Condensates in Neutron Star Interiors

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Abstract

The pulsars, now identified as neutron stars, are one of the most fascinating astronomical objects studied so far. The detailed observation of various pulsars over the past 50 years presents clues to the understanding of its internal structure, esp the study of cooling rates and ‘glitches’. Theoretically, various kinds of condensates are expected in the interior of neutron stars, including neutrons in a superfluid state, protons in a superconducting state and other exotic states like pion and kaon condensates. This essay focuses on various emergent states in the interior of a neutron stars in presence of a strongly interacting fermion system and their effects on the dynamics of the neutron star.
1 Introduction

On July 4th 1054, a supernova went off in the constellation of Taurus as observed widely by astronomers on Earth (and termed as nova stella, a new star). Centuries later, in 1731, John Bevis discovered the supernova remnant, which was later christened the Crab nebula (or M1, the first entry in the Messier catalogue). Finally, in 1968, a radio wave source was discovered in the nebula which pulsates with a period of 33 ms. The source was characterized as a compact object, less than 30 km in diameter and rotating at an incredible rate of 30 revolutions per second! [1, 2]

By now around 350 [1] such objects, known as pulsars, have been observed, with periods varying from a few milliseconds to a few seconds. Past 40 years of observations of these pulsating radio sources have revealed many interesting details about these objects. From the timing of the pulsation, it’s possible to keep track of the rotation, and by looking at the data over a long duration, one could see the gradual slowing down of rotation. Breaking that monotonic decline are the mysterious glitches, a sudden speed-up in the rotation speed of the neutron star, which relaxes afterwards to the slow declining trend. These glitches, alongside the temperature data from the observed radiation, provide significant clues for study of the physics of pulsars.

On the theoretical side, Thomas Gold, and later Baade and Zwicky identified these compact objects with neutron stars, the notion of which was floating around since the discovery of neutron in 1932. Back in 1933, Lev Landau and others had proposed [3] that if a star is massive enough, the electron degeneracy pressure would not be enough to counter the gravitational pressure and hence the star would collapse until the gravitational pressure is balanced by a degenerate neutron pressure.

Essentially the study of neutron star interiors is the study of behavior of material at extremely high densities, at times even exceeding the nuclear density ($3 \times 10^{14}$ g/cm³). At such densities, the system is a strongly interacting neutrons and protons liquid(fermions), where the interactions can lead to the emergence of various condensates. If such condensates exist in a neutron star core, they will affect the large scale properties of the star, and hence one could hope to look for evidence of such condensates in the neutron star interiors. [4–7]

In the rest of this essay we discuss the emergence of various condensates in the high density matter inside a neutron star and the effects of the presence of those condensates on the observables of the neutron star. The rest of this essay is organized as follows: §2 describes the observable phenomenon and known astrophysics of pulsars. §3 discusses the physics of nucleonic matter at such densities as expected in the neutron star interiors, their equation of state, interactions and condensate states. §4 and §5 discuss the observable effects of a condensate on the rotation rate and cooling rate, respectively. §6 presents an interesting perspective on the comparison of neutron star interiors and terrestrial sub-Kelvin superfluids, which leads to a novel way to simulate the properties of a neutron matter condensate in laboratory.
2 Pulsars

The pulsars are characterized by the emission of periodic pulses, generally in the radio waves or X-ray regime of the electromagnetic spectrum. The periods of known pulsars are quite stable and lie between 1.558 ms (PSR 1937+214) and 4.308 s (PSR 1845-19), with the accuracy of timekeeping at times rivalling the accuracy of atomic clocks [1]. Some of the properties as understood from astrophysical considerations are as follows:

