Vehicular Traffic: A Forefront Socio-Quantitative Complex System

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Abstract

We present the motivation for studying traffic systems from a physical perspective. We proceed to classify the myriad of theoretical approaches applied to the problem. Experimental methodology and observed phenomena are then presented. Next comparisons are made with relevant theoretical results. Finally, an attempt is made at evaluating the progress of theory in meeting its internally stated objectives.

1 Introduction

The study of vehicular traffic has strong ties with the field of non-equilibrium statistical physics. Most modern models of vehicular traffic treat individual vehicles as "particles" that strongly interact. Thus the general discipline is rightly described as "the study of interacting particles driven far from equilibrium." The nature of these interactions often departs widely from those found in the standard study of physical particles, yet successful mappings from the microscopic models to macroscopic phenomena carry the familiar form of the hydrodynamic equations. Tools familiar to theoretical physicists find ubiquitous application in the field: nonlinear ODEs, linear stability analysis, mean field theory, a form of Newtonian molecular dynamics simulation, cellular automata from a lattice formulation of the problem. A rich set of physical phenomena are involved in the dynamics of traffic flow: transitions between dynamical phases, phase segregation, criticality and self-organized criticality, metastability and hysteresis, etc. Indeed, the strong connection with more conventional physical systems has inspired the expansion of the field in recent years.

From a more practical perspective, understanding the fundamental laws governing traffic phenomena has the potential to benefit society at large. From the point of view of infrastructure, vehicular traffic constitutes the dominant method of travel and shipping, thus its optimization has direct economic benefits. As is typical in physics as a whole, there is a divide between theoretical investigation and practical application. In the case of vehicular traffic, the latter is the subject of the well established field of traffic engineering. Thus the questions pursued by the two camps are different, but often complementary [9]:

Physics:

- What are the dynamical phases of traffic?
- What is the nature of the fluctuations around steady states?
- How can the dynamical approach to steady state be described?
- What is the affect of quenched disorder (e.g. bottlenecks)?

Engineering:

- What is the relationship between traffic density and flux?
- What are the distributions of distance and time headways?
- What is the optimal placement and design of on and off ramps?
- Does a new lane significantly improve traffic flow?

For the purposes of this paper, we will concentrate our discussion on the microscopic states observed and modeled in vehicular traffic, with specific emphasis on the transition from free flow to congested traffic. In section 2, we present a brief overview of the historical development of the discipline. Section 3 is dedicated to synopses of four major theoretical approaches to the problem. We then proceed in section 4 to discuss empirical observations of traffic's dynamical states. In section 5 we discuss corresponding results from the theoretical models and make comparisons with the empirical findings. We conclude in section 6, emphasizing the relevance of the field to the modern study of physics.

2 Historical Background

Traffic studies can be traced as early as 1935 with the work of Greenshields [1]. Later, a flurry of research was localized in the 1950's-1960's. In this period, each of the four theoretical models presented in this paper (macroscopic fluid, gas-kinetic, car-following, and cellular-automata) find their beginnings. In 1955, Lighthill and Whitham [2] pioneered a 1D hydrodynamic theory of traffic. By postulating that traffic flux was a function of the density alone and using classic continuity arguments, they arrived at fluid dynamic equations in which traffic jams are modeled as shock waves. Though obscure today, work performed by Gerlough, contemporary with Lighthill and Whitham, used an early form of a cellular-automata algorithm [3]. Later work by Prigogine et al. (1960) pursued a gas-kinetic formulation based on the Boltzmann equation and were able to reproduce the formalism developed by Lighthill and Whitham [4]. In 1961, Newell proposed a car-following theory (where cars are 'particles' interacting through a Newtonian potential) known as the Optimal Velocity (OV) model, which is still in use today [5].

In somewhat of a gap between these early works and the more recently stimulated modern activity, Treiterer (1975) analyzed a series of aerial photographs and first observed the occurrence of the so-called "phantom jam" [6] (a traffic jam which emerges spontaneously from uniform traffic without an obvious cause like a traffic accident or lane closing, see fig.1) which became a major feature to qualify or disqualify modern traffic models. He also observed stop and go waves in congested traffic which would later be associated with a fundamental dynamical phase of traffic.

