Quantum Phase Transitions of Correlated Electrons in Two Dimensions: Connections to Cuprate Superconductivity

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ABSTRACT

The physics of the two dimensional electron gas is important to the high-T_c superconductivity in the cuprates where the behavior of electrons in the CuO_2 planes determines the properties of the material. Due to the filling of closely spaced atomic orbitals the electrons are strongly interacting at length scales where quantum effects are important. The ground state of such a system could have very different order as a function of some parameter, with these different phases connected by quantum phase transitions. Since the nature of the ground state is a crucial determinant of the behavior at finite temperature, the study of these phases and their excitations are important to high-T_c superconductivity. This paper starts with an introduction to quantum phase transitions including the important quantum-classical mapping described for the quantum rotor model. The coupled ladder antiferromagnet model is introduced and the quantum phases and excitations of this model are detailed. Finally, the magnetic transitions in the cuprate superconductors are described and shown to be of the same universal class as those of the coupled ladder antiferromagnet.

1 Introduction

1.1 Quantum Phase Transitions

Consider a system whose degrees of freedom reside on the sites of an infinite lattice, and which is described by a microscopic Hamiltonian containing two non-commuting operators. The Hamiltonian also involves a continuously variable parameter g that represents the essential tension between the competing ordering tendencies of the two non-commuting operators. The ground state of such a system can display a non-analyticity as a function of g due to a 'level-crossing' or an infinitesimally small 'avoided level-crossing' between the ground and the near excited states. Such a point displays many of the properties of a phase transition such as singularities and the divergences of length scales and is termed a Quantum Phase Transition (QPT).

Near the critical point, the characteristic energy scale in the system Δ (e.g. a energy gap to the first excited state) vanishes as $\Delta \sim J|\, g - g_c|^{z\nu}$. The characteristic length scale (e.g. the length scale that describes the decay of equal time correlations) diverges as $\xi^{-1} \sim J|\, g - g_c|^{\nu}$ ($\Rightarrow\ \Delta \sim \xi^{-z}$).

Since these singularities are in the ground state of the system, QPTs occur at temperature $T = 0$ and the fluctuations that drive the transitions are due the Heisenberg uncertainty principle, in contrast to the thermal fluctuations that drive classical phase transitions.

1.2 The quantum rotor

The N component quantum rotor is a degree of freedom at each site of a d dimensional lattice that can be thought of as a particle constrained to move on the N dimensional surface of a unit sphere Ref [1]. The orientation of each rotor is represented by a N dimensional unit vector $\hat{\mathbf{n}}_i$. The momentum $\hat{\mathbf{p}}_i$ of a rotor is constrained to be tangent to the surface of the sphere. The momentum and orientation at a given site obey the commutation relations $[\hat{n}_\alpha, \hat{p}_\beta] = i\delta_{\alpha\beta}$. Constructing the $N(N-1)/2$ components of the angular momentum

$$\hat{L}_{\alpha\beta} = \hat{n}_\alpha \hat{P}_\beta - \hat{n}_\beta \hat{P}_\alpha$$

For N=3, these reduce to the usual angular momentum operators $\hat{L}_\alpha = (1/2)\epsilon_{\alpha\beta\gamma}\hat{L}_{\beta\gamma}$. The commutation relations between these quantum operators at each site are $[\hat{L}_\alpha, \hat{L}_\beta] = i\epsilon_{\alpha\beta\gamma}\hat{L}_\gamma$ $[\hat{L}_\alpha, \hat{n}_\beta] = i\epsilon_{\alpha\beta\gamma}\hat{n}_\gamma$ $[\hat{n}_\alpha, \hat{n}_\beta] = 0$. The

Hamiltonian of this model is given by

$$H_Q = \frac{J\tilde{g}}{2}\sum_i \hat{\mathbf{L}}_i^2 - J\sum_{\langle i,j\rangle} \hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_j$$

Clearly, the first term which represents the kinetic energy is minimized when the orientation of each $\hat{\mathbf{L}}_i$ is maximally uncertain, while the nearest neighbor exchange term is minimized by a 'magnetically ordered' state in which the rotors have the same orientation.

