# Josephson Effects and a $\pi$-State in Superfluid Helium

Paul B. Welander

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**Abstract**

In this paper, I shall discuss the recent discovery of a metastable $\pi$-state in a $^3$He Josephson junction. This state is characterized by low frequency current oscillations and a nearly constant cross-junction phase difference of $\pi$. The $\pi$-state occupies a local minimum in the junction's free energy, and decays with an average lifetime on the order of a minute. Whether or not this state is an intrinsic one or systematic one is debated. For background, a brief review of the Josephson effects is included.

# Introduction

In 1962, Josephson suggested that electrical currents would develop between two bulk superconductors when brought into close proximity with one another [1, 2]. For these effects to manifest themselves, there must exist an appropriate tunnel junction, commonly referred to as a "weak link" [3]. According to broken symmetry arguments, the superconductors assume a complex order parameter, $\psi$, which can be interpreted as the macroscopic wave function of the condensate (superconducting electrons), and takes the general form $\sqrt{\rho_c}\, e^{i\phi}$, where $\rho_c$ is the condensate density and $\phi$ the condensate phase. The weak link allows the wave functions of each superconductor to overlap. Josephson predicted that the resulting current through these junctions would be related to the quantum phase difference, $\Delta\phi = \phi_1 - \phi_2$, by the equation

$$I = I_c \sin(\Delta\phi)\,, \tag{1}$$

where $I_c$ is the critical current of the junction. When a chemical-potential difference, $\Delta\mu$, is created across the weak link, the phase difference would then evolve according to the Josephson-Anderson relation [4]:

$$\frac{\partial(\Delta\phi)}{\partial t} = -\frac{\Delta\mu}{\hbar}\,. \tag{2}$$

These two equations are often referred to as the dc and ac Josephson relations, respectively. In a junction between two superconductors at identical temperatures, $\Delta\mu = -2eV$, where $-2e$ is the charge of an electronic Cooper pair and $V$ is the voltage across the weak link. Within a few years, both the dc [5] and ac [6] Josephson effects were observed in superconductors.

Because superfluid helium can also be described by a complex order parameter, the Josephson relations should therefore apply to two helium reservoirs separated by a weak link. The equations above would then describe a mass current across the junction, with $\Delta\mu = M\Delta P/\rho$, where $\rho$ is the liquid density, $\Delta P$ the pressure difference between the two reservoirs, and $M$ the particle mass ($m_4$ for a $^4$He atom; $2m_3$ for a $^3$He Cooper pair). Shortly after the successful experiments in superconductors, the first attempts were made at observing Josephson effects in helium [7, 8, 9, 10]. Unfortunately, these experiments failed to provide convincing evidence. To see why, let's examine the problem.

The difficulty in observing the Josephson effects in helium for many years involved the construction of a proper weak link. It was Likharev who first suggested in 1979 [3] that a suitable weak link must have cross-sectional dimensions, $d$, smaller than, or on the order of, the coherence length, $\xi = \xi_0(1 - T/T_c)^{-1/2}$, where $T_c$ is the superfluid transition temperature and $\xi_0$ is the zero-point coherence length. This criteria separates the temperature range below $T_c$ into two regimes: 1.) The Josephson regime ($\xi > d$), where flow of electrons or superfluid across the weak link obeys equations (1) and (2). 2.) The phase-slip regime ($\xi < d$), where the condensate exhibits super-flow. This regime carries the moniker "phase-slip" because when velocities in the weak link become larger than the critical velocity of the condensate, the order parameter phase, $\phi$, can 'slip' by an amount $2\pi$. Although the current in this regime is not sinusoidal in $\Delta\phi$, its behavior is still oscillatory with a periodicity of $2\pi$. This similarity, along with the fact that equation (2) still holds in the phase-slip regime, is what led to misguided interpretations of early experiments. It also led Likharev to define a new phenomenological current-phase relation:

$$I = I_c \sin\zeta\,, \qquad \Delta\phi = \zeta + \alpha\sin\zeta\,, \tag{3}$$

where $\zeta$ is an auxiliary phase angle and $\alpha$ is a non-ideality parameter. For $\alpha = 0$, the dc Josephson relation (eqn. 1) is recovered. In fact, the Josephson regime generally requires only $\alpha < 1$. For the case $\alpha > 1$, the current phase relation of equation (3) is both hysteretic and multi-valued in regions where $\Delta\phi \sim n\pi$, $n$ odd. This is the phase-slip regime.