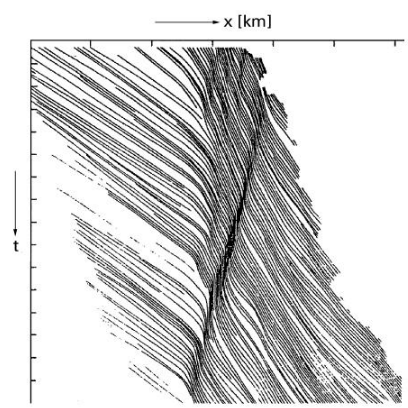

Figure 1: Emergence of a 'phantom jam.' The lines are single vehicle trajectories. (Reproduced from [17])

Work in traffic physics slowed until the early 1990's when new traffic data became available (largely due to induction loop detectors being installed on several major freeways) and the techniques of modern statistical physics were brought to bear on the problem. In this period, the works of Biham, Kerner and Kohnhauser, Nagel and Schreckenberg, Lee, Treiber, and others set the stage for the modern approaches to the problem.

3 Outline of Theoretical Models

The theoretical models that have been developed in the study of vehicular traffic can be sorted into four broad classes: hydrodynamic, gas-kinetic, car-following, and particle hopping (cellular automata) models. Models in the first two classes generally correspond to earlier work in the field, though some efforts in those areas are still being made. The microscopic kinetic and car-following models can be coarse grained to obtain macroscopic hydrodynamic models. Similarly, connections have been made between the Nagel-Schreckenberg (particle hopping) and Optimal Velocity (car-following) models [7]. Thus all the models, while conceptually different, contain unifying connections and therefore truly are different roads leading to similar 'Romes.' Before proceeding with our summary of these theoretical descriptions, it is helpful to present definitions of fundamental 'physical' quantities which characterize the problem: (1). *density* (ρ)-number of cars per unit distance, (2). *flux* (J)-number of cars passing a point per unit time, (3). *distance headway* (Δx)-distance between adjacent vehicles, (4). *time headway* (Δt)-time difference for adjacent vehicles to pass the same point.

3.1 Hydrodynamic Models

Traffic, when observed from a distance, appears to behave much as a continuum 1D fluid. Thus, early approaches to describe traffic flow where couched in the familiar language of fluid mechanics. All approaches rely on two simple statements of conservation, 'mass' and 'momentum':

$$\partial_t \rho + \partial_x J = 0 \quad , \quad J \equiv \rho v \tag{1}$$

$$d_t v = \partial_t v + v \partial_x v = F/m \tag{2}$$

From here, a mean-field description can be pursued. We expect that there will be fluctuations about the mean values of density and velocity:

$$v = \langle v \rangle + v' \quad , \quad \langle v' \rangle = 0 \tag{3}$$

$$\rho = \langle \rho \rangle + \rho' \quad , \quad \langle \rho' \rangle = 0 \tag{4}$$

Substituting these expressions into eqs. (1) and (2), averaging each equation, and approximating fluctuations about a quantity using the gradient ($\langle v' \rho \rangle \propto \partial_x \langle \rho \rangle$), we arrive at the mean field equations (the substitution $(v, \rho) \equiv (\langle v \rangle, \langle rho \rangle)$ has been made) [8]:

$$\partial_t \rho + \partial_x J = D \partial_x^2 \rho \tag{5}$$

$$\partial_t v + v \partial_x v = \nu \partial_x^2 + F/m \tag{6}$$

The original work of Lighthill and Whitham began with (5) under the assumption that $D \approx 0$ [2]. The enabling assumption that they made (and which is the basis of most macroscopic traffic models) is that traffic flux is a function of the density alone. Under this assumption, a single non-linear equation is obtained:

$$\partial_t \rho + \frac{dJ}{d\rho} \partial_x \rho = 0 \quad , \quad J(\rho) = \rho v(\rho) \tag{7}$$

This equation admits solutions of the form $\rho(x,t) = \rho(x - ct)$ ($c \equiv \frac{dJ}{d\rho}$), or density waves. From the theory of characteristics, solutions of (7) with crossing characteristics will have discontinuities (shocks) which propagate with speed $\frac{\Delta J}{\Delta \rho}$ (the differences spatially straddle the shock discontinuity). For $D \neq 0$, waves decay as e^{-Dk^2}, where k is the wavenumber. Thus traffic jams enter this formalism as propagating shock fronts which decay in time back to the uniform state.

