

Physics 569 Term Paper

High Density Quark Matter and Color Superconductivity

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Abstract:

Due to the nature of attractive interaction in quantum chromodynamics (QCD), quark matter is expected to be formed a cooper-pair at high density and low temperature. Here I'll describe some possible quark pairing phases in high density quark matter and showing the correspondence superconductivity in BSC theory. Based on the current understanding of the color superconductivity (CSC) phase, we might have a clear view to understand the color superconductivity inside the neutron stars.

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1 Introduction

Quantum Chromodynamics (QCD) is the fundamental theory of study the strong interaction between quarks and gluons. Since quarks are color-charged fermions, QCD is a non-abelian gauge theory under $SU(3)$ gauge. It has well tested and agreed with many experimental evidence over 30 years. However the phase diagram for QCD matter has not been well studied yet. The phase of the quark matter is depended on the temperature T and the chemical potential μ . For high temperature region ($T \gg \mu$), hadrons break into quarks, anti-quarks and gluons. This is called quark-gluon plasma (QGP) phase which corresponding to the early Universe in microseconds after the big bang. This new state of matter can also be created in experiments in the Relativistic Heavy Ion Collider (RHIC).

Another interesting region is at low temperature and large chemical potential region ($T \ll \mu$). This is the phase for the compressed nuclear matter and might exist in the interior of neutron stars. The phase corresponds to the color superconductivity phase. For the matter in highest density, we expect it is color-flavor locked (CFL) phase. In CFL phase, the mass of the up, down and strange quark is zero and these quarks form the conventional zero momentum spinless Cooper pair. When the chemical potential μ decrease, other possible phases need to be considered. The detail descriptions will be discussed in the following section.

The phase diagram of quark matter is shown in figure 1. Large temperature corresponds to the quark-gluon plasma (QGP) phase and can present the quark matter phase in the early universe and heavy-ion collisions. For low temperature and large chemical potential region, it is color superconductivity phase and it can exist in the neutron stars.

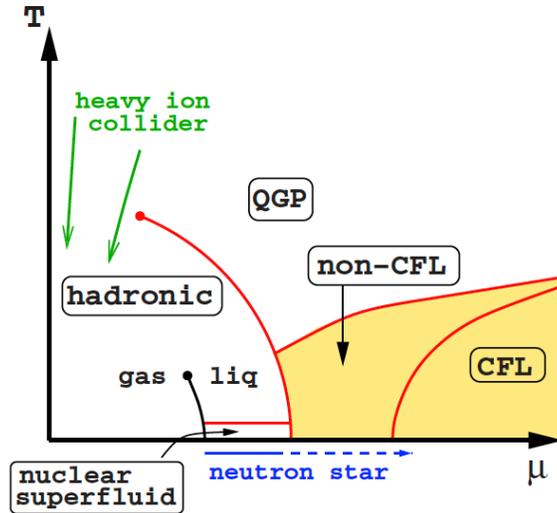


Figure 1: (from ref. [1]) Phase diagram for quark matter.

2 Color Superconductivity

Let's start at the baryons with typical size of about $1 \text{ fm} = 10^{-13} \text{ cm}$. At sufficient high density and low temperature, baryons overlap with each others and the quarks can move across the different neighboring baryons. The QCD interactions between the quarks become weak and are almost free near the fermi surface.

At zero temperature the free energy of quark matter can be written as $\Omega = E - \mu N$, where μ is the chemical potential, N is the number of quarks and E is the total energy of the system. If there is no interaction between quarks, to add or subtract a quark near fermi surface ($E_f = \mu$) is zero. However if we turn on the weak attractive interaction between quarks, the total energy will become lower as we adding or subtract particle pairs around fermi surface. These pairs of quarks form condensation and become the ground states of the system. This phenomenon is the same argument of superconductivity invented by Bardeen, Cooper, and Schrieffer (BSC).

In the superconductor, the attractive interaction between electrons is due to the phonons which is depended on the band structure of the metal and not easy to get the accurate calculation. In contrast, gluon plays an important role for the attractive interaction between quarks in QCD. The QCD Lagrangian is given by

$$\mathcal{L} = \bar{\psi}(\not{v}\gamma^\mu D_\mu + \hat{\mu}\gamma_0 - \hat{m})\psi - \frac{1}{4}G_a^{\mu\nu}G_{\mu\nu}^a \quad (1)$$

where ψ is the quark spinor in Dirac space (a $4N_c N_f$ component spinor) and $\bar{\psi} = \psi^\dagger \gamma_0$. $\hat{\mu}$ is the chemical potential and the quark mass $\hat{m} = \text{diag}(m_u, m_d, m_s)$. The covariant derivative is defined as $D_\mu = \partial_\mu + igT_a A_\mu^a$. Here g is the strong coupling constant, A_μ^a are the gauge field, and $T^a = \lambda^a/2$ are the generators of SU(3) gauge with the Gell-Mann matrices λ^a . The field tensor is defined as $G_a^{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$ and f^{abc} is the SU(3) structure constants.

