting order parameter fluctuates to zero at some point along the wire, allowing the relative phase across the point to slip by 2π, resulting in a voltage pulse; the sum of these pulses gives the resistive voltage. In a theory developed by Langer, Ambegaokar, McCumber, and Halperin (LAMH) , such phase slips occur via thermal activation as the system passes over a free-energy barrier proportional to the cross-sectional area of a wire. Experiments on 0.5 μm diameter tin whiskers confirmed the theory .

Subsequently, Giordano observed in thin In and PbIn wires a crossover from the LAMH behavior near *Tc* to a more weakly temperature dependent resistance tail at lower temperatures. He attributed this tail to phase slips occurring via macroscopic quantum tunneling (MQT), or QPS, through the same free-energy barrier. However, interpretation of these results has been complicated by the possibility of granularity in these metals that could give rise to a similar temperature dependence. Additionally, Sharifi *et al.* found in homogeneous Pb wires a systematic broadening of the transition with decreasing cross-sectional areas of the wires that could not be explained by the LAMH theory, but no crossover to a more weakly temperature dependent regime was observed. Thus it is controversial whether such quantum phase slips have been observed in experiments. Theoretically, it is also a subject of debate whether resistance arising from QPS is actually observable. Certain theories imply that QPS should be important when the wire diameter is about 10 nm, but such thin wires are extremely difficult to fabricate by conventional electron beam lithography.

2.Theory

In this section we will briefly overview the field of 1D superconductivity, with the emphasis on the question of phase slips in thin superconducting wires. In particular, we shall describe the LAMH theory, which accounts for the resistance in a thin wire below TC in terms of thermally-activated phase slips. The experiments that confirmed the LAMH theory will also be described briefly. We will then turn our attention to the question of quantum phase slips (QPS), by outlining various experimental data as well as different theoretical predictions surrounding the controversy whether QPS can be, or up to the present time, have been, observed experimentally.

2.1 Thermally activated phase slips:

Theoretical study of the processes, which can suppress superconductivity in 1D wires dates back to the 1960s, when Langer, Ambegaokar, McCumber and Halperin (LAMH)(1967, 1970) developed the theory of thermally-activated phase slips, which explained resistances and dissipation in thin wires below T_C.

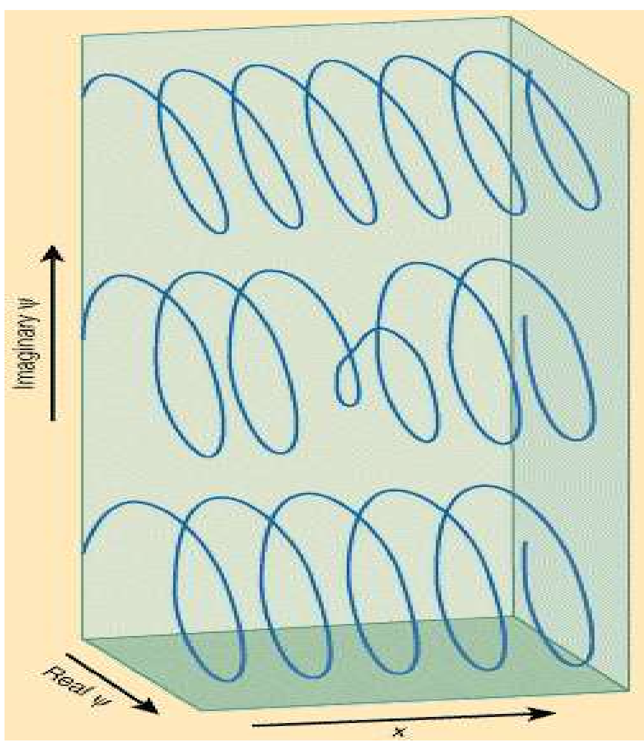

Figure 1*. A phase-slip process (Schön, 2000). During a phase slip, the phase difference of the order parameter is reduced by 2π. The top image shows the order parameter before the phase slip, the middle image at some instant during*

the process when the phase is being reduced, and the bottom image after the process.