For studying Josephson effects in helium, a suitable weak link consists of a small aperture in a thin film separating two bulk regions of superfluid. Here, 'small' means cross-sectional dimensions on the order of 100 nm. For $^3$He, $\xi_0 \sim 65$ nm, while for $^4$He, $\xi_0 \sim 1$ Å. Even with a 100 nm hole, one must go very close to $T_c$ for $\xi$ to diverge the necessary amount. Development of micro-orifices and the use of SQUID based Helmholtz resonators as sensitive mass flow devices eventually allowed for the observation of Josephson effects in $^3$He [11, 12]. And finally, 39 years after Josephson's original paper, the effects which bare his name were observed in $^4$He [13].

## Discovery of a New State

In 1998, a group at Berkeley, Backhaus et. al., reported the discovery of a metastable Josephson tunnelling state in superfluid $^3$He [14]. In this state, a quantum phase difference of $\pi$ is maintained across the weak link. The apparatus employed in the experiment is a volume of superfluid $B$-phase $^3$He partitioned in two by a wall containing both a flexible membrane and a weak link (Fig. 1). The weak link consisted of an array of 4,255 100-nm-diameter holes with a spacing of 3 $\mu$m in a 50-nm-thick SiN membrane. The flexible membrane was coated with a superconducting film, which allowed for motion detection via a superconducting quantum interference device (SQUID), with a resolution of $10^{-14}$ m$/\sqrt{Hz}$. The pressure difference between the two $^3$He reservoirs was controlled by applying a voltage

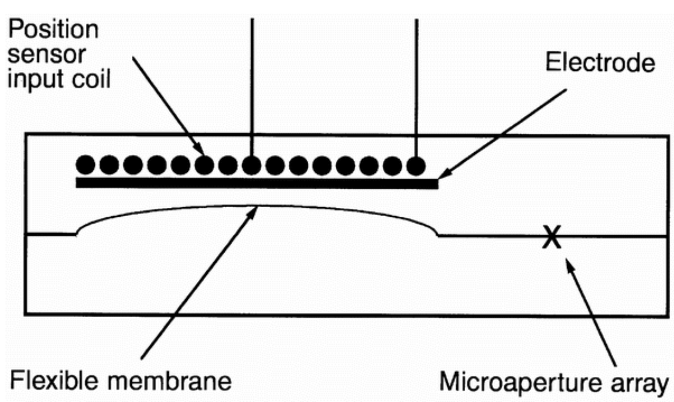


Figure 1: A schematic diagram of the experimental cell employed by Backhaus et. al. A container filled with $^3$He-$B$ is partitioned by a wall containing a flexible membrane and weak link array. [14]

between the membrane and an electrode mounted on an opposite wall. The experimental cell was attached to a nuclear demagnetization refrigerator that could cool the helium to temperatures less than 200 $\mu$K.

Determination of the mass current through the weak link is done by analyzing the position of the flexible membrane as a function of time, $x(t)$. The instantaneous mass current is then given by

$$I(t) = \rho A \frac{\mathrm{d}x}{\mathrm{d}t}$$

$A$ is the area of the membrane. The phase difference across the weak link varies in time according to the Josephson-Anderson phase evolution equation:

$$\Delta\phi(t) = \frac{-2m_3}{\rho\hbar} \int_0^t \Delta P(t')\mathrm{d}t' = \frac{-2m_3 k}{\rho\hbar} \int_0^t x(t')\mathrm{d}t' \tag{4}$$

where $k$ is the stiffness of the membrane. When $\Delta P$ is a constant, equation (1) reduces to the form

$$I(t) = I_c \sin(\omega_j t)$$

where $\omega_j$ is the Josephson frequency, $2m_3\Delta P/\rho\hbar$. Backhaus et. al. reported that mass currents of this type were observed only at temperatures above 0.48 $T_c$, where $T_c$ is the superfluid transition temperature of $^3$He. It is at this temperature where the coherence length becomes comparable to the aperture size, and a transition is made from the phase-slip regime to the Josephson regime.