Considering only one-gluon exchange under quark-quark scattering process [2]. Considering only one-gluon exchange term, the quark-quark scattering amplitude is proportional to the color tensor:

$$\sum_{\alpha}^{N_C^2-1} T_{aa'}^{\alpha} T_{bb'}^{\alpha} = -\frac{N_C+1}{4N_C}(\delta_{aa'}\delta_{bb'} - \delta_{ab'}\delta_{a'b}) + \frac{N_C-1}{4N_C}(\delta_{aa'}\delta_{bb'} + \delta_{ab'}\delta_{a'b}) \quad (2)$$

The attractive interaction comes from the first term (anti-triplet channel). This antisymmetric channel corresponds to the Cooper pair in dense QCD as shown in Fig. 2. Since the interaction is stronger than the conventional BSC superconductor, the superconductivity in QCD is more robust. This antisymmetric pair in quark matter breaks the local color symmetry SU(3). So this mechanism is called color superconductivity. Due to the symmetric breaking we can expect there are Goldstone bosons associated with this symmetry breaking pattern.

The quark pair condensation can be written by the anomalous self-energy.

$$\langle \psi_{ia}^\alpha C \gamma_5 \psi_{jb}^\beta \rangle \sim \epsilon^{\alpha\beta n} \epsilon_{\alpha\beta n} \quad (3)$$

where α, β are the color indices (blue, red, green), i, j are the flavor indices (up, down and strange) and a, b are the spin indices (up and down). This is antisymmetric both in color and flavor indices. Here we only discuss the most symmetric case, color-flavor locked phase (CFL) and two-flavor pairing phase (2SC).

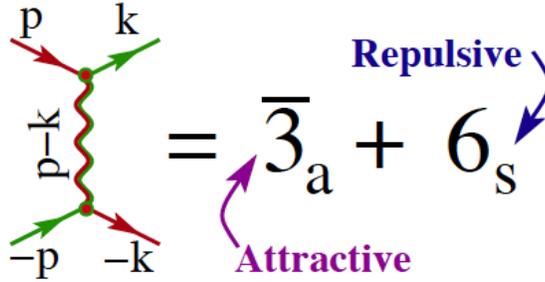


Figure 2: (from ref. [2]) The diagram of one-gluon interaction. The first term corresponds to the attractive interaction from antisymmetric antitriplet. The second term is the repulsive interaction from a symmetric sextet channel.

2.1 Color-flavor locked: CFL phase

For the ultra-high-density quark matter, the up, down, and strange quark behave similar. The most symmetric and most attractive pairs is the color-flavor lock phase. This phase can be form by quarks with all three colors and three flavors of zero-momentum spinless Cooper pairs. The quark-quark self energy is [3]

$$\langle \phi_i^\alpha C \gamma_5 \phi_j^\beta \rangle \propto \Delta_{CFL} \epsilon^{\alpha\beta A} \epsilon_{ijA} + \Delta_{CFL} \kappa (\delta_i^\alpha \delta_j^{beta} + \delta_j^\alpha \delta_i^\beta) \quad (4)$$

where α, β are the color indices and i, j are the flavor indices. Δ_{CFL} are the gap parameter.

Because of the color-flavor locking, this CFL phase breaks the chiral symmetry. That is $SU(3)_C \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_{C+R+L}$. It also breaks the global baryon number symmetry $U(1)_B$, and makes CFL phase a superfluid. Due to the above symmetry breaking, we can expect the massless Goldstone Boson.

As the density decreases, the mass difference between quarks increases. Two-flavor pairing (2SC) become the next phase below the CFL density.