2.1 Mass, radius and internal structure

The basic concern for calculating the limiting mass of a neutron star is same as that for any astrophysical system: hydrostatic equilibrium, which in this case means balance of gravitational pressure (trying to collapse the system) and the neutron degeneracy pressure (trying to keep it from collapsing). The first such calculation by Tolman, Oppenheimer and Volkoff, the so called \textit{TOV limit}, was calculated as 0.7 M\(_\odot\) (where M\(_\odot\)= 2 \times 10^{30} \text{ kg} \) is the solar mass). That value is somewhat troubling as it is smaller than the Chandrashekhar limit of 1.4 M\(_\odot\), the limiting mass of a white dwarf supported by electron degeneracy pressure. Troubling because one expects a neutron star to form only when the mass exceeds Chandrashekhar limit and hence the gravitational pressure can overcome the electron degeneracy pressure. Better models for the nucleon-nucleon interaction and hence the equation of state (leading to stiffer E.O.S) set the mass limit to around 2.7 M\(_\odot\), although the actual equation of state is still an open problem. Indeed, in practice, most of the known pulsars’ mass range in 1 - 2 M\(_\odot\). The typical radii are of the order of 10 km, leading to a density of the order of nuclear density [5].

The structure of a neutron star is composed of an outer solid crust, \(\sim1 \text{ km}\) in thickness, followed by an outer core and an inner core. The crust mostly consists of a lattice of bare nuclei, immersed in a degenerate electron gas. As one moves inwards, the nuclei become increasingly neutron rich, until nuclear densities are reached and the nuclei and surrounding neutrons merge into one uniform liquid composed mostly of neutrons with a small concentration of protons and electrons. In the inner core, for densities exceeding that of a nucleus, there are several exotic possibilities, involving existence of pions, kaons, hyperons and even, in extreme densities, quark matter [4,5].

It is generally believed that pulsars have large magnetic field of the order of \(10^{12} - 10^{14} \text{ G}\). This is due to \textit{flux freezing} during the collapse of the main sequence star, most of which have a surface magnetic field of 100 G. The magnetic field plays an important role in the emission of a highly directional beam that we observe, but the actual mechanism of emission is still controversial.
2.2 Rotation and glitches

The pulsars are the fastest rotating objects observed so far. The high rate of rotation is because of the conservation of angular momentum. When a slowly rotating star (the period for Sun is 25 days) collapses to form a neutron star, there is no way for it to lose its angular momentum while it contracts from a radius of about one million kilometers to a radius of \( \sim 10 \) kilometers. The collapse leads to an increase in period inversely proportional to the moment of inertia and hence radius squared. For instance, if sun were to collapse decreasing its radius by a factor of \( 10^4 \), the period will become \( 25 \text{ days} / 10^8 = 22.4 \text{ ms} \), which is in the expected order of magnitude.

The pulsars exhibit a slow but uniform increase in period of the order of \( \dot{P} = 10^{-15} \text{ s s}^{-1} \), indicating the slowing down of the rotation. This is observable in the timing data and indicates the decay with a time scale of a million to a billion years. The shorter period pulsars tend to have higher slow-down rates and hence small characteristic time of decay [1].

One of the most curious phenomenon associated with pulsars is glitches, the occasional speed up of the star by \( \Delta \Omega / \Omega \sim 10^{-6} - 10^{-8} \). Generally the rise of angular velocity is almost instantaneous, on the time scale of few hours or less, which relaxes afterward in a period of days to years. These glitches provide an indirect evidence for a superfluid core in the neutron stars (§4).

2.3 Cooling of neutron stars

A neutron star, when formed, has a temperature exceeding \( 10^{11} \) K, which would cool by emitting radiation via various emission processes, and the aim of studying the cooling is to be able to understand the cooling curve, a plot of temperature vs time, as a function of the parameters of the star, viz, mass, radius, period and magnetic field. In case of young pulsars (< \( 10^4 \) years), the thermal radiation is greatly obscured by nonthermal processes in the magnetosphere of the star, therefore the observations mostly rely on middle aged (\( 10^4 - 10^6 \) years) stars. The surface temperature detected in those cases is generally of the order of \( 10^6 \) K [8].