Thus far we have not specified $J(\rho)$. The Greenshields model is obtained by setting $J(\rho) = v_{max}\rho(1-\rho)$, which has qualitative similarities to empirical findings. Under the transformation $(x,t) \to (v_{max}t' - x', t')$, (5) becomes the deterministic Burger's equation [8]:

$$\partial_{t'}\rho + 2v_{max}\rho\partial_{x'}\rho = D\partial_{x'}^2\rho \tag{8}$$

A considerable advantage of the Greenshields model is that the Burger's equation has been extensively studied and can be further mapped onto the readily solved diffusion equation by means of a Cole-Hopf transformation [9].

Several qualitative features of vehicular traffic are still missing from the formalism developed to this point. First, cars cannot accelerate to new speed instantaneously (as is possible in the preceding), which requires the inclusion of momentum conservation (eq. 6). Second, the 'force' which acts on the vehicles needs to be defined. This is usually approximated by a 'relaxation' term and an 'interaction' term. Exponential relaxation toward a desired speed can be achieved by the term $\frac{1}{\tau}[V(\rho)-v]$, where $V(\rho)$ is the (driver) desired velocity, and τ is the relaxation time. The interaction term usually takes the form $-\frac{c_0^2}{\rho}\partial_x\rho$ (c_0 is the characteristic speed of jam propagation, 15km/h), which can be mathematically motivated from the relaxation term just described [8]. The interaction term captures the qualitative feature that traffic slows in response to increasing density. Thus we arrive at the Kerner-Kohnhauser model [10]:

$$\partial_t\rho + \partial_x(\rho v) = D\partial_x^2\rho \tag{9}$$

$$\partial_t v + v\partial_x v = \nu\partial_x^2 v - c_0^2\rho^{-1}\partial_x\rho + \frac{1}{\tau}[V(\rho) - v] \tag{10}$$

By using methods of non-linear analysis, Kuhne showed that the KK model has quasi-periodic behavior, just as is observed in real traffic [11].

3.2 Kinetic Models

The early work on kinetic models of vehicular traffic can be largely attributed to Prigogine, *et al* [4]. Inspired by the Boltzmann equation, he proposed:

$$\partial_t f + \partial_x f = -\tfrac{1}{\tau}[f - \rho F_{des}(v)] + \left(\frac{\partial f}{\partial t}\right)_{int} \tag{11}$$

The term involving $F_{des}(v)$ describes collective relaxation of the microscopic distribution $f(x,v,t)$ to $F_{des}(v)$, a spatially uniform steady state.

Later work showed that such collective relaxation was unphysical. To resolve the unphysical features of the Prigogine model, Paveri-Fontana [12] expanded phase space to include the desired speed of the individual driver, $g = g(x, v, v_{des}, t)$:

$$\partial_t g + v\partial_x g - \partial_v[\tfrac{1}{\tau}(v_{des} - v)g] = (\partial_t g)_{int} \tag{12}$$

$$(\partial_t g)_{int} = f\int_v^\infty dv'(1-P)(v'-v)g - g\int_0^v dv'(1-P)(v-v')f \tag{13}$$

$$f \equiv \int_0^\infty dv_{des} g \qquad , \qquad P \equiv \text{`}\mathit{Probability\ of\ passing}\text{'}$$

This formulation relies on the assumption of 'vehicular chaos,' namely that driver-driver correlations are small. This limits the validity of the description to low densities, which prevents a derivation of the macroscopic equations from this microscopic model [9].

The mapping from micro to macro was later successfully performed by Helbing, who developed a gas-kinetic theory starting from the master equation [7]. His Boltzmann-like formulation takes the following form:

$$\partial_t f + \partial_x(vf) + \partial_v[\tfrac{1}{\tau}(v_{des} - v)f] = \tfrac{1}{2}\partial_v^2(D_v f) + (\partial_t f)_{int} \quad , \quad f = f(x, v, t) \tag{14}$$

Here D_v is a velocity dependent diffusion coefficient and $(\partial_t f)_{int}$ represents the interactions between particles.

3.3 Car-Following Models

Car-Following models are so-named because the theoretical description is couched in terms of Newtonian dynamics, where the trajectory of each individual vehicle is traced or 'followed.' Early work ('Follow-the-Leader' models) postulated that driver response (acceleration) is related to the relative velocity of a vehicle and its downstream partner:

$$\ddot{x}_n(t + \tau) = S_n[\dot{x}_{n+1}(t) - \dot{x}_n(t)] \tag{15}$$

where τ describes the response time of the driver and S_n represents the 'strength' of the response. Forms have been proposed to make S_n more realistically depend on the nearest-neighbor proximity (the distance headway) to avoid 'collisions':

$$S_n = \frac{\kappa}{x_{n+1}(t) - x_n(t)} \tag{16}$$

Such a prescription results in a set of n coupled, non-linear, ordinary differential equations, from which it is very difficult to make analytical progress [9].