1.3 Quantum to Classical Mapping

It is possible to map models of quantum transitions in d space dimensions to models of finite temperature classical transitions in $D = d + 1$ dimensions. This mapping enables the use of the machinery of classical phase transitions to study quantum critical phenomena. The mapping of the classical XY chain ($D = 1$) to the $N = 2$ point quantum rotor ($d = 0$) is detailed below and the results are extended to the $N = 3$ point rotor. Starting from the classical XY chain Hamiltonian with $\mathbf{n}_i = (\cos\,\theta_i, \sin\,\theta_i)$,

$$H = -K\sum_{i=1}^{M} \mathbf{n}_i \cdot \mathbf{n}_{i+1} - \sum_{i=1}^{M} \mathbf{h} \cdot \mathbf{n}_i \quad \Rightarrow \quad H = -K\sum_{i=1}^{M} \cos(\theta_i - \theta_{i+1}) - h\sum_{i=1}^{M} \cos(\theta_i)$$

The partition function for this system (with periodic boundary conditions) in the transfer matrix formalism is given by

$$\mathcal{Z} = \int_0^{2\pi} \prod_{i=1}^{M} \frac{d\theta_i}{2\pi} \langle \theta_1 \mid \hat{T} \mid \theta_2 \rangle \langle \theta_2 \mid \hat{T} \mid \theta_3 \rangle \cdots \langle \theta_M \mid \hat{T} \mid \theta_1 \rangle = \mathrm{Tr}\hat{T}^M$$

$$\langle \theta \mid \hat{T} \mid \theta' \rangle = \exp\left(K\cos(\theta - \theta') + \frac{h}{2}(\cos\theta + \cos\theta') \right)$$

Scaling Limit: Taking the scaling limit of this classical system involves identifying the length scales of the system and performing an expansion in ratios of these length scales, keeping only the lowest order terms of the small-to-large length scale ratios. Taking the continuum limit with length variable τ and total length $L_\tau = Ma$, the scaling limit is achieved at large coupling K when the correlation length is much larger than the lattice spacing a. The resulting length ratios are ξ/a and $1/\tilde{h} = a/h$, the inverse of the field per unit length. Expressing K in terms of ξ/a (from the classical result $\xi = 2Ka$) and taking $a \to 0$ keeping τ, L_τ, $\hat{h}$, ξ fixed gives the scaling limit Hamiltonian as

$$H_c[\theta(\tau)] = \int_0^{L_\tau} d\tau \left[\frac{\xi}{4} \left(\frac{d\theta(\tau)}{d\tau} \right)^2 - \tilde{h} \cos \theta(\tau) \right]$$

The partition function now becomes a functional integral

$$\mathcal{Z}_c = \sum_{p=-\infty}^{\infty} \int_{\theta(L_\tau)=\theta(0)+2\pi p} \mathcal{D}\theta(\tau) \exp(-H_c[\theta(\tau)])$$

The summation over the number of windings p occurs because θ is now a continuous function or τ. This functional integral can be interpreted as the path integral of a particle constrained to move on a unit circle with angular coordinate θ and p representing the number of times the particle winds around the circle from imaginary time $\tau = 0$ to $\tau = L_\tau$. The quantum Hamiltonian for this particle is (with $\Delta = 1/\xi$)

$$H_Q = -\Delta \frac{\partial^2}{\partial \theta^2} - \tilde{h} \cos \theta$$

Mapping: The quantum-classical mapping is completed by: identifying τ as the imaginary time and the the change $a \to \tau$ as distance to time transformation; writing the Hamiltonian in terms of quantum operators; identifying the characteristic energy Δ as inverse correlation length $1/\xi$ and from the partition function, $1/L_\tau$ as the temperature T in the quantum model. The quantum angular momentum operator is $\hat{L} = \frac{1}{i}\frac{\partial}{\partial\theta}$ which obeys the commutation relation $[\hat{L}, \hat{n}_\alpha] = i\epsilon_{\alpha\beta}\hat{n}_\beta$. The Hamiltonian H_Q is just the point $N = 2$ rotor in an external field

$$H_Q = \Delta \hat{L}^2 - \tilde{\mathbf{h}} \cdot \hat{\mathbf{n}}$$

In terms of this Hamiltonian and its eigenvalues ϵ_μ, the partition function and correlation function are respectively,

$$\mathcal{Z}_c \;=\; \mathrm{Tr} \exp(-H_Q/T) \;=\; \sum_\mu \exp(-\epsilon_\mu/T)$$

$$\langle \mathbf{n}(\tau) \cdot \mathbf{n}(0) \rangle \;=\; \frac{1}{\mathcal{Z}_c} \mathrm{Tr} \left(e^{-H_Q/T} e^{H_Q\tau} \hat{\mathbf{n}} e^{-H_Q\tau} \cdot \hat{\mathbf{n}} \right) \;=\; \frac{1}{\mathcal{Z}_c} \sum_{\mu,\nu} |\langle \mu | \hat{\mathbf{n}} | \nu \rangle|^2 \, e^{-\epsilon_\mu/T} e^{-(\epsilon_\mu - \epsilon_\nu)\tau}$$