In this picture, we consider a constant voltage which is applied between the two ends of a superconducting wire. By the Josephson relation h, one expects the corresponding phase difference to increase in time. This in turn results in a continuously increasing supercurrent Is, since Is $\propto \nabla\tilde{\varphi}$Eventually the critical current is exceeded and superconductivity becomes energetically unfavorable. However, a steady current in the superconducting state is possible, if some thermal fluctuations cause the magnitude of the superconducting parameter to go to zero at some point along the wire, allowing the relative phase across the point to slip by 2π, so that $\Delta\varphi$and hence Is is reduced. Such a phase slip induces a voltage pulse. These voltage pulses sum to give rise to the measured resistance. Figure 1, illustrates the phase slip process(Schön, 2000). The details of the LAMH theory are discussed below.

LAMH Theory:

The LAMH theory applies to narrow superconducting channels in which thermal fuctuations can cause phase slips, i.e. jumps in the phase difference of the order parameter by 2π. Thermal activations of the system over a free energy barrier ΔF occur at a rate given by $\Omega(T)e^{-\Delta F/kT}$ where the attempt frequency

$$\Omega(T) \ = \ (L/\xi(T))(\Delta F/kT)^{1/2}(\tau_{\rm GL})^{-1} \qquad (1)$$

is inversely proportional to the relaxation time of the time –dependent Ginzburg-Landau (GL) theory $\tau_{GL} = 8k(Tc - T)/\pi h$ (*L* is the wire length, $\xi(T)$ is the GL coherence length, and *Tc* is the mean field critical temperature of the wire).The free energy barrier for a single phase slip is given by

$$\Delta F \ = \ (8\sqrt{2}/3)(H_c(T)^2/8\pi)(A\xi(T)) \qquad (2)$$

which is essentially the condensation energy density multiplied by the volume $A\xi(T)$ of a phase slip (where *A* is the wire's cross sectional area). A current *I* causes a nonzero voltage (averaged over the rapid phase slip processes),

$$V \ = \ (\hbar\Omega(T)/e)e^{-\Delta F/kT}\sinh(I/I_0) \qquad (3)$$

Where I_0=4ekT/h the limit of low currents $I \ll I_0$ Ohm's law is recovered

$$R_{\rm LAMH}(T) \equiv V/I \approx R_q(\hbar\Omega(T)/kT)e^{-\Delta F/kT} \ . \qquad (4)$$

Where $R_q = h/(2e)^2$

2.2 Macroscopic Quantum Phase Slips:

It has been suggested that, in addition to thermal activation, phase slips can also occur via quantum tunneling (van Run *et al*., 1987, Saito and Murayama, 1989). Such mechanism for phase slippage is known as *macroscopic* quantum tunneling (MQT) because it involves the collective motion of the electron condensate in a volume ~ξA which typically contains 10^8 or more electrons. This process has been studied extensively (Chakravarty and Stein, 1982, Caldeira and Leggett, 1981, Schmid, 1983) and verified experimentally in other systems, notably in Josephson junctions (Penttila *et al*., 2001, Penttila *et al*., 1999). However, little quantitative theoretical work had been done on MQT of phase slips in thin superconducting wires, until Giordano's results (Giordano, 1988, Giordano and Schuler, 1989, Giordano, 1994) provided possible experimental evidence for these processes. In his works, Giordano also presented a phenomenological model and arrived at a functional form for the wire's resistance due to MQT of phase slips. Subsequently more rigorous theories were developed (Chang, 1996, Duan, 1995, Zaikin *et al*., 1997b, Golubev and Zaikin, 2001), but these theories do not agree on whether MQT of phase slips in superconducting wires is actually observable in experiments . In this section Giordano's work has been briefly summarized and some other thoritical models has been has introduced.

Giordano's Model:

The physical idea underlying Giordano's treatment of MQT of phase slips is quite simple. He considers the motion of a damped particle moving in a tilted washboard potential, and assumes that the particle can be treated as a simple harmonic oscillator in a well, with a small oscillation frequency ω_0.

