Abstract
This essay describes the observations of the magnetization $M$ above the critical temperature $T_c$ in cuprates. The nonlinear behavior of $M$ suggests the existence of a "vortex liquid" region above $T_c$, which is a region of a strong superconducting fluctuations. This conclusion is consistent with the Nernst signal and other experimental data. At least in two families of cuprates, the low field magnetization exhibits non-analytic divergent behavior $M \sim H^{1/\delta}$, $\delta > 1$. The possible explanations are based on the Kosterlitz-Thouless (KT) transition and incompressibility of vortex liquid are considered in this paper.

Introduction
The phenomenon of superconductivity is truly one of the most important discoveries of the XX century. It took about half of the last century to discover the microscopic theory of superconductivity and it seems that the mystery of high-temperature superconductors might also remain unsolved for a long time. Even though it is two decades after the discovery of cuprates, there is still debate about the origin of the highly anomalous "normal" state above the superconducting transition temperature. The main questions are: i) what is the extent of the temperature and magnetic field regime in which superconducting fluctuations are observed; ii) how to characterize the regime of superconducting fluctuation theoretically. It is worth noting that the region of strong superconducting correlation is also called "vortex liquid". The reason that these problems are difficult is that other types of order parameters fluctuations are also significant, thus it is highly complex to disentangle the contribution from the different kinds of fluctuations.

The evidence of incipient order above $T_c$ and of a single-particle gap that persists into the normal state was obtained from STM and ARPES measurements [2]. However
it is quite difficult to tell the difference between a superconducting gap and a density-wave gap.

Numerous experiments that making use of the Nernst effect have demonstrated an enhanced Nernst signal in hole-doped cuprates at temperature $T$ significantly above the superconducting transition temperature. The Nernst effect is a thermoelectric phenomenon in which a voltage transverse to a temperature gradient is created in a conducting sample subjected to a magnetic field. The region of the enhanced Nernst signal for $T > T_c$ is supposed to be a continuous extension of the vortex-liquid state below $T_c$. That is why this region is referred as “vortex liquid”. In this state the enhanced Nernst signal originates from phase slippage caused by singular phase fluctuations of the pair condensate [1]. According to the phase-disordering scenario [6], the disappearance of the Meissner effect at $T_c$ is caused by the loss of long-range phase coherence, rather than the vanishing of the pair condensate. However, the persistent short-range phase stiffness still supports vorticity and produces a large, strongly temperature dependent Nernst signal in some region above $T_c$. The Nernst signal, unfortunately, is highly sensitive not only to superconducting fluctuations, but also to any type of order that leads to a reconstruction of Fermi surface. That is why there are other plausible interpretation than the fluctuating vortex-liquid interpretation [1].

The method that is not subject to the above mentioned shortcomings is the measurement of the magnetization. Even if the superconducting correlation length is finite, the fluctuation diamagnetism can be large in comparison to the "background", because the diamagnetic response of an ordered superconductor is many orders of magnitude stronger than that of any other known state of matter. To top it all, as opposed to the Nernst effect, the magnetization is a thermodynamic quality, thus it is not subject to the uncertainties in interpretation that are inherent to dynamical and nonequilibrium properties.

Li et al.[1] reported the results of a major experimental study of the magnetization of several important families of cuprate high-temperature superconductors over a broad range of temperatures and magnetic fields in their recent paper. They found that for a wide range of temperatures above $T_c$ there is a strong, nonlinear in $H$, diamagnetic response. From the data, they extracted the field-dependent onset temperature $T_M$, below which the diamagnetic response was observed. The nonlinearity and the high magnitude of the diamagnetic response is strongly suggestive about its superconducting nature. To top it all the onset temperature $T_M$ of the diamagnetic response demonstrate a strong correlation with the onset temperature of the Nernst signal and thus cleans up an ambiguity in the interpretation of the Nernst effect experiments. Li et al. also found an interesting feature of the low field magnetization in this and one of their previous papers [3]. It exhibits a non-analytic behavior, in some
Figure 1: Figure from Li et al. [1]. Magnetization curves of sample LSCO 09 with Sr content $x = 0.09$ and transition temperature $T_c = 24K$ measured in magnetic fields $H$ up to 14 T. (A) The (total) effective magnetization $M_{eff}$ vs. $H$ at temperatures $4.2 \leq T \leq 200K$. (B) The diamagnetic magnetization $M_d$ vs. $H$ at temperatures $4.2 \leq T \leq 30K$. (C) Curves of $M_d$ vs. $H$ at $22K \leq T \leq 80K$ displayed in expanded scale. In LSCO 09, the diamagnetic signal persists to more than 60 K above $T_c$. In (B) and (C), the bold curve is measured at the separatrix temperature $T_s = 22 K$.

range of temperatures above $T_c$ that leads to a divergent susceptibility.