The classical transfer matrix becomes an operator that evolves the state by an imaginary time a given by $\hat{T} \approx \exp(-aH_Q)$. Finally, in this quantum-classical mapping the universal functions for the free energy density $\mathcal{F}$ and the correlation function are

$$\mathcal{F} = \frac{1}{L_\tau}\Phi_{\mathcal{F}}\left(\frac{L_\tau}{\xi}, \tilde{h}L_\tau\right) \rightarrow T\Phi_{\mathcal{F}}\left(\frac{\Delta}{T}, \frac{\tilde{h}}{T}\right)$$

$$\langle \mathbf{n}(\tau)\cdot\mathbf{n}(0)\rangle = \Phi_n\left(\frac{\tau}{L_\tau}, \frac{L_\tau}{\xi}, \tilde{h}L_\tau\right) \rightarrow \Phi_n\left(T\tau, \frac{\Delta}{T}, \frac{\tilde{h}}{T}\right)$$

Thus the quantum mechanics of a single $N = 2$ rotor can be mapped onto the statistical mechanics of a XY chain. A similar mapping holds for the single $N = 3$ rotor with with $\hat{\mathbf{L}}$ now a three component operator giving the Hamiltonian and parition function:

$$H_Q = \frac{\Delta}{2}\hat{\mathbf{L}}^2 - \tilde{\mathbf{h}}\cdot\mathbf{n} \qquad L_\alpha = -i\epsilon_{\alpha\beta\gamma}n_\beta\frac{\partial}{\partial n_\gamma}$$

$$\mathcal{Z}_c(\tilde{\mathbf{h}} = 0) = \mathrm{Tr}\exp(-H_Q/T) = \sum_{l=0}^{\infty}(2l+1)\exp\left(-\frac{\Delta}{2T}l(l+1)\right)$$

1.4 Relevance of Quantum Phase Transitions

Although the theory of quantum phase transitions describes the behavior of the ground state of a system, it is of fundamental importance in determining the finite temperature properties of an interacting system. This is because most interacting systems are described in terms of emergent particles or quasiparticles which are excitations above the ground state of the system. Therefore the ordering and properties of the ground state clearly determine the nature of the quasiparticles including their dispersion, range of interactions, occupancy and lifetime. When the ground state is in a quantum critical region $g = g_c$, at higher temperatures the nature of these quasiparticles becomes extremely complicated due to the competing tendencies of the different states. Understanding the physics of this quantum critical region enables the mapping out of the physics at $\mid g - g_c \mid \neq 0$ and $T \neq 0$.

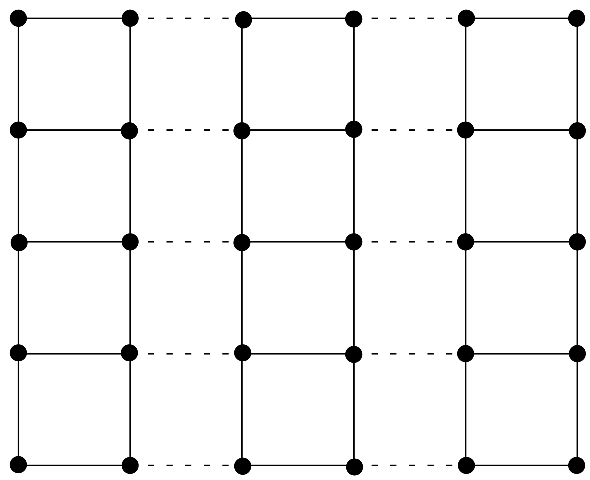

Figure 1: The coupled ladder antiferromagnet. Full lines indicate the A links while dashed lines indicate the B links