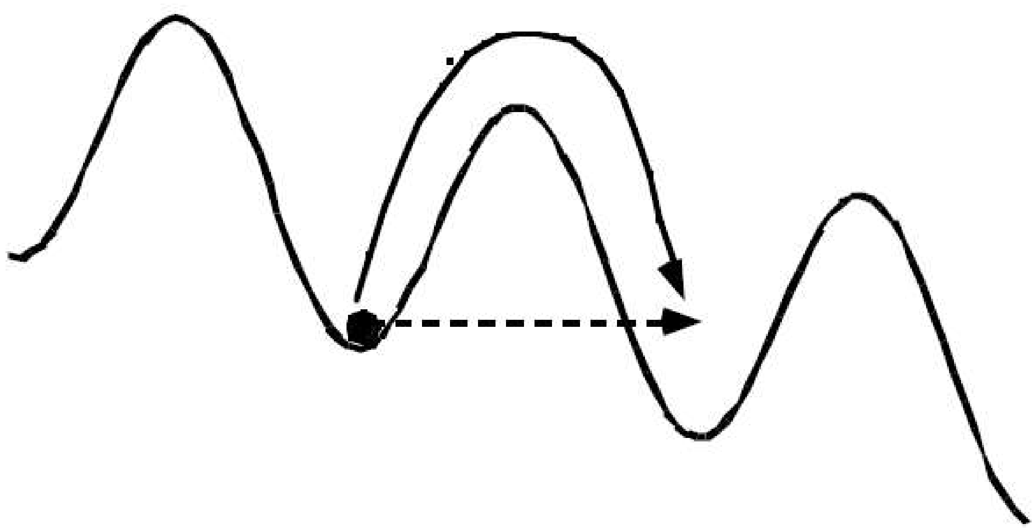

Figure2. *Tilted washboard potential. A particle trapped in one of the minima can reach the lower energy state by either thermal activation (solid line) or quantum tunneling (dashed line).*

Then the tunneling rate can be written in the form (Caldeira and Leggett, 1981)

$$\Gamma_{MQT} = B\sqrt{\frac{V_o\omega_o}{\hbar}}e^{-aV_o/\hbar\omega_o} \tag{5}$$

where V_0 is the barrier height, and B and a are constants. Giordano argues that V_0 should be identified with ΔF the free energy barrier in the LAMH theory: and since Γ_{GL} is the only time scale in the time dependent GL theory, it is natural to identify ω_0 with Γ_{GL}.

Therefore,

$$\Gamma_{MQT} = B\sqrt{\frac{\Delta F}{\hbar\tau_{GL}}}e^{-a\Delta F\tau_{GL}/\hbar} \tag{6}$$

Following the standard procedure used to obtain resistance from phase slip rate and inserting the factor L/ξ(which is the number of difference places along the wire that phase slips can occur), one obtains,

$$R_{MQT} = B\frac{L}{\xi}R_q\sqrt{\frac{\Delta F}{\hbar\tau_{GL}}}e^{-a\Delta F\tau_{GL}/\hbar} \tag{7}$$

one can easily see that apart from numerical constants, R_{MQT} can be obtained from the LAMH form by simply replacing the energy scale kT by $\hbar/\xi_{GL}$, which is also physically reasonable.

3.Sample Fabrication:

In recent years there has been substantial controversy regarding whether macroscopic quantum tunneling of phase slips in thin superconducting wires is observable or had actually been observed in experiments. One of the reasons that this question has not been settled is that it is extremely difficult to fabricate sub-20 nm homogeneous wires using conventional techniques, such as electron beam lithography, step edge lithography or stencil mask evaporation. But recently new fabrication has made it possible to make wires with diameter ~ 10nm. (Jeanie *thesis*).

The idea behind this new technique is quite simple: nanowires can be made by depositing metal onto free-standing carbon nanotubes or ropes, which act as mechanical templates, as shown in Figure 3. The cross-sectional area of a wire is determined by the width of the underlying nanotubes/ropes, and by the amount of material deposited. Because of the small sizes of isolated carbon nanotubes, which can be as small as 1.2 nm in diameter, and by using amorphous materials that form continuous films at a few nm in thickness, nanowires as thin as 8 nm can be fabricated. [Recent experiments (Tang *et al*., 1998, Sun *et al*., 1999, Wang *et al*., 2000) reported the synthesis of 4 Å-diameter nanotubes in channels of microporous aluminophosphate crystallites AFI. However, these nanotubes are thought to be unstable outside the AFI matrix, thus unsuitable for our purposes.] Moreover, by using titanium as a sticking layer (Zhang and Dai, 2000),different metals can be deposited to make different wires, such as ferromagnetic or superconducting ones. Therefore this method is relatively straightforward and versatile.

Measurement of these superconducting nanowires also proves to be challenging, as they are extremely fragile and sensitive to electronic noise and high frequency radiation.