In this paper, we, at first, consider the experiment of Li et al. and its implications. Second, we discuss the divergent behavior of the magnetic susceptibility in the low fields and two possible explanations of it. One is based on the Kosterlitz-Thouless transition in a layered superconductor with zero Josephson coupling between planes [5]. The other was proposed by P. Anderson [4] and it treats the vortex liquid as an "incompressible superfluid". After that we discuss the results and suggest how the phase diagram of a cuprate superconductor should look like.

**Torque magnetometry experiment. Nonlinear diamagnetic response above $T_c$ and its implications [1, 3]** The magnetization is measured by making use of torque magnetometry technique. In this method the sample is glued to the tip of a thin cantilever, with $H$ applied at a small angle to the crystal $c$-axis. Because of the 2D electronic dispersion in cuprates, the diamagnetic currents are largely confined to the
a-b plane, which makes torque magnetometry a very sensitive probe of diamagnetism in cuprates. The deflection of the cantilever gives a torque signal which corresponds to the effective magnetization $M_{eff}$.

In order to show the analysis of the data, let’s first examine the dependence of $M_{eff}(H)$ for the sample of La$_{2-x}$Sr$_x$CuO$_4$ with $x = 0.09$ (LSCO 09), shown on Fig.1. For this sample $T_c = 24K$. One can note that at temperature $T > 100K$, $M_{eff}$ is strictly linear in $H$ and paramagnetic in sign (see panel A). This behavior corresponds to the dominance of the anisotropic Van Vleck paramagnetic susceptibility $\Delta \chi_p$, which has a weak $T$ dependence given by $\Delta \chi_p = A + BT$, with $A \gg BT > 0$ at 200 K. Below 100 K, $M_{eff}$ begins to display a weak diamagnetic contribution. The temperature at which the diamagnetic contribution appears is identified as the onset temperature $T_{onset}$. $M_{eff}$ becomes increasingly nonlinear in $H$ as $T$ decreases from 60K to $T_c$ (24 K). Below $T_c$, the diamagnetic term becomes so dominant in magnitude that $M_{eff}$ has large negative values, in spite of the paramagnetic Van Vleck term. Li et al. assume that the paramagnetic background $\Delta \chi_p H$ follows the trend that is seen at $T > T_{onset}$ and consequently the diamagnetic term $M_d$ is related to $M_{eff}$ by

$$M_{eff}(H) = M_d(H) + \Delta \chi_p H = M_d(H) + (A + BT)H.$$ 

Hereafter, we consider mostly $M_d(H)$, subtracting the background Van Vleck term from $M_{eff}$.

Now we further inspect $M_d(H)$ for LSCO 09, which are displayed at selected $T$ in Fig. 1 B (4.2-30 K) and Fig. 1 C (20-80 K). As shown in Panel B, $M_d$ is nonlinear in $H$ over a broad range of temperature. The curve at 22 K has a flat profile and corresponds to a "separatrix" temperature $T_s$. Below $T_s$, $M_d$ takes on very large, negative values at small $H$. It is noteworthy that the low temperature curvature of $M_d$ vs. $H$ curves changes from negative below $T_s$ to positive above $T_s$. Panel C displays $M_d$ vs. $H$ curves of LSCO 09 sample at $T \geq T_s$ in expanded scale. The curves remain diamagnetic, displaying strong nonlinearity. Such a nonlinear diamagnetic response above $T_c$ suggests the presence of local supercurrents as well as finite pair amplitude in the pseudogap state.

In order to check, if the diamagnetic signals above $T_c$ can be suppressed by an intense magnetic field, Li et al. make extended measurements on LSCO 09 up to 33 T. In this case, $M_d - H$ curves display a broad minimum. For higher $H$, $M_d$ tends to zero as $H$ is increased, which is a sign of superconducting fluctuation. It should be noticed that the minimum the $M_d - H$ curve increases rapidly with $T$ (from 8 T at 25 K to 33 T at 40 K).