2 Correlated Electrons in Two Dimensions

2.1 The coupled ladder anti-ferromagnet

The coupled ladder anti-ferromagnet is an important model in the interacting 2D electron system, particularly the cuprate superconductors. Although it may not represent the microscopic dynamics of the high-T_c materials, there is experimental evidence and theory arguments that show that the universality class of the critical point of this model and the magnetic ordering transitions of the cuprate superconductors is the same. For a system of antiferromagnetically coupled $S = 1/2$ Heisenberg spins, the Hamiltonian is Ref [3]

$$H_\ell = J \sum_{i,j \in A} \mathbf{S}_i \cdot \mathbf{S}_j + \lambda J \sum_{i,j \in B} \mathbf{S}_i \cdot \mathbf{S}_j \tag{1}$$

the $\mathbf{S}_i$ are spin-1/2 operators on the sites of the coupled-ladder lattice (Fig 1), the A links form the ladders while the B links couple the ladders. Also $J > 0$ and $0 \leq \lambda \leq 1$. The parameter λ describes the essential tension between two ordering tendencies of the ground state. This model has two distinct phases at the extremities of the value of λ. At $\lambda \approx 1$, the system is a square lattice Heisenberg antiferromagnet with a long range spin ordering called the 'Néel' order. This ground state has broken spin rotation symmetry and there is a

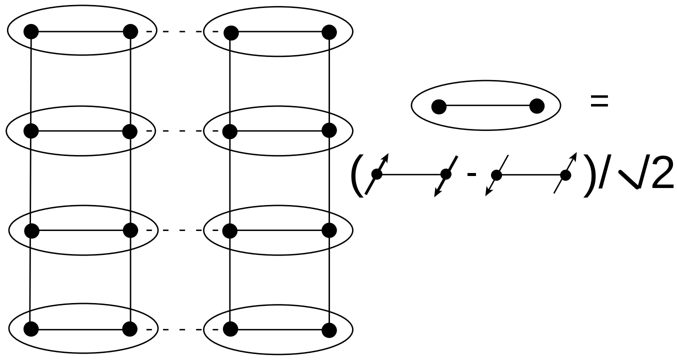

Figure 2: The quantum paramagnet ground state for $\lambda \approx 0$. The ovals represent singlet $S = 0$ pairs.

staggered non-zero expectation value

$$\langle \mathbf{S}_i \rangle = \eta_i N_0 \mathbf{n}$$

where $\mathbf{n}$ is the direction of broken symmetry in spin space and η_i is ± 1 on the two sub-lattices. The excitations above this ground state are the 'magnons' or spin waves of spatial deformation of $\mathbf{n}$. The important feature of these excitations are that they are 'gapless', and there are two polarizations with excitation energy $\varepsilon_k = (c_x^2 k_x^2 + c_y^2 k_y^2)^{1/2}$, with c_x, c_y the spin-wave velocities in the two spatial directions.

For $\lambda \approx 0$, the ground state is paramagnetic because the ladders are decoupled and spin-rotation symmetry is preserved. The neighboring spins on a ladder form $S = 0$ spin singlets that preserve the lattice symmetries (Fig 2). Excitations are formed by breaking this singlet bond to form the threefold degenerate $S = 1$ state. This broken bond can hop from site to site along the lattice and thus constitutes the quasiparticle excitation called the 'spinon'. The energy of this excitation is 'gapped' and is given at low k by,

$$\varepsilon_k = \Delta + \frac{c_x^2 k_x^2 + c_y^2 k_y^2}{2\Delta},$$

The phase diagram of the coupled ladder antiferromagnet is given in Figure 3

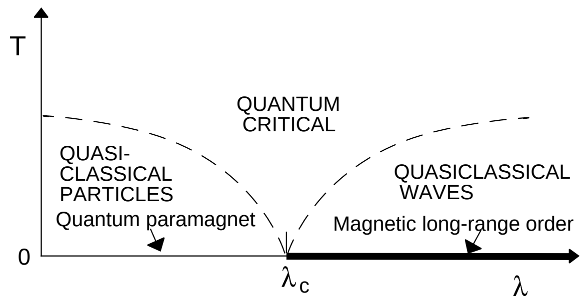

Figure 3: Phase diagram of H_ℓ for $T > 0$ and $0 \leq \lambda \leq 1$

2.2 Quantum-classical mapping for the coupled ladder antiferromagnet

The coupled ladder model can be mapped onto the ϕ^4 theory of a Heisenberg ferromagnet in $D = 3$. The quantum partition function is expressed as a coherent state path integral where the coherent states are a overcomplete basis that give for the spin operator $\langle \mathbf{N} \mid \hat{\mathbf{S}} \mid \mathbf{N} \rangle = S\mathbf{N}$ and represent the minimum uncertainty spin states localized as much in the $\mathbf{N}$ direction as quantum mechanics allows. For a single spin with the Hamiltonian, $H_0 = -\mathbf{h} \cdot \mathbf{S}$, the coherent state path integral represents the partition function as an integral over all closed curves, $\mathbf{N}(\tau)$, on the surface of a unit sphere, where each curve represents a history of the precessing spin in imaginary time, τ