However, the discovery here is that a new state of the weak link emerges at temperatures below 0.48 $T_c$. At these temperatures, the dc Josephson relation (Eqn. 1) no longer holds, though phase evolution still obeys equation (4). When $\Delta\phi$ is zero, the membrane exhibits a normal mode of oscillations similar to that of a mechanical pendulum. The oscillator is excited by a few sine-wave cycles of an applied pressure, frequency-matched to the low-amplitude resonant frequency of the pendulum modes. (Fig. 2) The objective is to pump the oscillator into regimes where $\Delta\phi$ becomes non-zero and approaches values of $\pm\pi$. The driving force is then turned off and the resulting behavior studied. If the oscillator is only moderately driven, the amplitude will slowly decay after the excitation ends. However, if the oscillator is driven to a higher level, after excitation it collapses to a different mode with lower amplitude and frequency. (Bottom trace, Fig. 2) After several cycles – minutes on the time scale – without further drive, an abrupt return to the original state (frequency and amplitude) occurs. The authors reason that the kinetic energy that appears to go missing in the intermediate state is instead stored in some other hidden degree of freedom.

After further analysis of the data, the true nature of this intermediate state becomes apparent. Using equation (4), $\Delta x$ is converted into a plot of $\mathrm{d}(\Delta\phi)/\mathrm{d}t$ versus $\Delta\phi$. (Fig. 3) During the excitation stage, the phase-difference oscillation is centered about $\Delta\phi = 0$. Collapse to the intermediate state shifts the center of oscillations to $\Delta\phi = \pi$, while the return to the original state has variations in $\Delta\phi$ centered at $\Delta\phi = 0$ or $2\pi$. Because of its associated $\Delta\phi$ value, the authors call this new intermediate, metastable state the "$\pi$-state." Likewise, the simple pendulum mode is referred to as the "zero-state" since its oscillations are centered at $\Delta\phi = 0$. (The zero-state is equivalent to the "$2\pi$-state.")

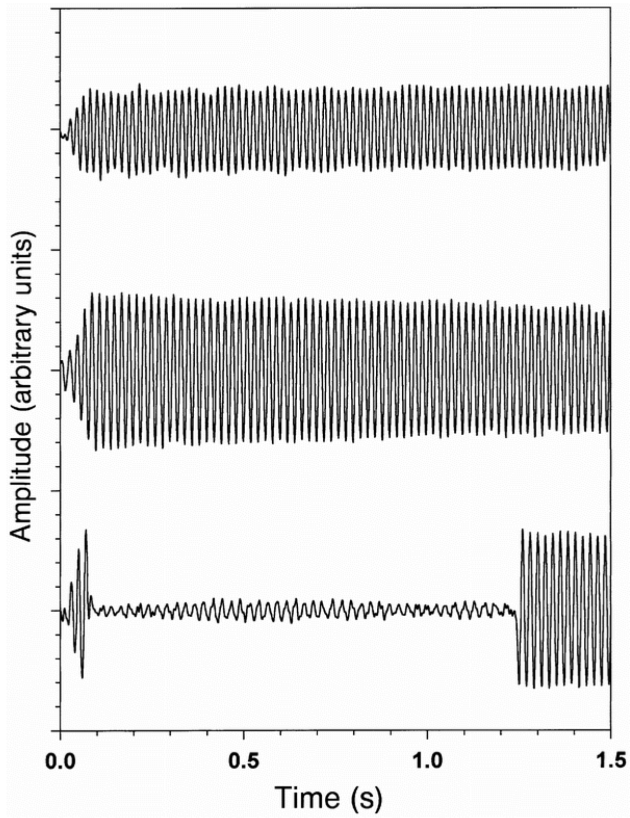


Figure 2: Oscillation of the flexible membrane resulting from application of 4 cycles of drive at the pendulum-mode resonant frequency, at $0.3T_c$. The amplitude of the drive increases from top to bottom. The top two curves are pendulum-like, while the bottom curve exhibits characteristics of the newly discovered $\pi$-state. A perturbation causes this metastable state to decay back to its original state. [14]

