Emergent Behavior in Automobile Traffic
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Automobile traffic is controlled on the local scale by driver desires and interactions. This complex system can exhibit a phase transition by spontaneously changing from a quickly moving freeway into a trickling traffic jam seemingly out of nowhere. Statistical mechanical and Cellular Automata models taking into account local interactions are able to account for this emergent behavior.
Introduction

As soon as automobiles became a commodity governments became interested in the study of traffic. Traffic jams, both on freeways and on city streets, have become a major source of headaches for governments and citizens alike. It is a problem that is only getting worse with time. In the US traffic on federal highways and interstates increased 38% during the 1990’s while road capacity only increased 8%. In Atlanta, GA daily commutes have increased by 20% in the same time period. In Britain a recent study estimates that traffic costs the nation 3 billion pounds annually. By studying the dynamics of traffic flow scientists hope to be able to suggest methods for reducing the occurrence of costly traffic jams.

The study of traffic is also of intellectual interest to physicists. In recent years models making use of the principles of statistical mechanics and fluid mechanics have had success in modeling traffic flow. Similar concepts and techniques can be applied to other self governing systems such as crowds of pedestrians and behavior of herding animals.

This paper is written with the intentions of describing several basic properties of traffic flow and the methods used to study them. Phenomena such as phantom traffic jams, synchronized flow and stop and go waves. Phantom traffic jams seemingly come out of nowhere. Smooth flowing traffic will suddenly congeal for no apparent reason such as an accident or a matter of curiosity that was difficult to ignore. I have always been interested in this emergent phenomenon myself and was curious to see if anyone could prove my theory that they were caused by some idiot weaving in and out of congested traffic. For most models transitions from free flow to congestion can be shown to be caused by the formation of a metastable state at high vehicle density that is unstable to certain kinds of fluctuations.

In the Methods sections I will be discussing the basic concepts and methods for collecting data as well as give a brief review of a couple of models used to simulate traffic flow. In the results section I will first discuss the basic observed phenomenon of free flowing traffic and three states of congested flow that it may transition to. I will then discuss a model based on a stochastic differential equation and one based on Cellular Automata and how they are able to reproduce some aspects of this behavior.

Methods

A. Measurements

The most important aspect of the study of traffic is the collection of data. Like any other experimental observation there are different methods for doing so, each with its advantages and drawbacks. Obviously the best data are obtained by detailed aerial surveillance; however this is far too costly to be effective. A somewhat scary method of data collection is called car following and consists of nothing more than a car equipped with detectors measuring other drivers’ behavior as it follows them[1, 3].

The most common method of data acquisition is by means of detectors placed along roads. Single induction loop detectors can measure the number of crossing
vehicles, \( \Delta N \), which cross the measured cross section in the time interval \( \Delta T \) as well as the transit time for each vehicle, \( \alpha \), \( t_{1,\alpha} - t_{0,\alpha} \) and the vehicles length \( l_\alpha \). From these measurements the following quantities are then determined

\[
Q(\mathbf{x}, t) = \frac{\Delta N}{\Delta T}
\]

**Vehicle flow:**

**Time headway** (brutto time separations) \( t_{0,\alpha} - t_{0,\alpha-1} \)

**Average Velocity** \( V(x,t) = \langle v_\alpha \rangle \)

**Vehicle density**

\[
\rho(\mathbf{x}, t) = \frac{Q(\mathbf{x}, t)}{V(\mathbf{x}, t)}
\]

**Headway** (brutto distance) \( d = v_\alpha \Delta t_\alpha \)

**Clearance** (netto distance) \( s = d_\alpha - l_{\alpha-1} \)

There is one major concern with this method of measuring the average velocity is not the same if computed temporally or spatially. This is because fast vehicles cross the measured section of the freeway more frequently than slow ones due to they skew the averages. If the density is instead computed with the harmonic average velocity \( \langle 1/v_\alpha \rangle \) better results can be obtained, but they are more sensitive to errors for slow vehicles. However both methods produce similarly shaped plots for velocity vs. density and the harmonic average can be used for much higher densities. [1]

**B. Stochastic Self Driven Models**

The dynamics of self driven many-particle systems can be modeled using the following stochastic equation:

\[
\frac{d\mathbf{v}_\alpha(t)}{dt} = \frac{\mathbf{v}_\alpha^0(t)\mathbf{e}_\alpha^0(t) + \xi_\alpha(t) - \mathbf{v}_\alpha(t)}{\tau_\alpha} + \sum_{\beta \neq \alpha} \mathbf{f}_{\alpha\beta}(t).
\]

Here the term \( \mathbf{v}_\alpha \mathbf{e}_\alpha \) is the desired velocity of the particle, \( \xi_\alpha \) is a random fluctuation and \( -\mathbf{v} \) is the dissipative force, \( \tau \) is just a scaling factor and the last term is just the interparticle forces. This is basically the equation describing Brownian motion with interparticle forces with the addition of the self driven term. The driver tries to achieve the desired state while subject to external forces, random “collisions” and external constraints.