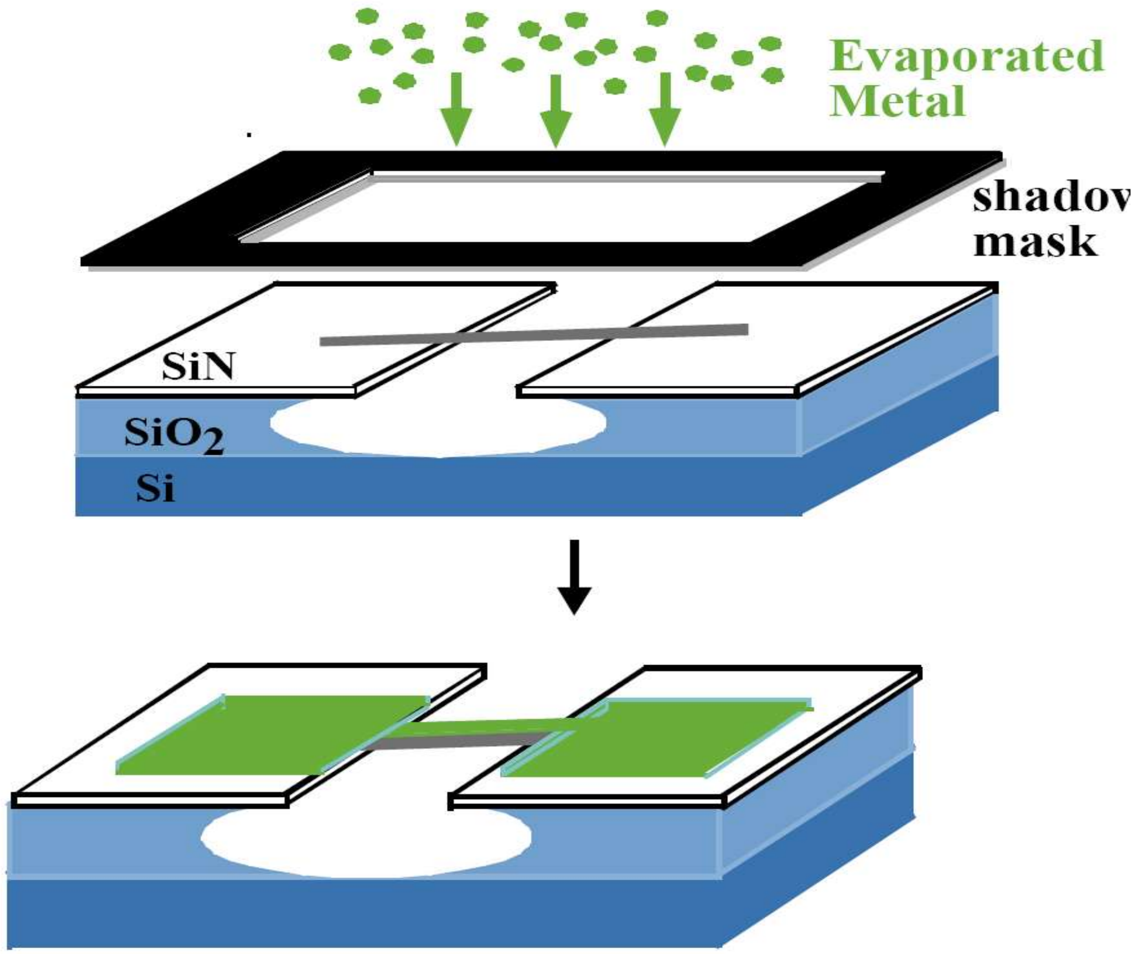

Figure 3. *Schematics of the fabrication process. Nanowires are formed by depositing metal atoms on carbon nanotubes, which are suspended across a slit on the SiN surface. The nanotubes act as mechanical support. The SiO layer is etched to give an undercut to prevent the "walls" from being coated with metal during deposition process. The electrodes are defined either by depositing metals through a shadow mask (shown here), or by photolithography and etching.*

Figure 4 is a schematic diagram of a typical sample. The sample is current biased through electrical leads I_+ and I_-, and the voltage drop across V_+ and V_- is measured, including both the nanowire and a segment of the co-evaporated thin film electrodes. The voltage drop across a segment of the thin film (V_+ and V_f) can be measured at the same time. This allows us to separate the different behaviors of twodimensional thin films from a one-dimensional wire.