The basic mechanism for cooling of the interior of a neutron star is the urca processes, the simplest mechanisms for neutrino emission. The reactions are

\[
\begin{align*}
n & \rightarrow p + e^- + \bar{\nu} \\
p + e^- & \rightarrow n + \nu
\end{align*}
\]

where the emitted neutrino/antineutrino carries off a part of the energy, the neutron star being transparent to neutrinos. The rate of emission by these processes strongly depends on the concentration of neutrons and protons in the core as well as their state [8]. Hence, study of cooling curves provide indirect evidence for the presence of condensates in neutron star cores (§5).
3 Neutron star interiors: High density matter

The physics of neutron star interiors is essentially the physics of strongly interacting baryons at high density of the order of nuclear density $\rho_0$ (small interparticle separation) and low temperatures. The temperature in the inner core can be as high as $10^9$ K, which corresponds to an energy of $\sim 0.1$ MeV, much smaller than the nuclear excitation energies which are at least a few MeVs.

3.1 Journey inbound

As one moves to the interior of a neutron star, the density increases and the electron Fermi energy rises. For low densities ($\rho \sim 10^7$ g/cm$^3$), the ground state of matter is a $^{56}$Fe lattice (probably bcc) surrounded by a relativistic electron gas [7]. As density increases beyond $8 \times 10^6$ g/cm$^3$, the electrons at the top of the Fermi sea begin to be captured by the nuclei [4], leading to an excess of neutrons via the inverse beta decay $p + e^- \rightarrow n + \nu_e$. Moving inwards, the matter becomes increasingly neutron rich, starting from $^{56}$Fe and going through a sequence of nuclei with a closed neutron shell (neutron number $N = 30, 50, 82$), eventually reaching $^{118}$Kr at the neutron drip point at density $\rho_{\text{drip}} = 4.3 \times 10^{11}$ g/cm$^3$.

Beyond the neutron drip point, the lattice begins to get permeated by a neutron gas alongside the electron gas. This is because if the nucleus is modelled as a potential well, the neutron start occupying the free continuum states lying spatially outside that potential well. The nuclei localized at lattice sites end up having too many neutrons with an atomic number of $Z = 40 - 50$. In a laboratory, these nuclei with a neutron excess will be unstable, but at such densities beta decays are forbidden as electrons are degenerate with a high Fermi energy, thereby blocking the formation of more electrons by beta decay due to Pauli’s exclusion principle. At densities $\sim \rho_0/3$, Coulomb forces make a spherical nucleus unstable. Due to the competition between Coulomb and surface energies, the nuclei become elongated and lead to unusual elongated structures, the so called pasta nuclei. Finally, at density $\sim \rho_0/2$, the nuclei dissolve to form a uniform liquid consisting of primarily neutrons alongside some 5% of protons and electrons and a few muons. [4,7]

Beyond the nuclear density $\rho_0$, the hadronic and quark degrees of freedom become important and the physics there is not well understood. Higher baryon resonances can form and are stabilized against beta decays, by reactions such as

$$n + e^- \rightarrow \Sigma^- + \nu_e$$
$$n + e^- \rightarrow \Delta^- + \nu_e$$

The mesons mediating nuclear interactions, esp pions, can also become dynamic degrees of freedom at such densities. Too little is known about the interaction of mesons and higher hadrons to conclude anything substantial about the behavior of matter at such densities, esp for densities exceeding $2\rho_0$. 

4
3.2 Equation of state

The biggest (and yet not completely resolved) question in the study of neutron star interiors is the equation of state of matter at such high densities. The basic problem is to model the nucleon-nucleon interaction and to study its effect in a many-body system, leading to an expression for the pressure $P$ as a function of the density $\rho$.

For densities up to $\rho_0$, the nucleon-nucleon interaction can be extracted from the p-p and p-n interaction experiments data with energies up to 300 MeV. The interactions are generally tensor in nature, with strong dependence on the spin, isospin and orbital angular momentum of nucleons. Most of these models are best fits, e.g. Paris, Urbana-v$_{14}$ (UV14) and Argonne-v$_{14}$ (AV14) two body potentials, where 14 refers to the different components included in the interactions. But generally E.O.S. with these two-body interactions are not stiff enough, i.e., don’t give a high enough pressure for a given density. To correct for that, the three body interactions need to be taken into account, which become repulsive and hence important at higher densities [7].