$YBCO$. Optimally-doped YBCO (YBa$_2$Cu$_3$O$_7$ with $T_c = 92K$) is distinguished as the cuprate with the smallest resistivity anisotropy and the largest interlayer (c-coupling)
Figure 2: Figure from Li et al. [1]. Contour plot of the diamagnetic magnetization $|M_d(T, H)|$ of UD Bi 2201, with La content $y = 0.7$ and $T_c = 12K$ (arrow). The spacing between adjacent contour lines is 10 A/m for $T < T_c$. The upper critical field $H_{c2}$ (defined by extrapolating $M_d \to 0$) is plotted as open circles.

energy. Because the coherence-length anisotropy $\xi_a/\xi_b = 3 - 5$ is only moderate, the vortices have the largest stiffness modulus along $c$ among cuprates. Here $\xi_a$ and $\xi_b$ are the coherence length along the axes $a$ and $c$, respectively. Optimally-doped YBCO should be the least susceptible to the phase disordering mechanism for the distraction of long-range phase coherence at $T_c$ and hence the best candidate for Gaussian fluctuations among cuprates [1].

Nevertheless, the experiments suggest that $T_c$ in optimally-doped YBCO is also dictated by large phase fluctuation. The $M_d-H$ curves are similar to those in LSCO 09. Only the magnitude of $M_d$ is different at comparable $H$ and $T$. The curves displaying significant diamagnetism surviving to intense fields at temperatures up to 40 K above $T_c$, which is a strong evidence that we are observing the phase-disordering mechanism, rather than Gaussian mean-field fluctuations.

Bi 2201, Bi2212. The nonlinear diamagnetic signals above $T_c$ are also observed in a single-layer $\text{Bi}_2\text{Sr}_{2-y}\text{La}_y\text{CuO}_6$ (Bi 2201) family. In this family, the transition temperature $T_c$ depends on the La content $y$. The optimal doping (OD) corresponds to $y \sim 0.44$. Specimens with $y > 0.44$ are underdoped (UD), while those with $y < 0.44$ are overdoped (OD). The measurements were made in OD Bi 2201 samples and in
both OD and UD regions. Above $T_c$, the $M_dH$ curves in Bi 2201 are also similar to those in LSCO (Fig.1) and YBCO samples, except that the magnitude of $M_d$ and field scales are slightly smaller than for LSCO. Above $T_c$, $M_d$ attains a broad minimum at fields below 20 T, and then approaches zero at $H \geq 40T$. Like the curves for LSCO and YBCO, the the low-field $M_d$-$H$ curves change a curvature sign across $T_c$. In order to suppress $M_d$ we need to apply about 20 T. Above $T_c$, the complete suppression of $M_d$ requires very high fields - comparable to those needed below $T_c$. An interesting weak field region is discussed under “fragile London rigidity”.

Contour plot. An excellent way to view the nonlinear diamagnetic magnetization is the contour plot of $M_d$ in the $T-H$ plane. Fig. 2 displays the contour plot in single-layered UD Bi 2201 ($y = 0.7, T_c = 12K$). The value of $|M_d|$ is as indicated at selected contours. With $H$ fixed (e.g at 10 T), $|M_d|$ decreases monotonically as $T$ is raised from 4 K to 60 K. Just as in the Nernst signal, the diamagnetic signal in the T-H plane bulges out to temperatures high above $T_c$, with no obvious discontinuities or changes in slope. The highest temperature at which $M_d$ is resolved is $\sim 50K$ (the onset temperature in this sample). The absence of a boundary implies that the vortex-liquid state below $T_c$ evolves continuously to the diamagnetic state above $T_c$.

Onset temperature. An important question is how high in temperature does the diamagnetic signal extend above $T_c$. We plot $M_{eff}$ measured in fixed H (14 T) versus $T$. Fig. 3(a) displays these plot for samples of Bi 2201 and LSCO in panels A and B, respectively. The high temperature part of the dependences may be fitted to the Van Vleck anisotropy term $\Delta \chi_p = A + BT$ (a straight line). We can determine $T_{onset}$ quite accurately, provided a set of points above $T_{onset}$ is sufficiently dense. In optimally doped samples of each family, $T_{onset}$ extends above $T_c$ by factors of 1.3 (YBCO), 1.4 (Bi 2212), 2.1 (LSCO) and 2.5 (Bi 2201). This is in dramatic contrast to the fluctuating diamagnetism which is observed in disordered low-$T_c$ superconductors. For instance, the sample of disordered Mg$_{1-x}$B$_2$Al$_x$ ($x = 0.25, T_c = 25K$) has a broad transition width of $\sim 15K$ and sizeable diamagnetism exists in the narrow interval 28 - 32 K above its $T_c$. But even though the profile of $M_d$ vs. $H$ is roughly similar, the factor $T_{onset}/T_c < 1.3$.