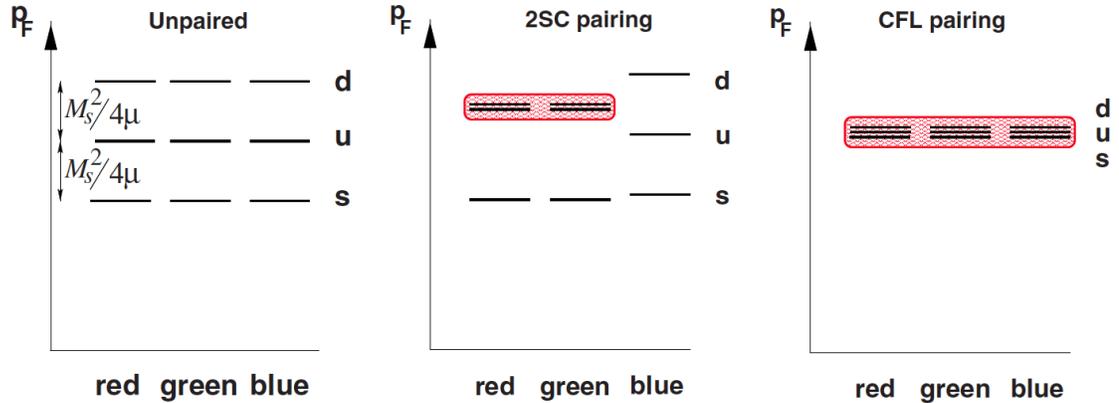


Figure 3: (from ref. [1]). Quarks with various colors and flavors split apart of the Fermi momentum. As the chemical potential decreases, the splittings increases. The CFL phase is all colors and flavors pair with the same Fermi momentum (Right panel). The 2SC phase is up and down quarks and two colors pair locking together with the same Fermi momentum (Middle panel).

2.2 Two-flavor pairing: 2SC phase

Below the CFL phase, there are only two out of three colors (red and green) and two out of three flavors (up and down) pairing. This is the 2SC phase

as shown in the middle panel in Fig. 4.

The quark-quark self energy in 2SC phase is

$$\langle \phi_i^\alpha C \gamma_5 \phi_j^\beta \rangle \propto \Delta_{2SC} \epsilon^{\alpha\beta 3} \epsilon_{ij3} \quad (5)$$

The symmetry breaking pattern is $SU(3)_C \rightarrow SU(2)_{rg}$. However the global symmetries are still maintained in 2SC phase. The baryon number is conserved. Hence there is no superfluid property in 2SC phase.

2.3 Gap equation

The easy way to study the quark pair in dense quark matter is by applying Nambu-Gorkov spinors

$$\Psi = \begin{pmatrix} \psi \\ \psi_C \end{pmatrix} \quad (6)$$

where Ψ_C is the charge conjugate spinor with the charge conjugate conjugate matrix $C = i\gamma^2\gamma^0$. Then the free propagator is

$$S_0^{-1} = \begin{pmatrix} [G_0^+]^{-1} & 0 \\ 0 & [G_0^-]^{-1} \end{pmatrix} \quad (7)$$

where $[G_0^\pm]^{-1} = -i(i\gamma^\mu\partial_\mu \pm \mu\gamma_0)$. As considering the diquark condensation, the propagator gets altered by the gluon and has off-diagonal term

$$S = \begin{pmatrix} G^+ & F^- \\ F^+ & G^- \end{pmatrix} \quad (8)$$

where the fermion propagators for quasiparticle and charge conjugate part are

$$G^\pm = \{[G_0^\pm]^{-1} + \Sigma^\pm - \phi^\mp([G_0^\mp]^{-1} + \Sigma^\mp)^{-1}\phi^\pm\}^{-1} \quad (9)$$

And the anomalous propagators are

$$F^\pm = -([G_0^\pm]^{-1} + \Sigma^\mp)^{-1}\psi^\pm G^\pm \quad (10)$$

where Σ is the self-energy presented diagrammatically in Fig. 3. By solving Eq. (6) and Eq. (7), we can get the gap equation

$$\phi^+(K) = g^2 \int_Q \gamma^\mu T_a^T F^+(Q) \gamma^\nu T_b D_{\mu\nu}^{ab}(K - Q) \quad (11)$$

in momentum space, and $D_{\mu\nu}^{ab}(K - Q)$ is the gluon propagator. This gap equation is similar to the gap equation of conventional BSC superconductor with the color and flavor indices.

Since the anomalous self-energy ψ^+ can be written with the corresponding gap functions Δ^\pm ,

$$\phi^+ = \sum_{e=\pm} \Delta^{(e)} \mathcal{M} \Lambda_k^{(e)} \quad (12)$$

where \mathcal{M} is a matrix in color, flavor, and Dirac space and it is a quark pair dependence parameter. And $\Lambda_k^{(e)} = (1 + e\gamma_0\gamma \cdot \hat{k})/2$ is projector operators to the positive ($e = +$) or negative ($e = -$) energy.

