Superfluid Phase Transition in Gaseous Two Component Lithium-6 System: Critical Temperature

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Abstract

We describe briefly the phenomenon of supefluidity. Next we explain the general approach followed by Stoof et.al[1, 2] using field theoretical methods and mean field theory to estimate the critical temperature of the superfluid phase transition of a two component gaseous system of Lithium-6. Finally we describe the recent experiment which is considered to prove conclusively the existence of such a phase transition[3].

I. SUPERFLUIDITY

Superfluidity is a phase of matter characterized by the complete absence of viscosity. Thus superfluids, placed in a closed loop, can flow endlessly without friction. Superfluidity was discovered by Pyotr Leonidovich Kapitsa, John F. Allen, and Don Misener in 1937. The superfluid transition is displayed by quantum liquids below a characteristic transition temperature. Physicists at MIT in Cambridge recently created a new form of matter, a superfluid gas of atoms. They used the lithium-6 isotope(a fermion) and cooled it to about 50 billionths of a kelvin above absolute zero. The determination of the transition temperature to superfluid state in ^{6}Li is the subject of this paper.

To date there is no theoretical prediction or experimental measurement of the critical exponents in superfluid Lithium-6. One expects however that this phase transition belongs to the same universality class as the BCS superconductivity phase transition in metals because their hamiltonians are similar. It is widely believed that the underlying mechanism is the same in both phase transitions the difference being that superfluidity is a neutral system. The fact that a system belongs to a given universality class must be determined by measuring the critical exponents or theoretically by mapping the hamiltonian to a known class-hamiltonian. Technically, universality is a prediction of the renormalization group theory of phase transitions, which states that the thermodynamic properties of a system near a phase transition depend only on a small number of features, such as dimensionality and symmetry, and is insensitive to the underlying microscopic properties of the system. In case of Lithium-6 a formal prove has not been given.

On the basis of the analogy between supeconductivity Stoof[1] was the first to predict that there could be a phase transition in gaseous Lithium-6 for sufficiently dilute systems whic was within experimental reach usign the newly discovered Fershbach resonance. In this work I will explain how to use the methods of field theory and mean field theory to estimate the critical temperature of superfluid Lithium-6. Finally, I will describe what experimental evidence we have that support the existence of gas superfluid in ^{6}Li.

II. TRANSITION TEMPERATURE

The accepted belief within the scientific community is that a superfluid phase transition in two component Fermi gas is, some how, equivalent to the superconductivity phase transition in metals; the formation of pairs with opposite spin due to an effective attractive interparticle interaction and some kind of condensation of them being the fundamental mechanism in both cases. In metals the attractive interaction between electrons is due to exchange of phonon whereas in Lithium the attractive interaction between atoms is due to the bare atractive triplet interatomic potential.

Superfluidity is a many body effect. It should be emphasized that is not necessary to have a Bose-Einstein condensate (BEC) of any kind to attain superfluidity. BEC is, however, a sufficient condition for the existence of supefluidity. BEC and superfluidity are have some differences. For example, the "'condensate"' of pairs of fermions occurs at the same time as the transition temperature is reached, where as in BEC it occurs somehow continuously as the temperature is decreased below the critical temperature[4].

For superfluid Lithium-6, there are several approaches to calculate the transition temperature. Here we follow closely Stoof[1]. To start consider a collection of atoms in bulk gas of Lithium-6 atoms with two hyperfine spin states. Lithium is a fermion since it has total number of fermions equal to 7. If the fermions have equal spin state they do not interact to first order due to the exclusion principle. Hence we consider two different spin states of ^{6}Li with opposite spin interaction. In this field theoretical approach one of the first question to ask is what is the order parameter which describes the phase transition. By writing the equations of motion for the fermion fields the form: $V_0 \langle \psi_5(x)\psi_6(x) \rangle$ naturally arises to some approximation. Intuitively, its value in the normal state is zero because it is proportional to the probability that by destroying a particle in the system on the state 5 and another in the state 6(consider just as labels for spin states) and return to the same state is zero. But in the superfluid phase the pair of fermions may condensate and this parameter is, in principle, allowed to take non-zero values. Allowing that possibility in the equations is how one obtains the BCS classical theory of superconductivity. This is type of object is called anomalous green function. In a phenomenological two-component theory of fluid of Landau the order parameter may be taken to be the superfluid portion of density in order to explain certain kind of phenomena. Its value above T_c is cero and below nonzero. The critical exponents are expected to be differ from real ones since this approach is based on mean field theory. To date it is not known what is the size of the critical region for the superfluid ^{6}Li near T_c, so that we do not know whether mean field ideas should work. It turns out that they do to linear order in the fluctiations at least in predicting the critical temperature as explained bellow.