Beyond $\rho_0$, calculations based on these interactions are fundamentally limited as they ignore the hadronic and quark degrees of freedom, which become important at those densities. Firstly, the interactions of those modes is not well understood, as there are no experiments to observe those interactions. Even if one could assume some plausible forms of interactions, one can not continue to assume that the system can be defined in terms of well defined particles at the densities a few times $\rho_0$. Owing to these limitations, the equation of state for a neutron star core, esp in the deep interior, still remains an open problem [4].

3.3 Pairing in neutrons and protons

As the temperature drops below the superfluid transition temperature, the nucleons, both in the crust and in the core, is expected to form a condensate due to Cooper pairing. The neutrons and protons can pair up to give a superfluid and a superconducting state, respectively. The electrons, however, are too weakly interacting and hence remain normal.

The pairing, as it always is the case in superconductivity, comes from a weak attractive interaction. The nucleon-nucleon interactions are attractive at large separations, but are strongly repulsive as one decreases the separation under $\sim$ 0.5 fm. At low densities, $\rho < \rho_0/10$, the neutrons experience mainly the attractive part of this interaction, hence the pairing is in a symmetric $^1S_0$ state [9]. This state is the spherically symmetric singlet state (opposite spin pairing) and has a central maxima in wavefunction. As the proton density is always low, even in the inner core, the proton pairing is always expected to be in the $^1S_0$ channel, forming a s-wave superconductor. As the density increases, the repulsive core of the interactions becomes important, suppressing the neutron pairing in the $^1S_0$ channel. But at such densities,
the neutron pairing can occur in a triplet $^3P_2$ channel. This state has a nonzero angular momentum and is hence rotationally asymmetric.

The pairing gaps can be calculated using the BCS gap equation assuming the form of the attractive interaction potential and hence the gap to be dependent on the wavevector $k$ (as opposed to being constant for the usual BCS calculation). The anisotropic version of gap equation is

$$\Delta(k) = -\sum_{k'} \langle k|V|k' \rangle \frac{\Delta(k')}{2E(k')} \quad E^2(k) = \epsilon^2(k) + |\Delta(k)|^2$$

which can be simplified by expanding $\Delta(k)$ into spherical harmonics and writing an equation for each mode. Each of those modes correspond to one of the pairing channels, the important ones, as it turns out, being $^1S_0$ and $^3P_2$. (The superscript here represents the total angular momentum quantum number $L^2$ and the subscript represents the orbital part $L_z$.) The presence of tensor forces leads to one $\Delta(k)$ for each component, hence a matrix equation as opposed to a single component equation.

The calculations for pairing gap have been carried out analytically and numerically for many two-body-effective potentials. The pairing gap for $^1S_0$ channel is obtained, for most two-body interactions, to peak around 3 MeV at a Fermi momentum close to $k_F \approx 0.8$ fm$^{-1}$, but the average value is under 1 MeV. For $^3P_2$ channel, there is a significant disagreement among different models for two-body interactions, but the peak is around 1 MeV for most calculations at low densities and the average is $\sim 0.1$ MeV [9]. At higher densities (higher Fermi wavevector), the disagreements get worse, indicating that we still don’t have the correct model for the physics involved.

These results obtained under BCS theory are not entirely satisfactory, as the BCS approach essentially amounts to a mean field calculation. Hence, BCS completely ignores fluctuations due to strong interactions, esp the three-body and higher interactions, which play an important role in the core of neutron stars, esp at high densities [10]. Approximate calculations show that the screening between nucleons and other polarization effects due to many-body interactions lead to a substantial reduction in the pairing in $^1S_0$ channel, but the verdict is far from being final.