All of the techniques used in the study of diffusion processes immediately translates, however much care needs to be taken in modeling the “forces.” A typical stochastic traffic flow model will use a similar equation and try to reproduce measurable effects in computer simulations. Forces are chosen on a model by model basis and then the system is studied to see how fluctuations lead to breakdown. Different models give different results for the causes of such breakdowns. One model may show that a phantom jam can result from a lane change whereas another will show no such effect for the same density and flow. This is a common theme among all traffic models. No model can rule anything out, rather they can show that when certain things are taken into account certain situations tend to lead to certain outcomes. [1,9]
One variation of this model is using desired time headway to account for the self-driven aspect. In this model it is the driver's desire to keep a safe distance that motivates his/her behavior. So this model is a hybrid of a driven stochastic model and a car following one. While this seems reasonable, it is rather hard to model accurately. Different drivers have different ideas of what is safe and have different levels of ability at maintaining that distance. This behavior must be accounted for in the stochastic term so accurate data is very important to aid in the design of the model. Unfortunately for many of the finer features of traffic flow there simply isn’t enough data to draw conclusions.[4]

C. Cellular Automata

Modeling multilane traffic is rather difficult to do with stochastic models. Drivers change lanes for various reasons and that needs to be accounted for. In many instances fast drivers drive in one lane and slow ones drive in others. Also there is certainly a need for different driving in situations where there are exits or entrances in the right or the left lanes. Because of this Cellular Automata models are more useful in understanding how behavior effects traffic flow. In these models time is discretized and each “cell” (car), or possibly just one subset of cells is updated once each cycle. The evolution of the system is then governed by a series of rules for how to evolve each cell. [1,5,8]

The main problem with this approach is the lack of any quick reaction to other cars because many times they are not even updated at every time step in order to increase computational efficiency. Automata models also lack interaction detail because all that is included is drivers’ reaction based on rules not on what a driver near them just did. However even without that level of detail the results are generally good. The biggest advantage is in the ease of design of an automata model. Deciding on the rules is more intuitive then trying to sum them all up in the form of forces. It also seems like it would be easier to introduce really bad drivers every now and then. An interesting property of Automata models is that they must have some degree of randomness added to them in order to lead to any type of flow breakdown. Intuitively the randomness makes sense because drivers are people, but it is interesting that the rules alone do not create jams. The fact that lack of fluctuation fails to cause breakdown makes a strong case for the need to automate driving on freeways.

D. Other Models

There are over 100 models for traffic flow that have been suggested by people from many different fields of research. There are models based on physical principles of kinetic theory, fluid mechanics, mean field theory and even field theoretical models. They all have advantages and drawbacks. Also there are models based on car following theory. In these models driver behavior is controlled by what the driver in front of them is doing.

Of course there are many models that have a little of each type of consideration in them. It is the sheer number of approaches that reveals the richness of the field and it is interesting to see how different considerations on the microscopic behavior lead to different behavior on the macroscopic level.
Results and Discussion

One important feature that is determined from the data is known as the “fundamental diagram.” The fundamental diagram is an empirical fit of the relationship $Q=\rho V$. Measurements seem to confirm several common features exhibited in traffic flow. At low densities the velocity-density relation is constant at the average free velocity, but then velocity decreases with increasing density. The velocity decreases monotonically and then hits zero at the jam density. The flow rate has one max at intermediate densities which is presumably due to a balance between driver desire and physical constraints.

The most striking feature is the lambda-like appearance of the fundamental diagram. (fig 1) There are two branches of the diagram one representing free flowing traffic and the other appearing more scattered, representing congestion. The tip represents a critical density up to which there exists a metastable state of high flowing traffic for densities greater than some $\rho_{c1}$. These states break down into one of several types of congested states in a probabilistic manner. (fig 2)[1]

Figure 1. Traffic Flow plotted as a function of density (a) and as a function of time. The plots show two instances of flow rate exhibiting an abrupt decay.
Figure 2. Breakdown probability as a function of flow for different waiting times. Above \( Q_{c2} \) there is no capacity to maintain flow\[1\]

\[\text{Figure 3. Data taken by Treiter et al in 1974 of the emergence of phantom traffic jam. Broken lines are lane changes. The line density is an indication of vehicle density and the slope is the car's velocity. The abrupt change in velocity signifies the onset of the jam. The jam front propagates backward with constant velocity [1]}\]

Congested traffic can become jammed at or near the critical density. The phenomenon of a “phantom traffic jam” was first observed from aerial photography by Treiter in 1966. These jams form without the presence of any obvious cause such as an accident or a bottleneck. Figure 3 shows the position vs. time graph for many vehicles.
The most striking feature is the abrupt change in velocity of vehicles. This is the onset of the jam and it propagates backward with a constant velocity.

There have been studies to try to determine what causes a phantom jam. Daganzo was able to show that a “lane change in front of a highly compressed set of cars” can lead to phantom jam formation. However other studies have show that perturbations akin to lane changes don’t necessarily propagate like phantom jams do. A more complete model needs to be used in order to get a definitive answer, though it does seem feasible that poor driving could, under the right conditions, lead to the formation of a wide moving jam.