We will briefly describe the field theoretical approach to calculate the critical temperature. This will make clear how mean field theory is incorporated by expanding in the fluctuations of the fields to first order and using the low energy short range approximation for the triplet potential between Lithium-6 atoms(Houbiers[1, 2]).

In second quantized form the general hamiltonia for a collection of Fermions with two spin components is given by

$$H = \sum_{\alpha=5,6} \int d\mathbf{x} \psi_\alpha^\dagger(\mathbf{x})(-\frac{\hbar^2\nabla^2}{2m} - \mu'_\alpha)\psi_\alpha(\mathbf{x}) \tag{1}$$

$$+ \frac{1}{2}\int d\mathbf{x}d\mathbf{x}' V_T(\mathbf{x}-\mathbf{x}')\psi_\alpha^\dagger(\mathbf{x})\psi_{-\alpha}^\dagger(\mathbf{x}')\psi_{-\alpha}(\mathbf{x}')\psi_\alpha(\mathbf{x}) \tag{2}$$

Where $\psi_\alpha(\mathbf{x})$ is the fermionic destruction operator. α refers to the two spin components present in the gas. μ_{alpha} is the chemical potential. The interparticle potential can be approximated by a delta potential of the form $V_0\delta(\mathbf{x}'-\mathbf{x})$, where the constant $V_0 = 4\pi\hbar^2/m$ is the pseudo potential common in mean field tratement of weakly interacting gases. The necessity of this will bemos clear bellow. a is the s-wave scattering length from two body scattering. In principle atoms with diferent spin could interact via p-wave or higher but at low temperatures and densities it turns out that only s-wave give non vanishing contribution. Next we expand about the mean density to first order in the fluctiations, i.e

$$\psi_\alpha^\dagger\psi_\alpha = \left\langle \psi_\alpha^\dagger\psi_\alpha \right\rangle + \delta\psi_\alpha^\dagger\psi_\alpha \tag{3}$$

and,

$$\psi_{-\alpha}\psi_\alpha = \langle \psi_{-\alpha}\psi_\alpha \rangle + \delta\psi_{-\alpha}\psi_\alpha \tag{4}$$

we are therefore left with a effective hamiltonian,

$$H = \int d\mathbf{x} \sum_{\alpha=5}^{6} \psi_\alpha^\dagger(\mathbf{x})(-\frac{\hbar^2\nabla^2}{2m} - \mu'_\alpha)\psi_\alpha(\mathbf{x}) + \Delta_0\psi_5^\dagger(\mathbf{x})\psi_6^\dagger(\mathbf{x}) + \Delta_0^*\psi_6(\mathbf{x})\psi_5(\mathbf{x}) - \frac{|\Delta_0|^2}{V_0}$$
$$-\frac{4\pi a\hbar^2}{m} n_5 n_6 \tag{5}$$

where $n_\alpha = \left\langle \psi_\alpha^\dagger \psi_\alpha \right\rangle$ is the equilibrium value of the density of atoms in state α. The quantity $\Delta_0 = V_0 \left\langle \psi_\alpha^\dagger \psi_\alpha \right\rangle$ is the equilibrium value of the **order parameter**. By Fourier transforming the fermionic fields

$$\psi_\alpha(\mathbf{x}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} a_{\mathbf{k},\alpha}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}}, \tag{6}$$

we obtain the hamiltonian which is not diagonal in the operators $a_{\mathbf{k},\alpha}^\dagger$ and $a_{\mathbf{k},\alpha}$. Next, we apply a Bogoliubov transformation to make it diagonal in terms of new operators which correspond to imaginary particles(quasiparticles) whose energy is the energy of excitations of the system. The equilibrium order parameter is found from $\Delta_0 = V_0 \left\langle \psi_\alpha^\dagger \psi_\alpha \right\rangle$ by fourier transforming the fields and substituting the **a**'s for the Bogoliubov operators. We obtain

$$\frac{1}{V_0} + \frac{1}{V} \sum_{\mathbf{k}} \frac{1 - 2f(\epsilon)}{2\sqrt{\xi_k^2 + |\Delta_0|^2}} = 0 \tag{7}$$