In order to compare $T_{onset}^M$ obtained here with the onset temperature of the vortex Nernst signal $T_{onset}^{\nu}$, we plot the onset temperatures vs. doping $x$ in the phase diagram for LSCO and Bi 2201 (Fig.3(b)). Notably, in LSCO, $T_{onset}^M$ (open squares) is almost equal to $T_{onset}^{\nu}$ over the entire doping range. In Bi 2201 the temperature scales are also the same. The difference in the slopes might be explained by the fact that the measurements were made on different samples.
**Figure 3:** Figure from Li *et al.* [1]. (a) Plots of the temperature dependence of $M_{\text{eff}}(T)$ in Bi 2201 (Panel A) and in LSCO (B), showing the onset of diamagnetism as $T$ is decreased. In both panels, the value of $M_{\text{eff}}$ measured at $H = 14$ T is plotted vs. $T$ in samples with various doping levels $x$. In general, $M_{\text{eff}}$ at high $T$ varies weakly vs. $T$, as shown by the straight lines which are of the form $A + BT$. Relative to this linear background, $M_{\text{eff}}$ shows a strong downwards deviation starting at the onset temperature $T_{\text{onset}}^M$ (indicated by arrows). (b) Phase diagram comparing the onset temperatures for the Nernst and diamagnetism signals vs. doping $x$ in La$_{2-x}$Sr$_x$CuO$_4$ (Panel A) and in Bi$_2$Sr$_{2-y}$La$_y$CuO$_6$ (Panel B). The superconducting transition temperature $T_c$ (solid circles) is plotted with the onset temperature $T_{\text{onset}}^\nu$ determined by the Nernst effect (solid diamonds), and $T_{\text{onset}}^M$ determined by torque magnetometry (open squares). In Panel B for Bi$_2$Sr$_{2-y}$La$_y$CuO$_6$, a large La content $y$ implies small hole carrier concentration (UD regime).

**Fragile London rigidity.** One of the most interesting features of the vortex liquid state above $T_c$ is the fragile London rigidity, observable in the limit $H \to 0$. Let’s investigate closely how $M_d$ approaches zero in OPT Bi 2201 Fig.1 (Panel C). The $M_d$ curves display a strong curvature as $H$ approaches zero from either direction. As $T$ decreases from 38 K to $T_c$ (30 K), the zero-H slope rises sharply to a vertical line. The curve at $T_c$ (25 K is the closest one to $T_c = 25K$ on this plot) seems to approach a logarithmic dependence vs. $H$. However mechanical noise precludes accurate measurements for $|H| < 300$Oe, thus precludes to investigate this feature in this particular experiment.

In one of his earlier experiments [3], Li *et al.* used high-resolution SQUID magnetometry to extend measurements in Bi 2212 down to 10 Oe. They discovered that over a broad interval of $T$ (86-105 K) in OPT Bi 2212, the low-H $M_d$ follows the power-law
dependence

\[ M_d(T, H) \sim -H^{1/\delta(T)} \quad (H \to 0), \]

with an exponent \( \delta(T) \) that grows rapidly from 1 (at \( T \approx 105K \)) to large scales (> 6) as \( T \to T_c^+ \). This implies that the weak-field diamagnetic susceptibility

\[ \chi = \lim_{H \to 0} \frac{M}{H} \to -\infty \]

is weakly divergent throughout the interval in T where \( \delta > 1 \). However, this divergence is extremely sensitive to field suppression. London rigidity seems to reflect the increasing tendency of the phase-disordered condensate to establish long-range superfluid response as \( T \to T_c^+ \). This feature has no analog in bulk samples of low-\( T_c \) superconductors, but may exists in a finite T interval above the Kosterlitz-Thouless(KT) transition in 2D systems such as Mo\(_{1-x}\)Ge\(_x\) and InO\(_x\).