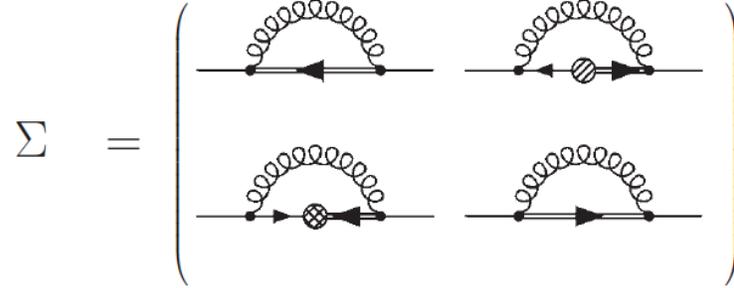


Figure 4: (from ref. [1]). The self energy from QCD in Nambu-Gorlov space. The curly lines correspond to the gluon propagator D . The quasiparticle propagators G^+ or (G^-) corresponds to the double lines with right (left) arrow.

2.4 Quasiparticle energy spectrum

What is the quasiparticle excitations spectrum in these 2 phases? From equation (12), the \mathcal{M} matrix can be written as

$$\mathcal{M}_{ij}^{\alpha\beta} = \psi_A^B \epsilon^{\alpha\beta A} \epsilon_{ijB} \gamma \quad (13)$$

And the \mathcal{M} matrix satisfies

$$\mathcal{M}\mathcal{M}^\dagger = \sum_r \lambda_r \mathcal{P}_r \quad (14)$$

The excitations spectrum is the pole for the full propagators, G and F from Eq. (8). From Eq. (9), (10) and (12), we can solve the poles from the propagators and hence the energy spectrum is

$$\epsilon_{k,r}^{(e)} = \sqrt{(ek - \mu)^2 + \lambda |\Delta^{(e)}|^2} \quad (15)$$

The spectrum is the similar form as the conventional BCS superconductor. All the parameters are labeled in Fig. 5.

Phase	ϕ_A^B	λ_1	λ_2	$(\mathcal{P}_1)_{\alpha\beta}^{ij}$	$(\mathcal{P}_2)_{\alpha\beta}^{ij}$
CFL	δ_A^B	4 (onefold)	1 (eightfold)	$\delta_\alpha^i \delta_\beta^j / 3$	$\delta_{\alpha\beta} \delta^{ij} - \delta_\alpha^i \delta_\beta^j / 3$
2SC	$\delta_{A3} \delta^{B3}$	1 (fourfold)	0 (fivefold)	$(\delta_{\alpha\beta} - \delta_{\alpha 3} \delta_{\beta 3})(\delta^{ij} - \delta^3 \delta^3)$	$\delta_{\alpha 3} \delta_{\beta 3} \delta^{i3} \delta^{j3}$

Figure 5: (from ref. [1]). The table of the parameters with respect to CPL phase and 2SC phase.

3 Color Superconductivity in Neutron Star

Neutron stars are the densest object in our universe. These compact stars may be the candidates of the quark matters. The typical mass of a neutron star is about the order of solar mass and the radius is about 10 km. The structure of the neutron stars is still not so clear. In recent model, the outer

crust of neutron stars is made of rigid lattice ions. The inner layer of the crust contains electrons, neutrons, nuclei. The next inner core should be the fluid consisting of neutrons and protons. Since the inner core has higher density, it may form by color superconducting quark matters.

What is the transition density from baryonic matters to quark matters? Based on the astrophysical observation, we believe the transition density is around $3n_0$, where $n_0 = 0.16$ nucleons/fm³ is the nuclear saturation density. As we discuss before, if there exist quark matters, the CFL phase with all quarks paired or other paired phases are all potentially possible color superconductivity phase in the core of neutron stars.

Based on the "strange quark matter hypothesis" [4] [5], the nuclear matter in the neutron stars is metastable at zero pressure. This means the ground state of the compact stars is the quark matter. This hypothesis points out some of the neutron stars we observed today may be the strange stars. In current study [6], the x-ray burst analysis shows that the strange stars have higher oscillation frequency than the neutron stars.

What is the signatures of superconductivity in neutron stars? We can consider the pairing energy first. The nuclear equation of state is affected by the pairing energy. But it is hard to detect. However, the gaps in the quasiparticles spectrum and the Goldstone bosons affects the transport properties, emissivity, heat capacity, viscosity and the conductivity of the neutron stars. By detect the gravitational waves and the cooling process by the neutrino emission, we may understand the colors superconductivity phases inside the neutron stars [1].

There are still many unanswered questions in this field. The detail profile of color superconducting quark matter of the neutron stars still has a long way to go. Examples are the effects of the crystalline phase, the vortices interac-

tions or the effect of high magnetic fields which are all the challenges for the particle physicists and astronomers. The ultimate goal is to have the fully understanding of the QCD phase diagram agreeing with both the observable data and theoretical predictions.

References

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