### 3.4 Pion condensation

The meson fields, esp the pions which mediate the interaction between nucleons, can condense into a macroscopic state at high densities. This proceeds via a macroscopic excitation of pion or kaon field when the density is high and hence the nucleons are interacting strongly, leading to a nonzero expectation value $\langle \pi \rangle$ of the pion field. A charged pion condensate can form by rotation of neutrons in the isospin space

$$|N'\rangle = \cos \theta |n\rangle + \sin \theta |p\rangle$$

$$|P'\rangle = \cos \theta |p\rangle - \sin \theta |n\rangle$$
where the condensation angle increases as the density increases above the critical density for condensation, $\rho_\pi$.

These quasiparticles are the collective modes of the system, and the pion condensation is essentially an emergent phenomenon as the density is increased [5]. The variation in charge by this mixing is compensated by a charged pion field, modulated spatially, which forms the condensate. The condensate is a broken symmetry state with a complex order parameter $\langle \pi \rangle$, hence it is superconducting. One could compare this to normal superconductors formed by Cooper pairing of electrons of opposite momenta and spin; in that picture, the pion condensate is a pairing of neutrons with proton-holes and protons with neutron-holes.

A crude estimate for the critical density for condensation gives $\rho_\pi = 2\rho_0$, but this is extremely sensitive to the strength of nucleon-nucleon repulsion, as the states $|N'\rangle$ and $|P'\rangle$ are bound states of a neutron and a proton-hole or vice versa. Another possibility is the formation of a condensate of kaons with a macroscopic expectation value of the charged K field via a similar mechanism [4].

### 3.5 Magnetic field and superconductors

The neutron stars have a surface magnetic field of the order of $10^{12}$ G. Since the critical field strength for the proton superconducting state is $H_c \sim 10^{15}$ G, one would expect it to expel the flux from the core, which will have dramatic effects on the magnetic field configurations. But the ohmic dissipation time scale for the current sustaining this field, $\tau_D = (4\pi\sigma/c^2)(R/\pi)^2$ where $\sigma$ is the conductivity and $R$ is the steller radius, turns out to be larger than the age of the universe for a neutron star, owing to conductivity 12 orders of magnitude larger than most metals. How is the flux expelled then? As it turns out, the magnetic field simply nucleates the superconducting state, leading to the formation of a state analogous to the type-II superconductors seen in laboratory [11].

### 4 Glitches

The presence of glitches in the timing data of pulsars provide the best evidence for a superfluid core in the neutron stars. If the core is superfluid, the rotation of the star will lead to formation of vortices with a quantized circulation

$$\oint \mathbf{v} \cdot d\mathbf{l} = \frac{h}{2m_n}$$

where $2m_n$ is the mass of the neutron pair. For a neutron star rotating at the observed angular velocities, the superfluid core will be densely permeated by such vortices, reaching a density of $2 \times 10^5$ cm$^{-2}$. Hence, on a macroscopic scale, the superfluid core can simply be assumed as a rotating rigid body with a classical moment of inertia.
The basic mechanism of glitches involves a discrepancy between the angular speeds of inner superfluid core and the outer crust. The speed up is expected to be triggered by a *starquake*, a cracking of the crust which decreases the moment of inertia of the crust instantaneously and hence leads to resharing of angular momentum and the observed speed up. The starquakes are plausible because as the star slows down, the centrifugal force at the equator decreases and hence the crust is under stress to acquire a less oblate (more spherical) equilibrium shape. At some point, the stress is enough to crack the crust and hence cause a starquake [1].

This mechanism can be modelled as a phenomenological 2-component model, with $I_n$ and $I_c$ being the moments of inertia of the neutral superfluid core and charged rest of the star. The period that we observe is the period of rotation of magnetic poles and hence that of the magnetic fields, and the charges (protons and electrons) are strongly coupled and hence corotating with the magnetic field. Hence, any variation in the period due to starquake, being in normal fluid, is rapidly communicated to the charged particles but the response of superfluid core is considerably slower due to weak frictional coupling between neutral and charged components.