The Kosterlitz-Thouless RG calculation of the magnetization\[5\]. In paper \[5\] Oganesyan \textit{et al.} report an RG calculation of the magnetization of a two-dimentional superconductor in a perpendicular magnetic field near its Kosterlitz-Thouless transition and at lower temperatures.

To compute the free-energy and thence magnetization \( M \) they map the two-dimensional vortex problem onto a two-component Coulomb plasma whose Hamiltonian is given by

\[ H = N_T E_{c0} + \pi \rho_{s0} \sum_{i<j} q_i q_j \ln r_{ij}^2 / a_0^2 + H_B. \] (1)

The number of vortices of charge \( q_i = \pm 1 \) is \( N_\pm \). The total number of vortices is \( N_T = N_+ + N_- \), and it is allowed to fluctuate by addition/removal of neutral vortex-antivortex pairs. However the net charge \( Q = (N_- - N_+) L^2 B / \phi_0 \) is constrained by the field B. \( E_{c0} \) is a bare vortex energy cutoff, \( a_0 \) is a “vortex core radius”, \( \rho_{s0} \) is the bare superfluid stiffness (inverse dielectric constant in the plasma language), and \( L^2 \) is the area of the system. The density of the field-induced vortices is \( n = B a_0^2 / \phi_0 \).

It is noteworthy, that Oganesyan \textit{et al.} generalize standard Kosterlitz RG method to non-neutral situation. The brief description of the main steps of their calculation without giving details is as follows. The couplings of the Hamiltonian is renormalized upon increasing the cutoff to \( a = a_0 b \) and integrating out neutral vortex-antivortex pairs with spacing less than \( a \). Under RG flow, the density of field-induced vortices grows as \( n(b) = n_0 b^2 \). For \( T \leq T_{KT} \) (\( T_{KT} \) is a critical temperature of the transition) thermally induced vortices become increasingly dilute and the system approaches a one component plasma. On the next step, the asymptotic behavior of magnetization derived from the exactly known free energy of a dilute one component plasma. The
simplified scaling relation for low-field $M$ when $T \leq T_{KT}$ looks like.

$$|M| \sim (T_{2D} - T)\ln(H_c/|H|)$$  \hspace{1cm} (2)

Actually, this relation is quite different from experimental data for $\delta(T)$, but it gives a divergent susceptibility.

Incompressible vortex fluid. [4]. According to P. W. Anderson [4], the nonlinear behavior of diamagnetic susceptibility is caused by the missing term in the conventionally accepted model Hamiltonian for quantized vortices in the Bose fluid. Anderson starts his consideration from a claim that it is appropriate to describe the phase above $T_c$ as a vortex fluid and that the system is similar to supersolid He. He infers the next properties of a vortex fluid state. In most range of observation it is dissipative. It means that the random motions of the vortices constitute a thermal reservoir into which energy may be dissipated, and current-current correlations decay with time. It is also incompressible in the sense that inserting an extra quantum of vorticity costs an energy which is divergent in the distances between such extra vortices. Other theories of vortex-mediated transitions such as Kosterlitz-Thouless and Willians theories discuss only the questions of adding/removing of pairs of vortices of opposite sign or vortex loops. Anderson takes into account the addition of net vorticity and suggests that the response of a vortex liquid to this is anomalous.

Here we consider a 2D model, that should be simply generalized to 3D. In this model the current of vortices is simply the sum of those due to the individual vortex points and the energy is the integral of the sum of the square of the sum over all vortices.

$$J_i = \nabla \phi_i = q_i \hat{\theta}_i / |r - r_i|, q_i = \pm 1; \quad U = \frac{1}{2} \int d^2r (\sum_i J_i)^2. \hspace{1cm} (3)$$

There must be a lower cutoff around the vortex points $a$ and an upper cutoff $R$ for the sample as whole. After the integration of the energy we obtain

$$U/2\pi = \sum_i q_i^2 \ln(R/a) + \sum_{i \neq j} q_i q_j \ln(r_{ij}) = (\sum_i q_i)^2 \ln(R/a) - \sum_{i \neq j} q_i q_j \ln r_{ij}/a \hspace{1cm} (4)$$

If the system of vortices is neutral ($\sum_i q_i = 0$) then the self-energy of the vortices which diverges logarithmically cancels and we have the standard Kosterlitz-Thouless interaction energy result.