Later work by Newell [6] reformulated the problem in terms of a desired (optimal) velocity that depends on the distance headway Δx. In the Optimal Velocity (OV) model, a driver accelerates from their current velocity to the optimal velocity in time τ:

$$\dot{x}_j(t + \tau) = V(\Delta x_j(t)) \tag{17}$$

Bando Taylor expanded this form and obtained [13]:

$$\ddot{x}_j = \frac{1}{\tau}[V(\Delta x_j) - \dot{x}_j] \tag{18}$$

One immediately notices that (18) corresponds to the classical motion of a particle with friction, driven by a 'force' proportional to $V(\Delta x)$. In general, the optimal velocity function $V(\Delta x)$ is required to: be monotonically increasing, be bounded from above, and have a point of inflection. The last requirement can be shown to be necessary to obtain density waves that are interpretable as traffic jams [7]. A popular form for $V(\Delta x)$ was proposed by Bando [13]:

$$V(\Delta x) = \tfrac{v_m}{2}[\tanh(\Delta x - x_c) - \tanh x_c] \tag{19}$$

By using a series expansion of the headway in terms of the density, Berg *et al.* have successfully obtained a macroscopic representation from the Optimal Velocity model [14]:

$$\partial_t v + v\partial_x v = \frac{1}{\tau}[V(\rho) - v] + \frac{1}{2\tau}V'(\rho)[\rho^{-1}\partial_x\rho + \frac{1}{3}\rho^{-2}\partial_x^2\rho - \rho^{-3}(\partial_x\rho)^2] \tag{20}$$

What distinguishes (20) from the results presented for the hydrodynamic and gas-kinetic models is that the coefficients represented in terms of observable microscopic variables, rather than the phenomenological constants of vehicular 'diffusion' and 'viscosity,' which are difficult to define. An unphysical feature of the OV model is that the collision frequency increases much too rapidly as the delay time τ is increased, since the optimal velocity function does not depend on Δv.

The so-called Intelligent Driver Model (IDM) attempts to rectify some of the unphysical features of the OV model by carefully describing the reactions of the individual drivers. Treiber *et al.* prescribe the following model of driver response [15]:

$$\dot{v} = a\left[1 - \left(\frac{v}{v_0}\right)^4 - \left(\frac{\Delta x^*}{\Delta x}\right)^2\right] \tag{21}$$

$$\Delta x^* = \Delta x_0 + Tv + \frac{v\Delta v}{2\sqrt{ab}} \tag{22}$$

Here v_0 is the desired velocity, Δx_0 is the safe distance headway, T is the safe time headway, a is the maximum acceleration, and b is the desired deceleration. As we will see in section 5, the IDM can produce quantitative agreement with empirical findings.

3.4 Particle Hopping Models

Unlike the models previously described, space and time are discretized in particle hopping models. Individual vehicles move on a lattice according to a set of predefined rules, thus particle hopping models are described in the language of cellular automata (CA). The simplest CA representation of vehicular traffic is the CA 184 algorithm developed by Wolfram. Here a vehicle moves ahead by one site if the site ahead of it is unoccupied and stops otherwise [7]:

$$x_j(t+1) = x_j(t) + min[1, \Delta x_j - 1] \tag{23}$$

A more realistic particle hopping model was developed by Nagel and Schreckenberg [16]. In this model, four steps are used in the vehicle update cycle: acceleration, deceleration, randomization, and movement. Vehicles increase their speed by 1 if below a maximum threshold (acceleration). If a vehicle would collide with its downstream neighbor, its speed is reduced to land on the site behind it instead (deceleration). The speed of a vehicle is reduced by 1 with probability p (randomization), which takes into account differences of individual drivers and the tendency to overreact when braking. Finally, the vehicles position is increased by its net velocity calculated in the preceding steps (movement). The algorithm can be expressed compactly as [7]:

$$x_j(t+1) = x_j(t) + max\Big[0\ ,\ min\big[v_{max}\ ,\ \Delta x_j - 1\ ,\ x_j(t) - x_j(t-1) + 1\big] - u_j(t)\Big] \tag{24}$$

where $u_j(t)$ is 1 with probability p and 0 with probability $1-p$. As we will see, overreaction during breaking in the NaSch model is an important feature in spontaneous jam formation. When modeling real traffic, the time step of the update cycle is usually associated with the shorteest relevant time scale of the system, ie. the reaction time of drivers (1s)[7]. One shortcoming of the NaSch model described here is that it does not generate metastable or hysteretic dynamics. This can be rectified by qualitatively incorporating the fact that drivers do not always immediately notice when they can begin to accelerate again once stopped. This results in 'slow-to-start' rules where a driver accelerates with probability q if accelerating from stopped [17].