$$Z_0 = \int \mathcal{D}\mathbf{N}(\tau)\delta(\mathbf{N}^2(\tau) - 1) \exp\left(-iS \int \mathcal{A}_\tau(\mathbf{N}(\tau))d\tau + \int d\tau S\mathbf{h} \cdot \mathbf{N}(\tau)\right)$$

Applying the above path integral to every site of the model, and taking a spatial continuum limit of the fields at each site $\mathbf{N}_j(\tau)$ into $\mathbf{n}(r, \tau)$, a unit length continuous order parameter field representing orientation in spin space, gives the coupled ladder partition function

$$\begin{aligned} Z_\ell = \int \mathcal{D}\mathbf{n}(r,\tau)\delta(\mathbf{n}^2(r,\tau) - 1) \exp\Bigg[&-iS \sum_j \eta_j \int d\tau \mathcal{A}_\tau(\mathbf{n}(r_j,\tau)) \\ &- \frac{1}{2g\sqrt{c_x c_y}} \int d^2r d\tau \left((\partial_\tau \mathbf{n})^2 + c_x^2(\partial_x \mathbf{n})^2 + c_y^2(\partial_y \mathbf{n})^2\right)\Bigg] \end{aligned} \quad (2)$$

where $c_x = JSa\sqrt{(1+\lambda)(3+\lambda)}$, $c_y = JSa\sqrt{2(3+\lambda)}$ are the velocities, and the coupling constant is $g = (2a/S)[(3+\lambda)^2/(2+2\lambda)]^{1/4}$. The tension parameter is now g with $g > g_c$ corresponding to the spin gap phase with $\lambda < \lambda_c$, and $g < g_c$ is the $\lambda > \lambda_c$ Néel phase

The first term in the action is the 'Berry phase' term which arises from the commutation relations of the underlying spins. This term which remains complex in both real and imaginary time adds a complex weight to the partition function and is of central importance in determining the phases and critical points of many spin systems. However in the case of the coupled ladder antiferromagnet model, this term has no consequences and so is dropped. For the remaining term, interpreting the imaginary time coordinate as a third spatial coordinate, the path integral Z_ℓ reduces to that of a 3-dimensional ferromagnet with $\mathbf{n}$ as the local magnetization and $g \sim T_{cl}$ the classical temperature. Performing a coarse-graining transformation from $\mathbf{n}$ to a spin field $\phi_\alpha(r,\tau),\ (\alpha = x, y, z)$ gives for Z_ℓ

$$Z_\ell = \int \mathcal{D}\varphi_\alpha(r,\tau) \exp\left[-\int d^2r d\tau \left\{\frac{1}{2}\left((\partial_\tau\varphi_\alpha)^2 + c_x^2(\partial_x\varphi_\alpha)^2 + c_y^2(\partial_y\varphi_\alpha)^2 + s\varphi_\alpha^2\right) + \frac{u}{24}\left(\varphi_\alpha^2\right)^2\right\}\right] \quad (3)$$

In this mapping to the ϕ^4 theory, the ferromagnetic order which appears for $s < s_c$ with expectation value $\sqrt{-6s/u}$ in the classical model corresponds to the antiferromagnetic Néel order in the quantum model. The response and correlation functions can be calculated along the imaginary time direction in the classical model and analytically continued to obtain the real time expressions. The response function to a staggered transverse applied field to the Néel order is

$$\chi_\perp(k,\omega) = \frac{1}{c_x^2 k_x^2 + c_y k_y^2 - \omega^2}$$

The poles of this response functions represent the two spin wave excitations and the dispersion $\left(c_x^2 k_x^2 + c_y^2 k_y^2\right)^{1/2}$ is the same as the result given before.