Backhaus et. al. also determine the free energy of the $^3$He weak link,

$$F(\Delta\phi) = \frac{\hbar}{2m_3} \int_0^{\Delta\phi} I(\Delta\phi')\mathrm{d}(\Delta\phi') \tag{5}$$

and find that there is indeed a local minimum at $\Delta\phi = \pi$. (Fig. 3c) Global minima occur at values of 0 or $2\pi$. With this picture in mind, the dynamics of the oscillator can be described as follows. Moderate driving of the membrane can excite the oscillator, but not enough to escape the free energy minimum centered at $\Delta\phi = 0$. Only for large excitations can the oscillator make a transition to the metastable $\pi$-state ($0 \rightarrow \pi$). The weak link can remain in this state, sometimes for several minutes, until a fluctuation causes it to decay back to the zero-state ($\pi \rightarrow 0$). Experiments were performed at various temperatures. At the upper temperature limit, 0.48 $T_c$, the $\pi$-state lifetime was measured at only 100 ms. On the other hand, much lower temperatures produced lifetimes of several minutes. Although the authors admit to knowing of no precise microscopic theory for this new state, they equate the $^3$He $\pi$-state to the superconducting $\pi$-junction. This analogy, they claim, reflects the '$p$-wave' symmetry of the superfluid $^3$He order parameter.

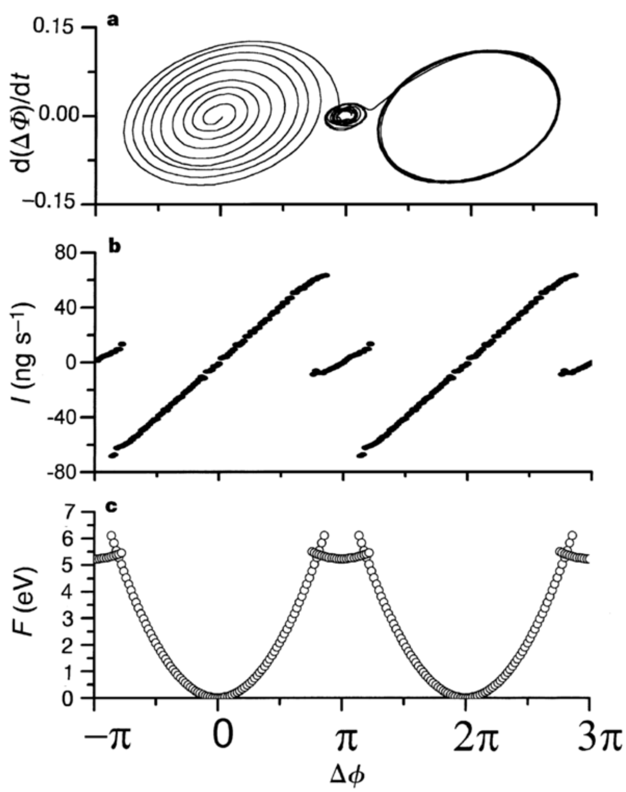


Figure 3: Oscillator characteristics. **a.** Trajectory of the oscillator in the $\mathrm{d}(\Delta\phi)/\mathrm{d}t$ versus $\Delta\phi$ plane at $0.28T_c$. During the drive stage of the experiment, oscillator motion is centered at $\Delta\phi = 0$. The low-amplitude motion at $\Delta\phi = \pi$ is a signature of the metastable $\pi$-state. After several oscillations, the weak link decays to an orbit about $\Delta\phi = 2\pi$. **b.** The measured current-phase relation for the microaperture array at $0.28T_c$. **c.** Weak link free energy, $F$, as a function of $\Delta\phi$ at $0.28T_c$. [14]

## Could it be? – Arguments Against

Shortly after the $\pi$-state discovery was announced, a significant challenge was mounted against its foundation. Avenel et. al. [15] argued that the array of 4,225 apertures that had been employed in the experiment could not be expected to behave as a single, coherent weak link. They also showed that the results of Backhaus et. al. could be explained without invoking the superfluid analogue of the "much-debated '$\pi$-junction'" in other systems.