4 Empirical Observations

Researchers in traffic phenomena have only a few sources for empirical data. In general, modern traffic data are gathered by induction loop detectors installed on overpasses which measure coarse grained distance and time headway distributions as well as short term temporally averaged density and velocity data. Despite the roughness of the data gathered, quantitative comparisons can often be made.

4.1 Traffic Jams

The most widely observed phenomena in traffic physics are traffic jams. The works of Kerner *et al.* ([18]-[21], see, e.g. [9] for more references) taken together constitute a detailed analysis of the formation and characteristics of traffic jams. Many of the findings are the result of case studies of individual traffic jam events. Kerner's works reveal that almost all traffic jams are caused by some sort of bottleneck which restricts the traffic flow (which is not surprising given common experience). The nature of the jams did not seem to depend on the details of the bottleneck, eg. whether it was a lane reduction, traffic accident, on-ramp, or uphill gradient [19]. This is encouraging from the standpoint of physics because it seems to indicate universal behavior. The notable exception to the bottleneck induced traffic jams are the so-called 'phantom' jams mentioned earlier. From Treiterer [6] and Kerner, we have come to understand spontaneous traffic jams as a kind of nucleation effect. Local fluctuations in the traffic density (often caused by driver overreaction during braking) become amplified as more cars brake behind the locally slowed pocket. As more cars are involved in the process (due to increased velocity upstream), a jammed state emerges.

Kerner observed that the jammed state is stable for long periods of time and that jam waveforms can propagate for long distances over changing road conditions (encountering on and off ramps and lane reductions) with little change. He also found that jam waves propagate at a characteristic speed of about 15km/h. This is primarily due to the fact that it takes an individual driver at least 2s to accelerate, thus drivers leave the jam sequentially and slowly. In fact the outflow density profile is essentially constant in time. This leads to the other major characteristic of traffic jams. Kerner found that the outflow from a jam is not only constant in time, but is universal: other jams under similar weather conditions will have the same outflow [19]. These universal characteristics of traffic jams make the system accessible to a physical description.

4.2 Phases of Traffic

In addition to the jammed state just described, the works of Kerner and others have revealed that traffic actually consists of many dynamical phases. Below is a summary of the states reported so far (naming may vary).

1. Free Traffic (FT) is characterized by high flow. Each individual vehicle is moving at a roughly constant speed and is relatively unaffected by its neighbors. In FT traffic in different lanes in nearly uncorrelated. Free flow breaks down at higher densities.
2. Synchronized Traffic (ST) is characterized by a high density and high flow. ST exists in density regimes closer to the jammed state, yet has a flux nearly the same as free flow. In synchronized traffic, cars all move with nearly uniform velocity and strong correlations exist between different lanes.
3. Oscillatory Congested Traffic (OCT) is a jammed traffic state that has periodic 'stop-and-go' waves. A 'stop-and-go' wave is a period pulse of lower density that propagates upstream.
4. Highly Congested Traffic (HCT) is the jammed state. In HCT vehicle flux is either zero or negligible.
5. Pinned Localized Clusters (PLC) are regions of high density locally confined to a road inhomogeneity (such as an on ramp).

As in condensed matter physics, the different phases of vehicular traffic can be identified in practice by their microscopic behavior. A revealing microscopic property is the distribution of time headways [9]. An example is displayed in figure 2. In the FT regime, one immediately notices two characteristic peaks in the distribution. The first peak occurs near $\Delta t = .8s$ and corresponds physically to 'platoons' of vehicles traveling at high speed close together. The second peak near $\Delta t = 1.8$ reflects the desire to maintain a safe time headway (driving schools encourage a Δt of about 2s). In ST, the short time peak is destroyed while the safe driving peak gains relative prominence. In OCT, no short time headways are observed and leaving only the safe driving peak [9]. In this manner different microscopic states can be identified either from empirical or simulation data. Different phases of traffic have also been found to