For $s > s_c$, the disordered phase of the classical model corresponds to the quantum paramagnet phase. The susceptibility for real frequencies in this case is:

$$\chi(k,\omega) = \frac{\mathcal{Z}}{\Delta^2 + c_x^2 k_x^2 + c_y k_y^2 - \omega^2}$$

There is a gap to the excitations. The dispersion relation given earlier is reproduced with a low k expansion of the pole in the above expression.

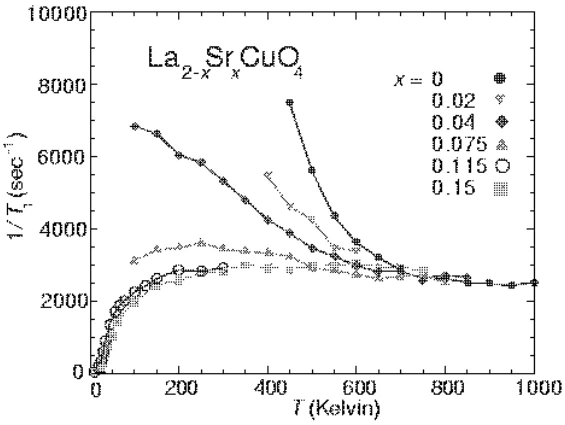

Figure 4: Measurement of the relaxation of the Cu nuclei (a measure of the spectral density of spin fluctuations) as a function of doping and T Ref [5]. At low doping, the behavior is of spin wave excitations while at high doping the behavior is the signature of gapped quasiparticle excitations

3 Connections to Cuprate Superconductivity

Although the coupled ladder antiferromagnet Hamiltonian H_ℓ may not be the microscopic model for High-T_c superconductors, measurements of spin fluctuations in these materials have crossovers very similar to that of the critical point of the model. This is described in the nuclear spin relaxation experiments Ref [5] shown and described in Fig 4

The tuning parameter for High-T_c superconductors is the doping. Therefore consider the phase diagram of the doped square lattice antiferromagnet. This is given in Fig 5 as a function of doping δ and a parameter N which is the number of components of each spin. The regions of the figure each represent the breaking of a specific symmetry: $\mathcal{S}$, the electromagnetic $U(1)$ gauge symmetry which is broken in any superconducting state. $\mathcal{M}$, the $SU(2)$ spin symmetry, broken in magnetically ordered states. $\mathcal{C}$, the symmetry of the space group of the square lattice. In this case, this symmetry is broken by a specific type of spin wave. In cuprate materials, superconductivity appears when the square lattice of the CuO_2 planes is doped with mobile carriers. There is close similarity between the transitions of the cuprate superconductors and the doped antiferromagnet. At low doping δ and high N, both have stripe like structures with the distance between stripes inversely proportional

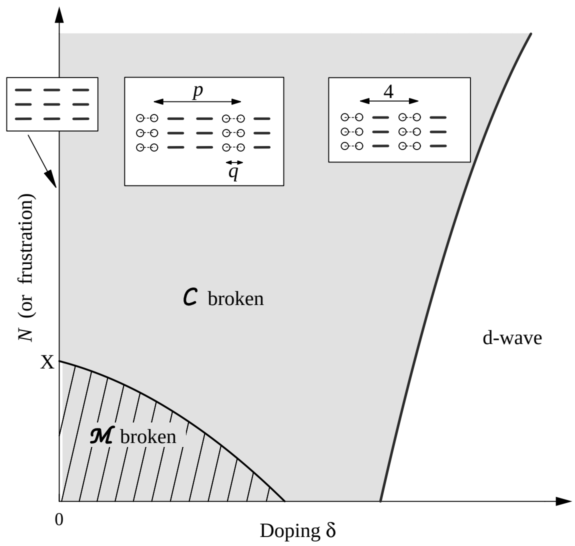

Figure 5: Ground state phase diagram of a doped antifeffomagnet. The labeled regions correspond to broken symmetry with $\mathcal{S}$ broken for all $\delta > 0$ and large N. The insets show the nature of $\mathcal{C}$ breaking at large N

to δ. The region of broken $\mathcal{M}$ symmetry is a Néel state with magnetic order, and known to exist in the cuprates. At the onset of this $\mathcal{M}$ symmetry breaking, the quantum critical behavior is quite similar to that of the coupled ladder antiferromagnet. At itermediate doping when the $\mathcal{C}$ symmetry is broken it is expected that there is coexistence of superconducting order and magnetic order in the form of a magnetic spin-density-wave (SC+SDW state). At large δ the $\mathcal{C}$ symmetry is restored and the state is pure d-wave superconductor (SD state).