After a starquake, the interaction between these two components is given by

$$I_c \dot{\Omega} = -\alpha - \frac{I_c(\Omega - \Omega_n)}{\tau_c} \quad I_n \dot{\Omega}_n = \frac{I_n(\Omega - \Omega_n)}{\tau_c}$$

where $\alpha$ is an external braking torque on the crust and $\tau_c$ is a characteristic decay time. Taking them to be constant over the time scale one’s interested in, the equations can be solved to given in terms of $I \equiv I_n + I_c$ and $\tau \equiv \tau_c I_n / I$ as

$$\Omega = -\frac{\alpha}{I} t + \frac{I_n}{I} \Omega_1 e^{-t/\tau} + \Omega_2 \quad \Omega_n = \Omega - \Omega_1 e^{-t/\tau} + \frac{\alpha \tau}{I_c}$$

where $\Omega_1$ and $\Omega_2$ are arbitrary constants. Here, as $t \to \infty$,

$$\Omega - \Omega_n = \frac{\alpha \tau}{I_c} = \frac{\alpha I_n}{I_c \tau_c} = \frac{I_n}{I_c} \frac{\tau_c}{T} \Omega, \quad \frac{1}{T} = -\frac{\dot{\Omega}}{\Omega} = \frac{\alpha}{I \Omega}$$

Hence, taking $I_c \sim I_n$ as a crude approximation, $T$ being observed from the slow decay rate of period and $\tau$ from the glitches, the final expression can be used to compute $(\Omega - \Omega_n)/\Omega \sim 10^{-5}$ for the Vela and the Crab pulsar. Now, using this as the boundary condition to the constants $\Omega_1$ and $\Omega_2$ to a pulsar data, one could test this model to see if the same fitting parameters are needed for various glitches. To the lowest order, the values are reasonably constant, lending credibility to the model. Hence, the data obtained so far supports the theoretical prediction that the star consists of 2 distinct components, and hence provides an indirect evidence for the existence of superfluids in neutrons star cores.

5 Cooling of Neutron stars

The cooling rate of a neutron star shows a strong dependence on the state of matter in the star interior. The cooling proceeds primarily via neutrino emissions, the rate
of which depends strongly on the low lying excitations of the state of matter. This is because for degenerate matter, only particles lying within $\sim k_B T$ of the Fermi level can participate in the reactions. The neutrino emissions processes can be classified in two types, fast and slow, as distinguished by their emissivities [8]

$$Q_{\text{fast}} = Q_f T_9^6, \quad Q_{\text{slow}} = Q_s T_9^8$$

where $Q_f$ and $Q_s$ are slowly varying functions of $\rho$, the density. The fast processes include Urca and other Urca-like processes involving hyperons. These emissions have density thresholds and are not expected to occur outside the inner core of the star. The slow processes include neutron bremsstrahlung and modified Urca process (Urca process requiring a spectator neutron for momentum conservation) and are expected to occur everywhere inside the core.

For a superfluid occurring due to Cooper pairing of fermions, one of the distinctive feature is the appearance of an energy gap at the Fermi level. This leads to the fermions close to the Fermi level being pushed away from it, and are hence rendered greatly inactive, as far as neutrino emissions are concerned. Thus, the primary effect of superfluidity is a strong (exponential) suppression of the fast Urca process.

But on the other hand, the Cooper pairing of neutrons initiates a new neutrino process [8], which is essentially annihilation of two quasi-baryons, the excitations of the system. For the normal fluid, the excitations are either particles or holes, and two of them can’t annihilate owing to conservation of particle number and a particle-hole annihilation is forbidden for kinematical reasons. However, for superfluid, the elementary excitations are a linear superposition of particle and hole states, hence two such entities can annihilate and emit neutrinos. This process peaks at a temperature of $\sim 0.8 T_c$ and is exponentially suppressed for temperatures lower than that. Even at its peak, the luminosity by this process cannot exceed the luminosity due to other processes not involving the superfluid. Hence, the presence of superfluidity decreases the luminosity.