Making use of the Likharev equations for superfluid flow (Eqn. 3, $I$ is plotted versus $\Delta\phi$ for the case $\alpha = 2$. (Fig. 4) For this scenario, the current is both hysteretic and double-valued in the region near $\Delta\phi = \pi$. If a large number of apertures are employed, statistics would suggest that in this double-valued region half of the holes would sit on the upper branch and half on the lower branch. In this case, the measured total current would tend toward zero, creating a third pseudo-branch represented by the dotted line in Fig. 4a.

Avenel et. al. solidify their arguments with a numerical simulation. The relationship between $\Delta\phi$, the membrane displacement, $x$, and the number of apertures, $N$, can be described by:

$$
\begin{aligned}
\Delta\phi &= \zeta_i + \alpha_i \sin\zeta_i\,, \\
\frac{\mathrm{d}x}{\mathrm{d}t} &= C\sum_{i=1}^{N} \sin\zeta_i\,, \\
\frac{\mathrm{d}(\Delta\phi)}{\mathrm{d}t} &= V\sin(\omega t) - x - \frac{1}{\omega_0 Q}\frac{\mathrm{d}x}{\mathrm{d}t}\,,
\end{aligned}
\tag{6}
$$

where $C$ is related to the small signal oscillation frequency, $\omega_0^2 = C\sum_{i=1}^{N}(1+\alpha_i)^{-1}$, $V\sin(\omega t)$ is the external drive of maximum amplitude $V$ at frequency $\omega$, and $Q$ is the quality factor of the resonator.

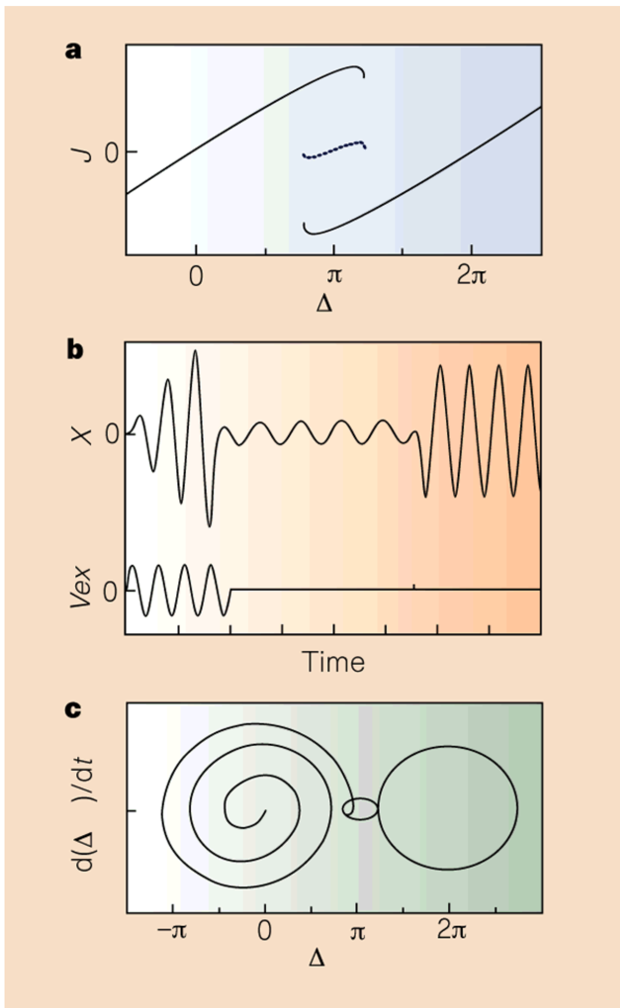


Figure 4: Numerical simulations. **a.** Current-phase relation for $\alpha = 2$. **b.** Time evolution of the membrane displacement amplitude $x$ in response to an external drive, $V_{ex}$, for 200 apertures and gaussian distribution for $\alpha_i$. **c.** Corresponding trajectory in the $\mathrm{d}(\Delta\phi)/\mathrm{d}t$ versus $\Delta\phi$ plane. [15]