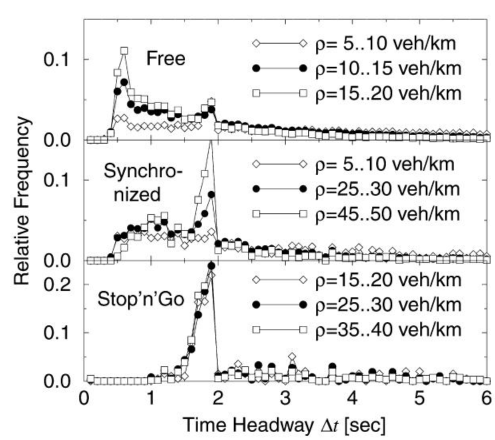

Figure 2: Distributions of Time Headway for Free, Synchronized, and Oscillatory Congested Traffic. (Reproduced from [17])

coexist. A striking example from Kerner is three phase coexistence of FT, ST, and jam waves (see figure 3). Here two jam waves propagate in parallel across regions of FT and ST. A remarkable feature is that the jam waves cross the transition between FT and ST with little to no disturbance.

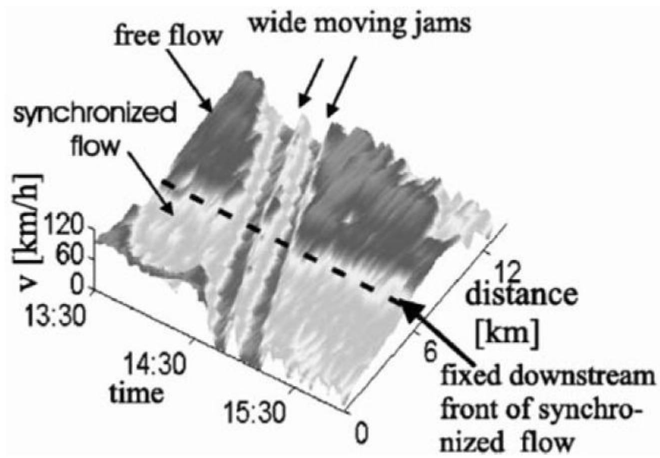

Figure 3: Spatio-temporal plot of 3 dynamically coexisting phases. (Reproduced from [21])

4.3 Phase Transitions

An often sought after relation in traffic physics is the 'fundamental diagram,' or flux-density relation which indicates a separation between free flow and congested states. A typical example of the flux-density relation can be found in figure 4. At low density, flux varies linearly with density, which means that the increase in density in this regime has not affected the average velocity. At higher densities, vehicles impede each other (vehicle-vehicle interactions become important) and congested traffic states form. It is now well established that the flux is discontinuous in density across the transition, thus it is a first order phase transition. Kerner has found that both of the transitions $FT \rightarrow ST$ and $FT \rightarrow HCT$ are first order [20]. Correspondingly, traffic states also exhibit hysteresis. Figure 4 contains a time trace of traffic data moving from free traffic to a congested phase and eventually back to free flow, thus forming a hysteresis loop. Though the primary order parameter governing the phase transitions is density, the transitions can be triggered by other means. Kerner found that a propagating jam interacting with a bottleneck can induce dynamical phase transitions. He observed a jam approaching a region of free traffic prior to, during, and after a bottleneck. After passing the bottleneck, a synchronized traffic state had formed prior to the bottleneck [21]. As noted before, he also found that the $FT \rightarrow ST$ transition can also be induced by a short-time localized fluctuation in the density (the 'phantom jam').

5 Theoretical Results

All of the model classes discussed in this paper are capable of reproducing free, synchronized, and jammed traffic states. For the rest of this section, we will primarily

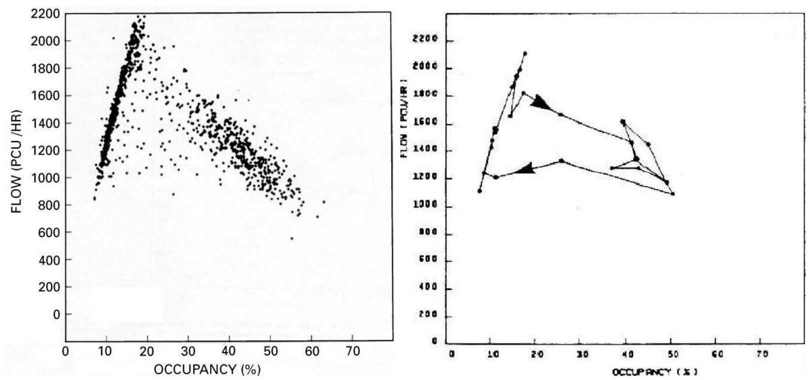

Figure 4: Flux-density relationship observed in traffic (left). Hysteresis in dynamic phase transitions (right). (Reproduced from [9])

focus on the modern car-following and particle hopping models, which are capable of comprehensive qualitative, and at times quantitative, agreement with empirical observations.