Where f is the fermi occupational function, ϵ is the energy of the quasiparticle which depends on $\mathbf{k}$. For the case considering here note that that the energy of the quasiparticle depends only on the magnitude of $\mathbf{k}$ and not on the direction this is a-wave pairing and is and assumptions that comes from expanding to fisrt ordern in the density fluctiuations. Where also assumed equal densities of the spin components. This expression contains an ultraviolet divergence due to neglect of the momentum dependence of the interaction. This is solved by replacing the interaction term V_0 with $T^{2B}(0,0,0)$. The critical temperature is found by the condition

$$\frac{m}{4\pi a \hbar^2} + P \int \frac{dk}{(2\pi)^3} \frac{2f(k)}{2[\epsilon(k) - \mu]} = 0 \tag{8}$$

which is free of divergences because the fermi distributions in the Cauchy principle-value integral. Therefore we can write

$$U_0 \int_0^\infty d\epsilon f(\epsilon) \frac{f(\epsilon)}{\epsilon - \mu} = 1 \tag{9}$$

where $N(\epsilon)$ is the density of states per unit volume for a single specie of the Fermi gas which is given by $m^{3/2}\epsilon^{1/2}/(\sqrt{2}\pi^2\hbar^3)$, assuming material in bulk. From this equation we see the mathematical necessity of an attractive interparticle potential. The factor $1/(\epsilon - \mu)$ is negative for energies in the rage 0 to μ approaching $-\infty$ as $\epsilon \to \mu^-$ therefore the integral in this rage is negative and for an non zero temperature for which this equation is satisfied

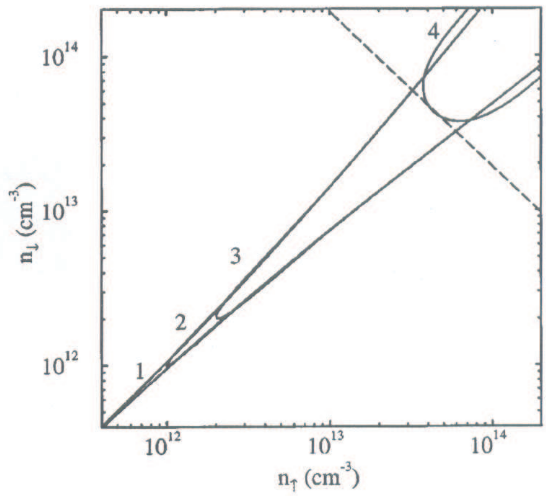

FIG. 1: Contours of the critical temperature as a function of the hyperfine densities n_5 and n_6 for $T = 0.01$ nK,$T = 11$ nK, $T = 37$ nK, and $T = 1725$ nK.

U_0 must be negative. For values of energy $\epsilon > \mu$ the integrand is suppressed for energies larger than $\sim kT$ above μ by the factor $[\exp((\epsilon - \mu)/kT) + 1]^{-1}$. By making some plausible approximations of the integrals one can obtain an analytical expression for the transition temperature. The result is

$$T_c \approx \frac{5\epsilon_F}{3k_B} \exp \frac{-\pi}{2k_F|a|} - 1 \tag{10}$$

Where $\epsilon_F = \hbar^2 k_F^2/2m$. The parameter $k_F a$ is a measure of the strength of the interparticle interaction in fermi systems, k_F is the fermi momentum. This result has been obtained by number of groups working in different areas of physics[1, 5, 6, 8]

III. EXPERIMENT

Since the realization of a Fermi degenerate gases by De Marco[9] there was a fierce competition to find superfluidity in dilute gases of fermionic atoms. In the following I will describe the experiment that showed conclusively evidence for superfluid Lithium-6[**?**]. They basically report observation of vortices lines in a trapped gas of ^{6}Li. They created a mixture of 50-50% of the lowest two hyperfine states of ^{6}Li. Next, they cooled the gas to temperatures of $T/T_c = 0.07$ in an optical trap with a cilintrical form. The trap potential