Equations (6) are solved for $N = 200$, gaussian distribution of $\alpha_i$ values with a mean of 2 and standard deviation of 0.1, and a $Q$-value of 1000. (Fig. 4b & c) $V \sin(\omega t)$ consisted of four periods at a frequency of $\omega = 1.1\omega_0$, and the amplitude was adjusted until the system could be driven into a low amplitude, metastable state. This intermediate state has properties remarkably similar to the $\pi$-state discovered by Backhaus et. al. In this model however, the kinetic energy that suddenly disappears – the same energy that Backhaus et. al. conjecture has gone into some other mysterious degree of freedom – is actually stored in circulating currents within the array. If a small perturbation at later times is introduced into the numerical model, the system returns to the zero-state. This calculation seems to suggest that this new $\pi$-state is not intrinsic to superfluid $^3$He, but rather a manifestation of the aperture array employed as the weak link. In light of their numerical work, Avenel et. al. make the following statement: "In conclusion, our analysis shows that direct measurements of the current-phase relationship in an array of holes should be interpreted with care... By themselves, (the data of Backhaus et. al.) do not reveal the existence of an exotic $\pi$-junction, hidden degrees of freedom, or any influence of textural effects."

## Having Doubts? – Further Proof

Since the publication of the original $^3$He $\pi$-state data, several theoretical works have proclaimed support for the intrinsic existence of this metastable state [16, 17, 18, 19]. Meanwhile, further experiments were performed by the Berkeley group and their new results were published in a follow-up article by Marchenkov et. al. [20]. For these experiments, a modified cell was employed – similar to the cell shown in Figure 1 – that utilized a more efficient acoustic shield. Other critical apparatus parameters were left unchanged.

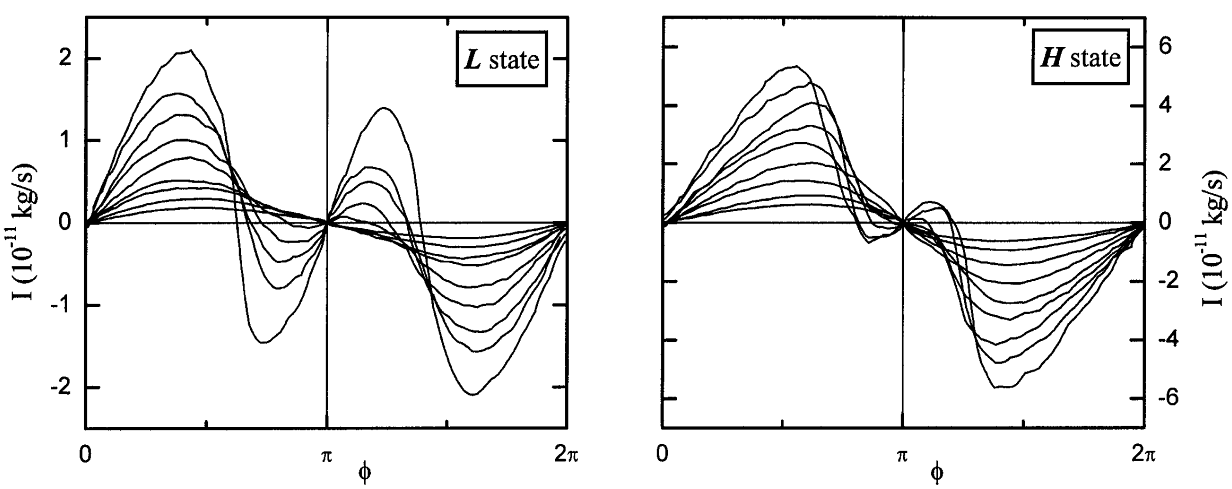


Figure 5: The **L** and **H** states of the superfluid $^3$He-$B$ weak link for temperatures 0.45¡$T/T_c$¡0.85. For large $T/T_c$, the current-phase relations are nearly sinusoidal. At lower temperatures, higher harmonics are observed, a signature of the $\pi$-state. [20]