Superfluidity also affects the specific heat of the star. Near the transition temperature, the specific heat shows a discontinuity typical of such transitions. Also, as the temperature decreases, the specific heat falls off as an exponential as opposed to a power law. This is because now a threshold energy $\Delta$ is required to break a Cooper pair. Hence, the net effect of superfluidity in the core is to decrease the cooling rate near the transition temperature but to increase it at lower temperatures [6].

The presence of pion condensation can significantly enhance the cooling rate of a neutron star [6]. This is because the nucleon eigenstates are now linear combination of proton and neutron states, which can decay into itself as opposed to a proton-to-neutron or a neutron-to-proton beta decays. For a state $N'$, we can have

$$N' + e^- \to N' + \nu_e$$
$$N' \to N' + e^- + \nu_e$$
The usual beta decays are forbidden because of momentum considerations: a neutron can decay into a proton only if the final state the proton is created in is free to begin with, owing to the Pauli’s exclusion principle. Hence, for a degenerate proton/neutron gas where all states up to the Fermi level are filled up, such beta decays are suppressed. For pion condensates, however, the excitations decay into themselves, hence, such processes can still proceed and contribute significantly to the cooling. The overall cooling curve is obtained by a complex interplay of all these processes, but one could still infer a lot from them, if one knows what to look for!

6 Comparison with terrestrial condensates

In the context of terrestrial phenomenon, the existence of condensates is essentially within the realm of low temperature physics, where low here refers to temperature of the order of a few Kelvins, achieved only by liquid Helium. Indeed, the entire class of high-$T_c$ superconductors is called so because of a transition temperature around 100 K, something that would be considered a low temperature for most commonplace applications! And in this sense, the superfluidity and superconductivity in neutron star cores would be at a very high temperature of $10^{10}$ K.

But for a physical system, it makes more sense to talk of the temperature scaled to a characteristic temperature (corresponding to a characteristic energy scale) of the system. When we take that viewpoint, with the characteristic temperature scale being the Fermi temperature, the picture looks something like the following [12]

<table>
<thead>
<tr>
<th>Condensates</th>
<th>$T_c$</th>
<th>$T_c/T_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conventional superconductors</td>
<td>5 K</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$^3$He</td>
<td>2.7 mK</td>
<td>$5 \times 10^{-4}$</td>
</tr>
<tr>
<td>High-$T_c$ superconductors</td>
<td>100 K</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Atomic Fermi gases</td>
<td>200 nK</td>
<td>0.2</td>
</tr>
<tr>
<td>Neutron matter</td>
<td>$10^{10}$ K</td>
<td>0.1</td>
</tr>
</tbody>
</table>

From this perspective, both neutron star matter and atomic Fermi gases can be considered high temperature condensates. This, coupled with the fact that it is possible to experimentally vary the interaction in the atomic Fermi gases using Feshbach resonance, make those system an interesting laboratory to simulate the physics of nuclear matter. The system can be driven into the so called unitarity regime, where it is both dilute and strongly interacting. In that regime, all length scales drop out of the problem and the system exhibits universal behavior, in the sense that the properties of the system depends only on the product of Fermi wavevector and scattering length. Quantum Monte Carlo simulations for nucleonic matter with
AV-14 interaction, performed by Gezerlis and Carlson [12], show that the E.O.S and pairing gap as a function of density are quite similar to that for the ultracold gas in the unitarity regime. Hence, ironically, these ultracold and ultra dilute systems may eventually turn out to be the best laboratories to simulate the behavior of matter at extremely high densities and temperatures!

7 Conclusion

Neutron stars present us with a fascinating system to study the behavior of matter at extreme densities. The presence of a strongly interacting fermion system leads to emergence of various condensates and emergent states. The really curious aspect is the fact that from the meager data we have, we can infer so much about the presence and behavior of such emergent states in the interior of the neutron star. One could only hope that following neutron stars from their birth in violent supernovae to their fading into oblivion would bring in more insights into their properties as well as a better understanding of matter at the smallest scales and extreme densities.

References