$$U_{K-T} = -2\pi \sum_{i \neq j} q_i q_j \ln(r_i - r_j)/a + \sum_j E_c, \hspace{1cm} (5)$$
where $E_c$ is a core energy. However if we have a mismatch in + and - vortices (which corresponds in our case to an external B field) there remains an additional to Eq.3 divergent term, proportional to the logarithm of the upper cutoff radius. Anderson shows that this additional term is proportional to $n_V \ln(R_c^2/a^2) = n_V \ln(1/n_V a^2)$, where $n_V$ is a density of ”extra” vortices and $R_c$ is approximately the distance between unpaired ”field” vortices. The crucial point, which makes the vortex liquid incompressible, is that the additional energy term is not screened out by the thermally excited pairs above $T_c$. It is noteworthy, that this is in contrast to seemingly similar systems in electrostatics, where we have screening. To sum it up, according to Anderson, ”the

Discussion

The boundary between the normal and superconducting state (Fig.4) is defined as a line, across which the resistance vanishes. The other lines on a phase diagram are, as far as we know, crossovers from one state to another, consequently, they are more fuzzy. The region labeled “vortex liquid” is identified as a region

![Phase diagram](image)

Figure 4: Figure from [2]. Schematic phase diagram of a cuprate superconductor plotted as a function of the applied field (H) and temperature (T). The region in which evidence of unusual, possibly 2D superconducting behavior is found in some cuprates is indicated by the dark blue region above $T_c$. $T^*$ denotes the crossover between the bad metal phase.

anomalous response is not a critical phenomenon but an intrinsic property of the vortex liquid phase, and it should persist as long as there is a finite core energy for vortices". 
of strong superconducting fluctuations because of several characteristic features of magnetization curves: the magnetization is opposite in direction to \( H \) (diamagnetic), it is large compared to (for example) the Landau diamagnetism in conventional metals, and it is nonlinear function of \( H \). \( T_M \) is identified as the point at which \( M_d \) vanishes, and where \( M_{eff} \) changes from being a linear function of \( H \) to nonlinear. The nonlinearity is an expected feature of a superconducting state. For small \( H \), \( M_d \) grows, but after some characteristic value of the field \( H_{min} \) it should tends to zero. In other words, a \( M_d \) curve exhibit a minimum at some nonzero field \( H_{min} \). Experiments suggest that \( H_{min} \to 0 \) as \( T \to T_c \) in all samples, considered above excepts OPT YBCO. The observation of \( H_{min} \) requires very high magnitudes of a magnetic field (up to 45 T), because \( H_{min} \) grows rapidly as we increase the temperature above \( T_c \). The important result is the smoothness of \( M_d \) across \( T_c \), which suggests that the region of the phase diagram above \( T_c \) corresponds to an extension of a "vortex-liquid" with somehow degraded, by a disorder, long-range correlation.

It is worth to emphasize, that the “vortex liquid” state has two features: 1) In two families of cuprates (Bi 2201 and Bi 2212), a nonlinear behavior of the low-field magnetization was observed : \( M \sim H^{1/\delta(T)} \), \( \delta > 1 \) for some range of temperatures \( T_{2D} > T > T_c \), with a strongly dependent exponent \( \delta \). If this behavior really extends to arbitrary small \( H \), this means that the susceptibility is divergent. At the present, we have measurements only down to 10 Oe [3]. Probably, we can make measurements for even lower values of the magnetic field by making use of the phase-locked cantilever magnetometry technique, which is currently employed by the group of R. Budakian at UIUC.

There are two main ideas of why the magnetization displays a non-analitic behavior. The first was proposed by Oganesyan et al. [5]. They noticed that a layered superconductor with zero Josephson coupling between planes displays has a diamagnetic magnetization at small \( H \) given by \( |M| \sim (T_{2D} - T)\ln(H_c/\ |H|) \) below the Kosterlitz-Thouless transition temperature. This scaling relation, at least, produces a divergent susceptibility. However, an apparent problem is that even weak but finite interplane Josephson coupling leads to 3D superconducting transition with \( T_c > T_{2D} \) [2].

2) It is no clear, why \( H_{min} \to 0 \) as \( T \to T_c \). We need more experimental data to find a precise value of critical exponents. The present in [1] data is consistent with the scaling behavior \( H_{min} \sim (T - T_c)^{2\nu} \), with \( 2\nu \sim 1 \). This is not consistent with 3D XY(Kosterlitz-Thouless) scaling, but looks reasonable for 3D critical points.
References