The primary result of the new data set was that their array of 4,225 apertures could be characterized by two distinct $I(\Delta\phi)$ relations (Fig. 5). Both display sinusoidal behavior near $T_c$, but with differing amplitudes. The higher critical current state – referred to as the **H** state – has more than double the amplitude of its cousin, the **L** state. The authors find that once below $T_c$, the particular state is randomly set and fairly robust. Performing detailed measurements of a family of $I(\Delta\phi)$ curves at different temperatures, and doing so over several cool-downs, they also find that all the data collapse onto well-defined curves. Furthermore, the resonant frequency about a particular stable $\Delta\phi$ value is given by

$$f^2_{\Delta\phi_{min}} = B\left(\frac{\partial I}{\partial(\Delta\phi)}\right)_{\Delta\phi_{min}},$$

where $B$ is a constant. In Figure 5 one notices that for temperatures near $T_c$, $(\partial I/\partial(\Delta\phi))_\pi < 0$, implying a complex frequency, $f_\pi$. At lower temperatures, $(\partial I/\partial(\Delta\phi))_\pi$ becomes positive-valued, and a metastable state near $\Delta\phi = \pi$ becomes possible.

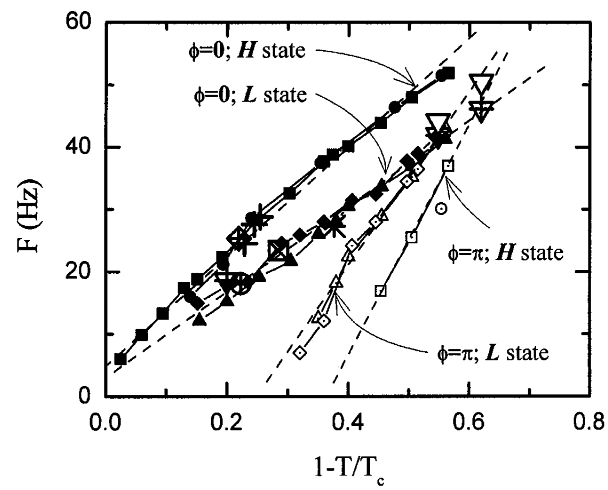


Figure 6: The two states of the $^3$He weak link as seen through oscillation frequencies at $\Delta\phi = 0$ and $\Delta\phi = \pi$. [20]

Figure 6 plots the frequencies associated with the **L** and **H** states at various temperatures. Data were taken from ten different thermal cycles, and the results have been grouped according to $\Delta\phi$ values (0 or $\pi$). That all the data fall onto two sets of distinct curves again supports the notion of two states (**L** and **H**) in the system. One can also make the following general statements: At all temperatures where a frequency could be reasonably deduced, $f_0$ for the **H** state is greater than that of the **L** state. On the other hand, $f_\pi$ for the **L** state is greater than that of the **H** state for all temperatures.

Marchenkov et. al. go on to make conjectures about the underlying physics of these two states. Due to the tensor nature of the order parameter, a sample of $^3$He-$B$ can be characterized by what's referred to as an **n** texture. An **n** texture is a spatially varying vector field that is oriented normal to any solid boundary. The suggestion is made by the authors that perhaps the two states, **L** and **H**, are associated with parallel or antiparallel

**n** textures on opposites sides of the weak link. The very existence of these states may well be the strongest indicator that the tensor nature of the $^3$He order parameter is indeed responsible for the existence of the metastable $\pi$-state.