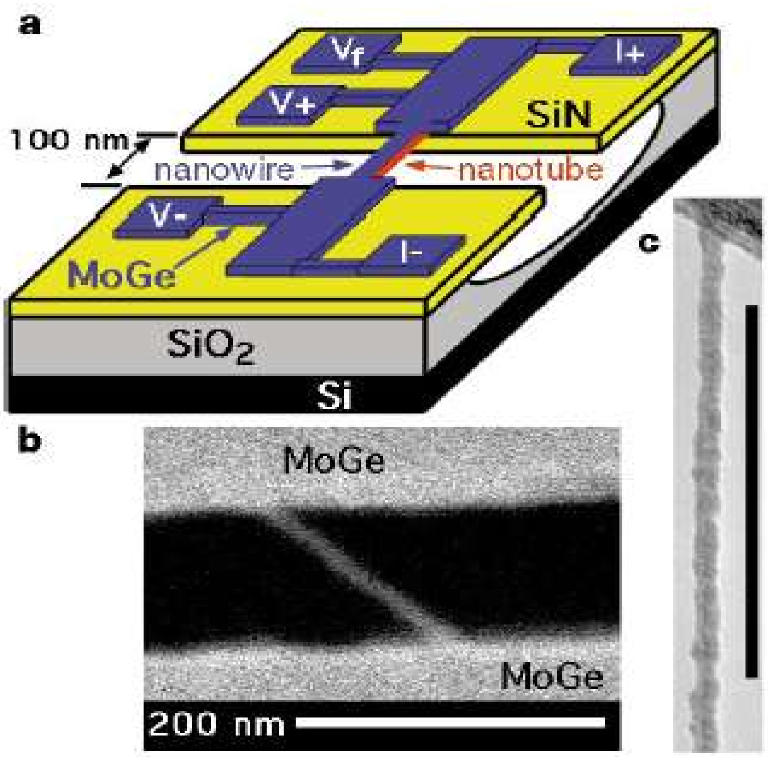

Figure** 4*. *Fabrication and imaging of nanowire.* ***a.Schematic of the sample.***b.*** SEM image of a sample, showing a free-standing nanowire which connects two Mo-Ge electrodes. Its apparent width, including blurring in the SEM image, is W < 10 nm. The scale bar (white) is 200 nm.* ***c.*** *One of the thinnest wires found under a higher-resolution transmission electron microscope. The wire width is WTEM < 5:5 6 1 nm. The scale bar (black) is 100 nm. (Bezryadin et al, Nature,2000)*

4. Recent Experiments on thin nanowires:

The excellent agreement between experimental data on tin whiskers and the LAMH theory confirms that resistance in a superconducting wire below T_C results from thermally activated phase slips. However, there are still some unresolved issues. For example, since the free-energy barrier ΔF is proportional to the cross-sectional area of a wire, the superconducting transition width in a thinner wire will be broader. It is not clear if the LAMH theory, based on the GL equation that is only valid close to T_C, is still applicable. Also, it is speculated phase slips can occur via quantum tunneling, instead of thermal activation. All of these issues become more relevant as the advancement in lithography techniques has made fabrication of much thinner wires possible. In particular, some of the recent experiments on 1D superconducting wires fueled the controversy whether quantum phase slips can be observed experimentally.