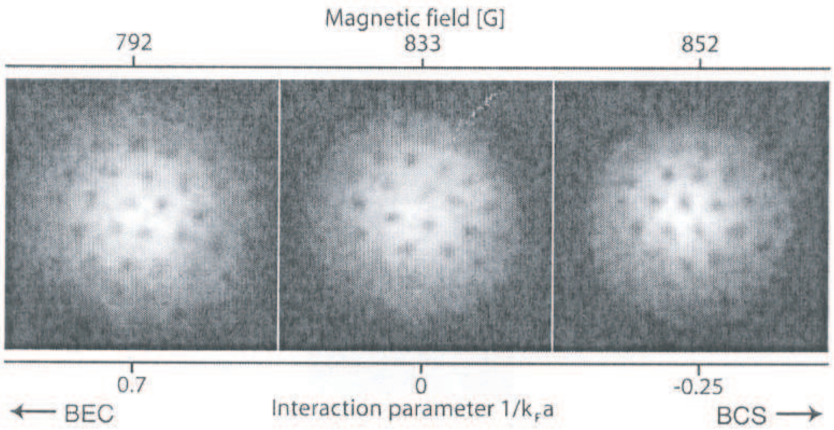

FIG. 2: Vortex littices in cold gas of ^{6}Li

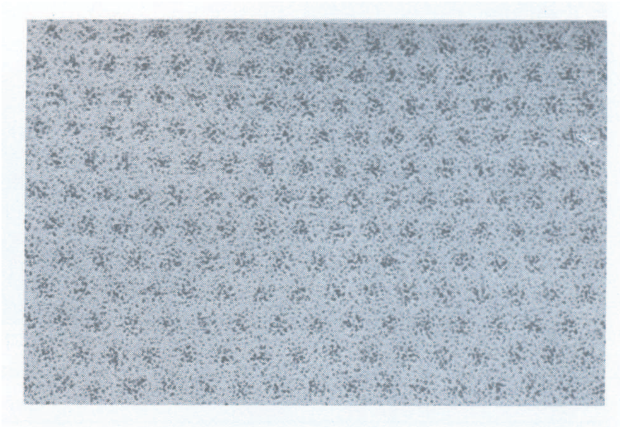

FIG. 3: Triangular array of vortex lines emerging through the surface of a $Pb_{.98}In_{.02}$ superconducting foil in a field of 80 gauss normal to the surface.

was rotated about its axis of symmetry using a combination of detuned cross lasers. Next they let the gas expand freely and the they took a picture of it from the z-axis of rotation. The contrast of the picture is sentitive to variations of density and so the picture reveled the ditribution of mass, see Fig.2. The experiement was made in the present of a fixed magnetic field. The B field was necessary because they wanted to take advantage of the Fechbach resonace by which the interaction energy on the atoms can be made to have any value, positive of negative.

The pictures revealed that particularly for attractive interactions(BCS side) i.e. negative scattering lengths arrays of triangular vortices were formed Fig. 2. This vortices are a reflection is a macroscopic wave function developed by the gas. Since BEC of molecules are instable in that regime. This macroscopic manifestation of quantum mechanics must be a supefluid state.

In general, a BCS-like phase transition is expected in any dilute gases of fermionic atoms when the interatomic interaction are attractive. In standard BCS theory an attractive interaction between the particles is an essential ingredient. The special characteristic of Lithium-6 is its triplet large attractive interatomic potential, the largest among the alkali metals. This interactions are enhanced via Feshbach resonce by choosing the right spin state of the hyperfine Zeeman splitting and tuning the B-field. The transition temperature turned out to be proportional to $\exp(-\pi/2k_F|a|)$, but the parameter $1/k_F|a|$ is, in general, large for dilute systems. It was predicted(although for deuterium(Leggett)) that the transition temperature was so low that it was unattainable experimentally.

As mentioned above, there has been no measurement of the critical exponents for superfluid ^{6}Li. Most expact them to be the same as for the BCS-like. There are several studies of critical exponents for a bose systems near the critical temperature. For example Holzmannand Baym[7] found by using field theoretical methods that Bose-Einstein condensation is a second order phase transition with critical coefficient $2\beta = 0.66$. Which is close to β for Helium-4.

IV. CONCLUSION

The discovery of the superfluid phase transition marked an important step towards understanding the nature of macroscopic manifestation of quantum mechanics. It helped probe the limits and aplicability of field theoretical methods and mean field ideas. It is possible that this will help understand high-temperature superconductors since compared with the Fermi temperature gaseous superfluid Lithium-6 is a high T_c superfluid as opposed to the known neutral counterparts He-3 and He-4 or charged as superconductos. Much is to be expected out of this field of research in the coming years.

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