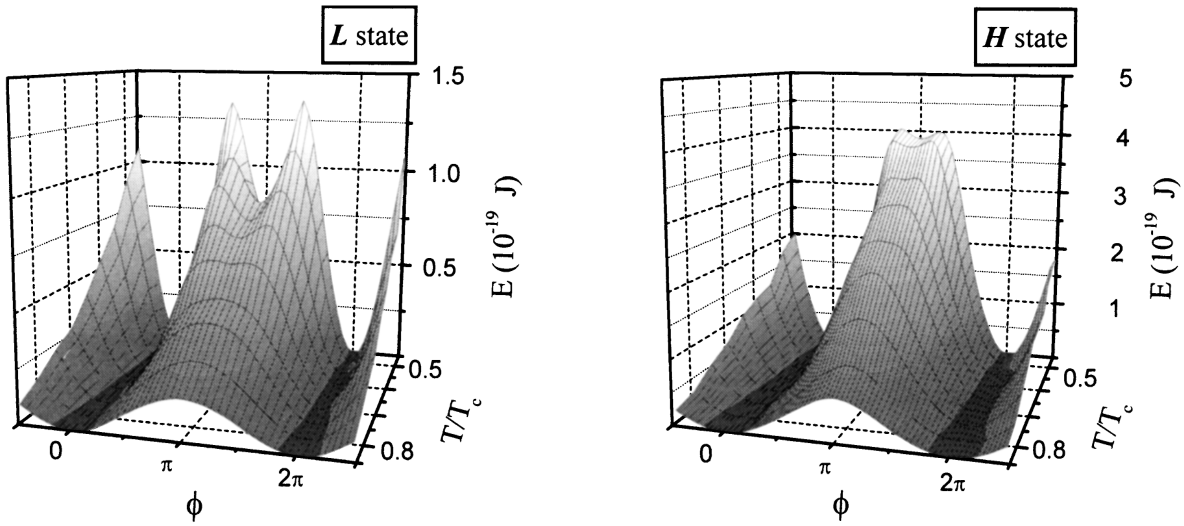


Figure 7: Energy of the superfluid $^3$He-$B$ weak link calculated from the current phase relations shown in Fig. 5. The **L** state develops a fairly deep minimum at $\Delta\phi = \pi$ while the **H** state's minimum is relatively shallow. Both depths are large compared to $k_BT$ but small compared to the macroscopic energy. [20]

It can also be seen from Figure 5 that the **L** and **H** states become somewhat distorted as the temperature is lowered. The $I(\Delta\phi)$ progress from sine-like behavior to that which displays higher harmonic terms. It is this very distortion of the current-phase relation that produces the metastable state near $\Delta\phi = \pi$. From these $I(\Delta\phi)$ curves, Marchenkov et. al. plot the free energy of the weak link (Eqn. 5) versus temperature and phase-difference, shown in Figure 7. The plot indicates that the free energies associated with these states do indeed have local minima at $\Delta\phi = \pi$ that develop continuously as a function of temperature. The **L** $\pi$-state develops for temperatures below about $0.65T_c$, while the **H** $\pi$-state begins to take shape below about $0.55T_c$. Therein lies the major accomplishment of this work. Due to the superior acoustic shielding, $\pi$-states were observed at temperatures up to $0.65T_c$, whereas the previous limit was $0.48T_c$. Furthermore, $I(\Delta\phi)$ is non-hysteretic at this higher temperature. The non-ideality parameter, $\alpha$, is less than 1, and the weak link begins to show Josephson behavior. This is a direct blow to the challenge put forth by Avenel et. al. Any theory for the existence of $\pi$-states requiring hysteretic phase slips no longer seems plausible.

## Conclusions

A brief review of Josephson effects in superconductors and superfluids was given. The discovery of a new metastable $\pi$-state was examined. Arguments for and against its existence were considered. Theoretical evidence for this state was mentioned only in passing in order to focus on the experimental results. In the final analysis, it is likely that these $\pi$-states are indeed an intrinsic property of superfluid $^3$He and may very well result from the symmetry of the order parameter.

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