Phantom, or wide moving jams are not the most common form of jams. The most common type of jam is one that is spread out over space and usually lasts for an extended period of time. These jams are also known as synchronized flow and are usually caused by a capacity drop.

Another phase of congested traffic is the incredibly frustrating stop and go wave. In this state cars speed up a little only to come to a stop again. Studies have determined that there is no fundamental frequency of these waves and that the duration of one period can be anywhere between 4 and 20 minutes. All indications are that these are nonlinear waves.\[1\]

Models have been successful in reproducing the observed behavior, but as of yet there is no one universal model. Interpretation of results are also a topic of debate. It appears as though different microscopic considerations result in different macroscopic phenomenon when different types of fluctuations arise. It is because of this feature that the existence of so many models may be very advantages to understanding the system as a whole. Below I will briefly discuss two models and the considerations they take into account.

Wagner used a stochastic, car following, model in terms of the time headway, T. He simulated the following equation for a German highway:

$$\dot{T} = \alpha(m_T(v) - T) + D(v)T\xi,$$

Alpha is just the inverse of the time that T decays to the desired brutto time separation of \(m_T\), \(\xi\) is Gaussian white noise and \(D(v)\) is its coupling strength. The desired headway for a driver is chosen from \(p(T)\) which is the probability distribution of headways. This distribution is actually a measured average as it is hard to measure the distribution of desired headway. Whether or not that even makes a difference is up in the air as so many other details are usually left up to the stochastic term to account for.
The results are similar to the measured ones. His model was tested for other features that are beyond the scope of this paper. The model had some short comings and did not even produce stable jams. However he did test for the existence of phase transitions under different boundary conditions. What he found was that the distribution of the headway times, $T$, underwent a transition in both closed and open systems as the density was increased. So like many other models when simple microscopic interactions are taken into account phase transitions occur that correspond to observable behavior. But also like many other models different inclusions or parameters lead to different results for similar conditions.[2]

The model employed by Nagel et al uses the two criteria for deciding if a car will change lanes. The driver must decide if it is safe to change lanes by measuring the room he or she has to make a lane change. The driver then changes lanes if the velocity in the lane they are in is less than their desired velocity and they see that they can move to the other lane to increase their speed. Each car is updated in its own lane by increasing its speed if it can and it is less than the desired speed. Or decrease speed if it is going to fast. Also there is a random speed decrease performed with probability $p$ to give the system more realism. On even time steps the left lane is updated based on the rules and on odd ones the right one is.[5]
Figure 5. Top Flow vs. Density. Bottom Velocity vs. Density[5]

Figure 6) X vs t plots for Nagel’s Automata rules for both the left and right lanes.

The results of Nagel’s simulations are in very good agreement with what is experimentally observed. The critical density appears to be very close to that shown in figure 1. There are corrections to his model that he implemented that give almost identical numbers to those in figure 1. The velocity vs. density relationship also exhibits the plateau then decreasing behavior. There were a couple of peculiarities that developed with this model. Basically every car wanted to get in the left lane and go fast but slow
cars that wanted to make the lane change couldn’t do it. Despite some unintended effects that were later partially compensated for Nagel was also able to observer the formation of a jam front with the model (figure 6). In addition they also incorporated slower moving trucks in their models to study the effects of the different speed limits that they are subject to in some European countries. It would be nice to determine whether such a law is harmful or helpful to traffic flow.

**Conclusion**

Traffic flow is a very complex system that is rich with interesting phenomena. Drivers interact with each other on a “microscopic” level and these interactions can lead to “macroscopic” phenomena such as phantom jams or stop and go waves. A free flowing traffic system can undergo a transition into one of several states of congested or jammed flow if the density or flow becomes too high. There reasons for making a transition into one state over another are not very well understood other than obvious bottleneck effects.

The system exhibits a universal feature, namely interactions or rules on a small scale lead to some type of order or, pattern formed, on a larger scale under the right conditions. In physics this idea has been known for a long time in terms of phase transitions. However only recently has it begun to be understood at a fundamental level. The phenomena of super conductivity, water changing to ice and traffic forming jams are all fundamentally related. Being essentially the same problem they can be studied using the same methods in order to fully understand them. Microscopic considerations are used to create a model that tries to incorporate all of the important interactions and details. When the models are tested they can then be analyzed for their effectiveness at describing the real world system.

It may seem like the approaches to modeling the problem are too many and the results are too varied, but they all shed some light on the paths for a dense flow of automobiles to become one of the many congested states of frustration. The great number of ideas that have been proved to be useful is a testament to the universality of order arising from rules under the right condition. Traffic and similar complex systems offer a great chance for physicists to take their tools out into other areas of interest and hopefully bring the ideas that develop back to tackle some of the unsolved problems in physics.

**References**


[8, ®] Steven Wolfram because he would probably have me sued for not mentioning his name in a paper that uses the term cellular automata.