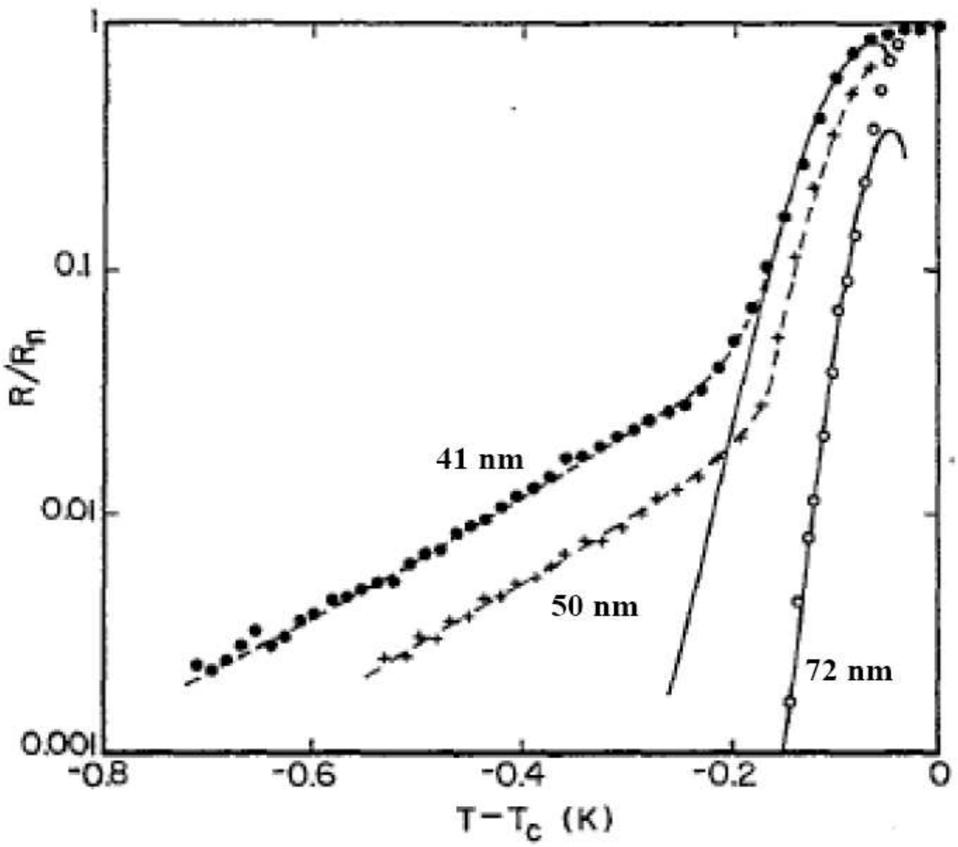

***Figure 5**: Normalized resistance as a function of T-T_C for thin indium wires with different diameters(Giordano, 1990). The solid lines are fits using LAMH theory. The dashed line is a fit which also takes quantum tunneling of phase slips into account.*

One of these experiments was done by Giordano (Giordano, 1990, Giordano, 1994) on a number of thin In and PbIn wires fabricated using step-edge lithography. In relative thick wires, which have diameters ~ 75 nm, the resistance below T_C can be satisfactorily described by the LAMH theory. However, for thinner wires about 40nm in diameter, he observed a cross-over from the LAMH behavior near T_C to a more weakly temperature dependent resistance tail, as shown in Figure 2. 3. In the thinnest wire (~ 16nm in diameter) measured, R/R_N saturated at about 0.2 at low temperatures. Giordano attributed such behaviors to phase slips which occur via microscopic quantum tunneling (MQT), instead of thermal activation, through the energy barrier. However, these results are controversial, because such cross-over has only been observed in a couple of samples, and these wires may be granular, especially since some of the wires are exposed to air for a few days to obtain smaller effective diameters.

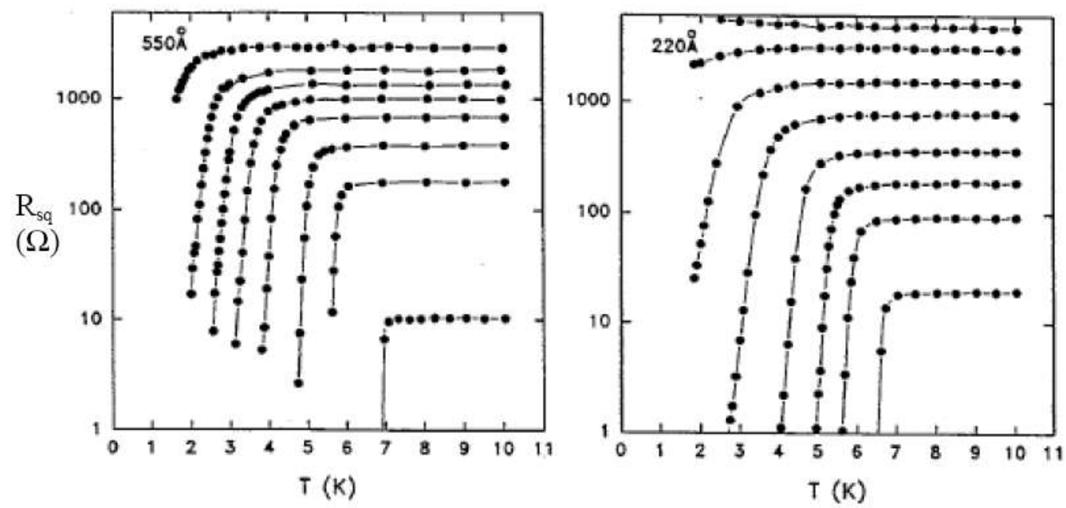

Figure 6. *Resistive transitions for 2 sets of Pb wires with different widths (550 Å and 220 Å, respectively). The different curves within each graph are measurements done at successive thicknesses as more metals are deposited. The lines are guides to the eyes. Sharifi et al(1990).*