5.1 Traffic Jams

In analytical theory, the invariant properties of traffic jams has inspired description in terms of classic non-linear differential equations such as the Burger's, Kortweg de Vries (KdV), and Modified KdV equations. Different models typically favored one particular equation. More recently, Nagatani has discussed how all three non-linear equations can be derived from the Optimal Velocity model and exist in different areas of a proposed (analytical and numerical) phase diagram. The various solutions of triangular shock (Burger's), soliton (KdV), and kink-antikink (MKdV) waves have qualitative similarities with empirical time trace data [7].

5.2 Phases of Traffic and Phase Transitions

The Modified Nagel-Schreckenberg, Optimal Velocity, and Intelligent Driver models are capable of reproducing the five phases discussed so far. Theoretical phase diagrams have been proposed for certain hydrodynamic, gas-kinetic, and car following models. A host empirical findings have been verified analytically or by simulation.

Similar to Kerner's observations, Knospe, using the NaSch model, has observed three phase coexistence [22]. These results were obtained by incorporating rules which reflect the desire for smooth driving into the NaSch model, which implies that the ST phase may emerge due to this microscopic tendency. Results from Nagel with empirical boundary conditions reproduce the empirical flux-density relations measured by Wiedemann [7]. The flux-density relation obtained from the OV model also recaptures the shape of the empirical curve along with the finite discontinuity at the critical point. Expansion of equation (20) (coarse-grained OV model) around the critical point results in a Time-Dependent Ginzburg-Landau equation, implying that the jamming transition is governed by the TDGL equation. Stability analysis and simulation imply that the free-congested transition is similar to the gas-liquid phase transition [7].

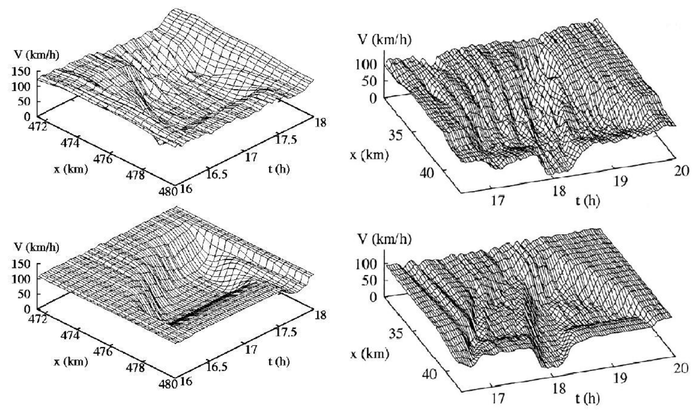

Figure 5: Traffic breakdown from a lane closing (left) and OCT on an uphill road section. Compare empirical measurements (top) with IDM results (bottom). (Reproduced from [15])

Some of the results from the IDM model have even more striking quantitative agreement with empirical findings. Figure 5 shows spatio-temporal plots of congested traffic from observation and for the IDM with empirical boundary data [15]. Such close agreement reassures us that the microscopic dynamics are being correctly considered. It should be noted that much more information could be presented here, but we instead refer to the relevant literature.

6 Conclusion

Vehicular traffic provides a rich set of ‘physical phenomena.’ Understanding the dynamic phases of traffic, as well as the causes of metastability and hysteresis are bound to have broader impacts than road design. As we have seen, the field of traffic physics is replete with connections to other fields of physical study. The appearance of macroscopic hydrodynamic equations and the Time-Dependent Ginzburg-Landau equation speak of deeper connections. Even the similarities between the formation of ‘platoons’ behind slow moving vehicles and Bose-Einstein Condensation raise questions [9]. The field of traffic physics is becoming well developed in the modern language of non-equlibrium statistical physics, and its further maturation should help in fundamentally understanding other important physical systems with similar qualitative features.

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