3.1 Magnetic transitions in d-wave superconductors

For the SC+SDW to SC transition, the coarsegrained spin wave field is taken to be

$$S_{j\alpha} = \Phi_\alpha(r_j)e^{iKr} + \text{c.c.},$$

Starting from the Hamiltonian and performing the Bogoliubov transformation obtains in succession

$$H_{tJ} = \sum_k \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} + J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$H_{BCS} = \sum_k \varepsilon_k c^\dagger_{k\sigma} c_{k\sigma} - \frac{J_1}{2} \sum_{j\mu} \Delta_\mu \left(c^\dagger_{j\uparrow} c^\dagger_{j+\hat{\mu},\downarrow} - c^\dagger_{j\downarrow} c^\dagger_{j+\hat{\mu},\uparrow} \right) + \text{h.c.}$$

where $c_{j\sigma}$ is the annihilation operator for an electron on site j, given in momentum space $c_{k\sigma}$, and ε_k is the dispersion of the electrons. The second term is similar to the J_1 term in the coupled ladder model with $S_{j\alpha} = \frac{1}{2} c^\dagger_{j\sigma} \sigma^\alpha_{\sigma\sigma'} c_{j\sigma'}$ here. The terms in the action involving the fields $\Phi_\alpha(r_j)$ are similar to those of the ϕ^4 theory described before. To account for the spin $S = 1/2$ fermionic excitations, viz. the Bogoliubov quasi particles, an additional term in the action is required involving the spinors Ψ_1 and Ψ_2 which is

$$\mathcal{S}_\Psi = \int d\tau d^2x \left[\Psi^\dagger_1 \left(\partial_\tau - i v_F \tau^z \partial_x - i v_\Delta \tau^x \partial_y \right) \Psi_1 \ + \Psi^\dagger_2 \left(\partial_\tau - i v_F \tau^z \partial_y - i v_\Delta \tau^x \partial_x \right) \Psi_2 \right],$$

The simplest possible coupling between the Φ_α and the the $\Psi_{1,2}$ is

$$\kappa \int d^2 r d\tau |\Phi_\alpha|^2 \left(\Psi^\dagger_1 \Psi_1 + \Psi^\dagger_2 \Psi_2 \right).$$

However, from a scaling analysis, it is seen that the coupling strength κ is irrelevant at the SC+SDW to SC critical point. Therefore the critical magnetic fluctuations are entirely associated with the the Φ_α with no involvement of the S=1/2 quasiparticles. This implies that the $S = 1$ exciton should be stable in the SC state near the transition.

This has been observed experimentally Ref [6]. The net consequence is that the quantum critical behavior of the coupled ladder antiferromagnet describes this quantum transition also.

In addition to the magnetic ordering transitions, a quantum critical point has been observed in what is the pure SC state. Working with the Hubbard model on the square lattice, the ground state in the half filling electron density is found to have $d_{x^2-y^2}$ symmetry while at low density it is found to have d_{xy} symmetry in the relative coordinate. This can be analyzed by introducing terms for the d_{xy} interactions in the Hamiltonian. The classical field theory for this model is not the standard ϕ^4 theory but involves a cubic term $\sim \phi^3$ as well. There is experimental evidence for this transition also Ref [7]. From the theory and experiments, a plausible $T = 0$ phase diagram for the cuprate superconductors is shown in Fig 6.

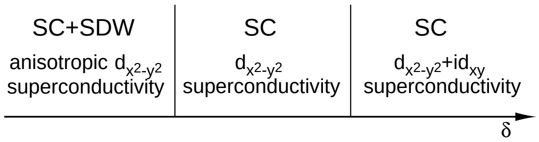

Figure 6: Conjectured $T = 0$ phase diagram for the cuprate superconductors as a function of increasing hole concentration δ

4 Conclusions

Quantum phase transitions which happen in the ground state of interacting systems have a profound influence on the finite temperature physics by determining the nature and properties of the quasiparticle excitations. In many cases it is possible to map the quantum mechanics of the ground state in d spatial dimensions to a classical model of finite temperature transition in $D = d + 1$ dimensions. The coupled ladder antiferromagnet model is of central importance in studying electronic systems in two dimensions and on square lattices. It is seen that the magnetic order transitions in the cuprate superconductors have similar universal properties as the model with theoretical arguments and experimental evidence in support.

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