In one of the more recent experiments, Lau(Lau *et al,*2001) has measured the resistance vs temperature of nanowires with nominal widths ranging from 10 to 22 nm and lengths from 100 nm to 1mm. With decreasing cross-sectional areas, the wires display increasingly broad resistive transitions. The data are in very good agreement with a model that includes both thermally activated phase slips close to *Tc* and quantum phase slips (QPS) at low temperatures, but disagree with an earlier model based on a critical value of R_N /Rq. Their measurements provide strong evidence for QPS in thin superconducting wires.

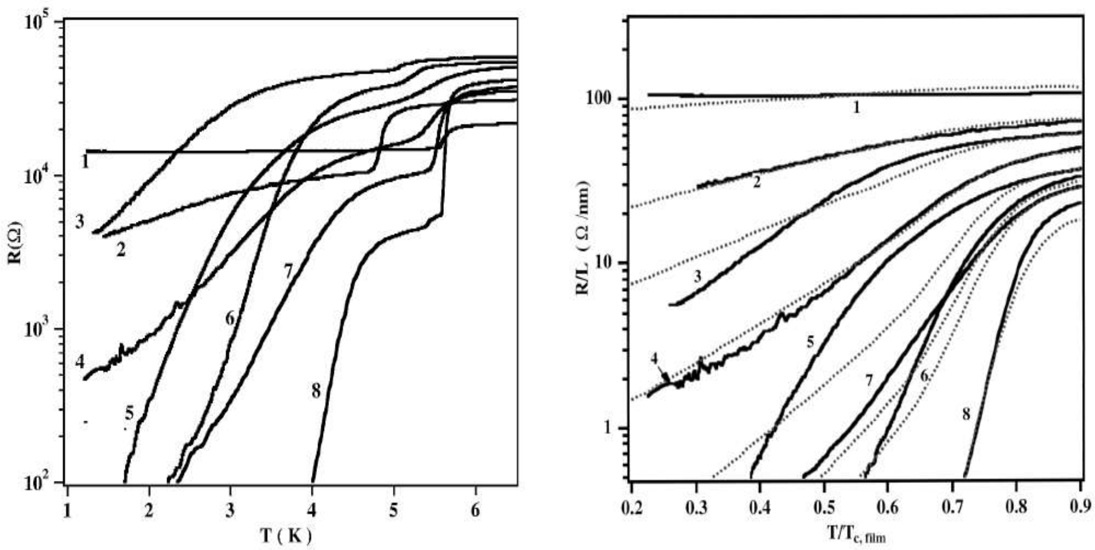

***Figure** 7.First graph shows Resistances as a function of temperature for different samples. In the second graph shows data for measured resistance per unit length normalized temperatures. The dotted lines are curves calculated using Eq. (7) and sample parameters.*

MQT causes phase slips even as $T \rightarrow 0$ and results in experimentally measurable resistance at all temperatures for sufficiently narrow wires. Therefore the total resistance in the superconducting channel will be $R_{\mathrm{LAMH}} + R_{\mathrm{MQT}}$. However, unless this is small compared to R_N, current carried by the parallel normal channel will significantly reduce the observed resistance. To take account of this in a simple way, they take the total resistance predicted by their model to be,

$$R = [R_N^{-1} + (R_{\mathrm{LAMH}} + R_{\mathrm{MQT}})^{-1}]^{-1} \tag{8}$$

Conclusion:

The most technically challenging aspect of the experimental study of nanowires has been the difficulty of fabricating homogeneous and ultrathin wires with conventional techniques. With recent advancement in lithographic technique and using carbon nanotubes or DNA as templates, nanowires with thickness down to 9nm diameter have been fabricated. The wires display resistive superconducting transitions, the widths of which increased with decreasing cross-sectional areas. Such resistive transitions can be explained by a combination of the activation of phase slips via thermal excitation close to T_C and macroscopic quantum tunneling at lower temperatures. Even though, thermally activated phase slips are well understood theoretically and experimentally, macroscopic quantum tunneling still remains controversial. Different experimental data seem to fit different theoretical models for MQT. In future, a lot of work needs to be done in the regime where MQT dominates over TAPS.

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